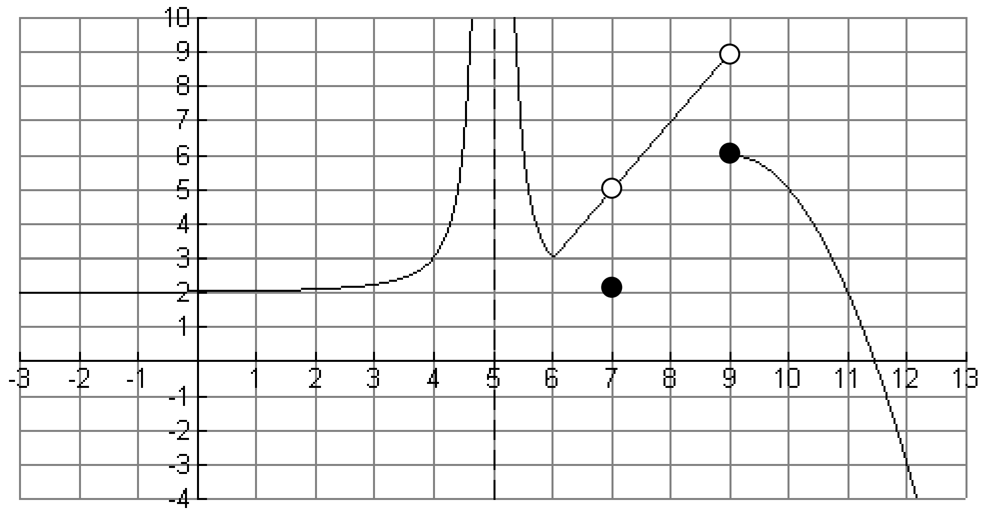
MATH 150 **MATH 150 Final Exam Review Problems**
Rockville Campus

1. The graph of the function $f(x)$ is shown. Use this graph to answer the questions in the chart below.



	Is $f(x)$ defined for this value of x? If so estimate $f(a)$.	Find $\lim_{x\to a} f(x)$ if it exists.	Is $f(x)$ continuous at this value of x?	Is $f(x)$ differentiable at this value of x?
a = 4				
a = 5				
a = 6				
a = 7				
a = 9				

2. Use the <u>definition of the derivative</u> to find $f'(x)$ given that $f(x) = 3x^2 - x + 5$.

3. The temperature of a person during an illness is given by $F(t) = -0.1t^2 + 1.2t + 98.6$ where F is the temperature in degrees Fahrenheit at time t in days.

(a) Find the rate of change of the temperature with respect to time.

(b) Find $F(1.5)$ and write a sentence in everyday language explaining the meaning of your answer in the context of this situation. (Interpret $F(1.5)$). Use appropriate units.

(c) Find $F'(1.5)$ and write a sentence in everyday language explaining the meaning of your answer in the context of this situation. (Interpret $F'(1.5)$). Use appropriate units.

(d) What was the highest temperature the person had over the first 7 days?

4. Differentiate the given functions:

(a) $f(x)=5x^3-7x^2+3x-9$	(g) $y=4e^{\sqrt{x}}$
(b) $g(x)=\dfrac{4}{x^3}-\dfrac{x^3}{4}+\sqrt[4]{x^3}$	(h) $f(x)=x^5e^{2x}$
(c) $f(x)=\sqrt[3]{x^2-1}$	(i) $f(x)=\ln(5x+3)$
(d) $f(x)=\dfrac{1}{\left(x^2+x+1\right)^6}$	(j) $f(x)=\ln(x^3-7)^4$
(e) $f(x)=x^5(3x-1)^7$	(k) $f(x)=[\ln(x^3-7)]^4$
(f) $f(x)=\dfrac{x^2+3}{3x^2-5}$	

5. Find the equation of the line tangent to the curve $y=\sqrt{x^2+16}$ when $x=3$.

6. Find $f''(x)$ if $f(x)=\dfrac{5}{3x+4}$.

7. Given $f(x)=x^4-12x^3$

(a) Find $f'(x)$ and $f''(x)$.

(b) Determine the interval(s) on which $f(x)$ is increasing or decreasing.

(c) Find any local/relative maximum or minimum points.

(d) Determine where $f(x)$ is concave up or concave down.

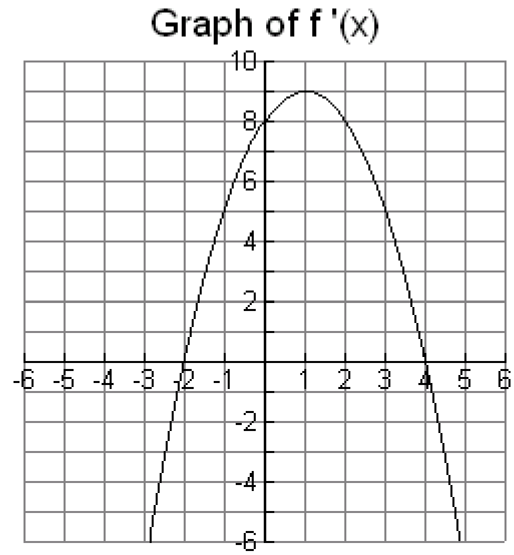
(e) Find any points of inflection.

(f) Sketch the graph of $f(x)$.

8. For a differentiable function $f(x)$, $f(3)=2,\ f'(3)=0,\ and\ f''(3)=5$. Which of the following statements is true?

(a) f has a local/relative maximum at (3, 2).
(b) f has a local/relative maximum at (3, 0).
(c) f has a local/relative minimum at (3, 2).
(d) f has a local/relative minimum at (3, 0).
(e) f has a point of inflection at (3, 2).
(f) f has a point of inflection at (3, 0).

9. The graph of $f'(x)$, the ***derivative*** of a function $f(x)$, is shown. Note that the graph of $f(x)$ is not given. Based on the graph of $f'(x)$, answer the following questions about $f(x)$.



(a) On what interval(s) is $f(x)$ increasing?

(b) On what interval(s) is $f(x)$ decreasing?

(c) At what value(s) of x does $f(x)$ have a local/relative maximum?

(d) At what value(s) of x does $f(x)$ have a local/relative minimum?

10. The revenue function for a certain product is $R(x) = 300\left(15x - x^{3/2}\right)$.

(a) Find the marginal revenue when 64 items are produced and sold.

(b) For what value of x is the revenue a maximum?

11. A manufacturer of cameras finds that the price at which it can sell x cameras per week is $P(x) = 500 - x$ dollars. The total cost of producing x cameras per week is $C(x) = 150 + 4x + x^2$ dollars.

(a) Find the revenue function $R(x)$.

(b) Find the profit $P(x)$.

(c) Find the production level which maximizes the profit.

12. A tool rental company determines that it will rent 500 jackhammers per day at a daily rental fee of \$30 per jackhammer. For each \$1 increase in rental price, 10 fewer jackhammers will be rented. What rental price maximizes revenue? Write your answer in a complete sentence.

13. A distributor of sporting equipment expects to sell 10,000 cases of tennis balls during the coming year at a steady rate. Yearly carrying costs (to be computed on the average number of cases in stock during the year), are \$10 per case and the cost of placing an order with the manufacturers is \$80. How many times per year should the distributor order cases of tennis balls, and in what lot size, in order to minimize the inventory cost? Write your answer in a complete sentence.

14. A rectangular garden of area 75 square feet is to be surrounded on three sides by a brick wall costing \$10 per foot and on one side by a fence costing \$5 per foot. Find the dimensions of the garden such that the cost of the materials is minimized. Write your answer in a complete sentence.

15. A closed box with square base is to be built to house an ant colony. The bottom of the box and all four sides will be made of material costing \$1 per square foot, and the top is to be constructed of glass costing \$5 per square foot. What are the dimensions of the box of greatest volume that can be constructed for \$72?

16. At what point or points does the function $f(x) = \ln(3x+1)$ have slope 4?

17. Let $y = e^{2x}\left(x^2 - 6\right)$.

(a) Find $\dfrac{dy}{dx}$ and write your answer in simplified, factored form.
(b) Find the critical numbers of the function.
(c) On what interval or intervals is the function increasing? On what interval or intervals is it decreasing?
(d) Does the function have a local/relative minimum? If so, what is it?
(e) Does the function have a local/relative maximum? If so, what is it?

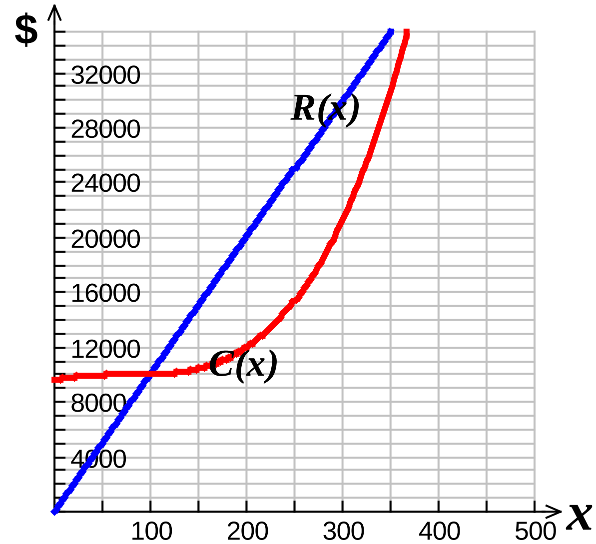
18. Find the absolute maximum and the absolute minimum of the function $f(x) = x + \dfrac{4}{x}$ on the interval [0.5, 4].

19. If the equation $y = 100e^{.03t}$ describes the population in millions, of a small country, where t is time in years, find
(a) The population when $t = 0$
(b) The predicted population when $t = 6$ years.
(c) The number of years it will take for the population to grow to 150 million.
(d) The number of years it will take for the population to grow to twice its original number, that is, the "doubling time".

20. The radioactive isotope iodine-131 has a half-life of 8 days.
(a) Find its relative growth rate to 4 decimal places.
(b) Find the amount remaining after 10 days if initially there are 5 mg.

21. A business wants to have \$200,000 in five years to pay for some new machinery. Interest is 3.2% compounded continuously. How much money should be invested now? Round to the nearest whole number.

22. A company has a cost function $C(x)$ that gives the cost in dollars of producing q units of a good. The company's revenue function $R(x)$ gives the revenue generated by selling x units of the same good. These two functions are shown on the graph. Use the graph to estimate the level of production that will maximize profit for the company.



23. Find the indefinite integral:

(a) $\int \left(x^3 - 6x^2 + 2x - 1\right) dx$ (b) $\int \frac{5}{x} dx$

(c) $\int (4 - 5e^{-5t} + \frac{e^{2t}}{3})\, dt$

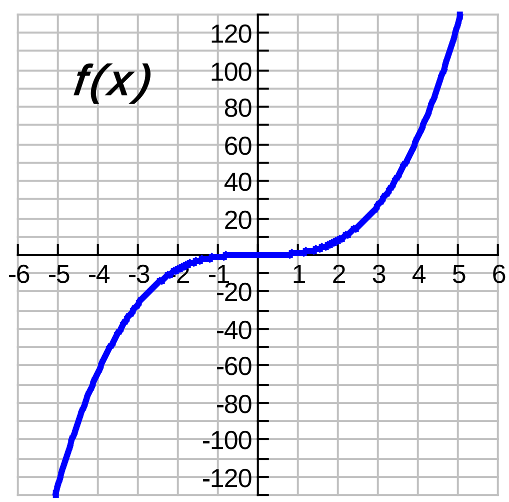
24. Find the function that has derivative $f'(x) = 3x^2 + \frac{1}{x} - 4$ and whose graph contains the point (1, 2).

25. Evaluate the given definite integrals. Give exact answers.

(a) $\int_2^3 (6 + \frac{1}{x^2}) dx$ (b) $\int_0^1 e^{4x}\, dx$

(c) $\int_1^4 \sqrt{x}\, dx$

26. Estimate the value of the integral $\int_{-5}^{5} f(x) dx$ for the function $f(x)$ in the graph.



27. Given the integral $\int_{-1}^{2} (x^2 + 4)\, dx$

(a) Calculate the area represented by the integral
(b) Sketch the area represented by the integral
(c) Find the average value of the function over the interval $-1 \le x \le 2$

28. On a hot summer afternoon, a city's electricity consumption is $-3t^2 + 18t + 10$ units per hour, where t is the number of hours after noon ($0 \le t \le 6$). Find the total consumption of electricity between the hours of 1 and 5 p.m.

29. The population of a country was 4.5 million in 1987 ($t = 0$) and 6.4 million in 1994. Assume that the population is growing at a rate proportional to its size.

(a) Find the exponential growth rate *k*, to four decimal places, and write the exponential growth function for which *P(t)* is the population in millions t years after 1987.

(b) Find $P(13)$ and write a sentence in everyday language explaining what this means in this situation. Use appropriate units.

(c) Find $P'(13)$ and write a sentence in everyday language explaining what this means in this situation. Use appropriate units.

(d) What was the ***average*** population between 1987 and 2007?

30. Find the area bounded by the curves $y = x + 1$ and $y = x^2 - 3x - 4$.

31. Let $p = D(x)$ be the price per unit (in dollars) that a company can charge if sells *x* units. Let $p = S(x)$ be the price per unit (in dollars) at which producers are willing and able to sell *x* units of a good. Given the demand and supply functions $D(x) = x^2 - 12x + 36$ and $S(x) = x^2 + 6x$, find

(a) The equilibrium point
(b) The consumer surplus at equilibrium
(c) The producer surplus at equilibrium

32. Find the future value of a continuous money flow if $4000 per year is invested at a constant rate compounded continuously for 5%, for 3 years.

33. Given $f(x, y) = 4xy^2 - 3x^3y + y^5$, find:

(a) $\frac{\partial f}{\partial x}(x, y)$ (b) $\frac{\partial f}{\partial y}(x, y)$

ANSWERS

1.

	Is $f(x)$ defined for this value of x? If so estimate $f(a)$.	Find $\lim_{x \to a} f(x)$ if it exists.	Is $f(x)$ continuous at this value of x?	Is $f(x)$ differentiable at this value of x?
a = 4	3	3	Yes	Yes
a = 5	Not defined	Does not exist	No	No
a = 6	3	3	Yes	No
a = 7	2	5	No	No
a = 9	6	Does not exist	No	No

2. $f'(x) = 6x - 1$

3. (a) $F'(t) = -0.2t + 1.2$

(b) $F(1.5) \approx 100.2^\circ$ After one and a half days, the person's temperature is about 100.2° F.

(c) $F'(1.5) = 0.9^\circ$ *per day* After one and a half days, the person's temperature is rising at a rate of about 0.9° *per day*.

(d) F(6)=102.2°F. After 6 days, the person's temperature reached a maximum of 102.2°F.

4.

(a) $f'(x) = 15x^2 - 14x + 3$	(g) $\dfrac{dy}{dx} = \dfrac{2e^{\sqrt{x}}}{\sqrt{x}}$
(b) $g'(x) = \dfrac{-12}{x^4} - \dfrac{3x^2}{4} + \dfrac{3}{4\sqrt[4]{x}}$	(h) $f'(x) = 2x^5e^{2x} + 5x^4e^{2x} = x^4e^{2x}(2x+5)$
(c) $f'(x) = \dfrac{2x}{3(x^2-1)^{2/3}}$	(i) $f'(x) = \dfrac{5}{5x+3}$
(d) $f'(x) = \dfrac{-6(2x+1)}{(x^2+x+1)^7}$	(j) $f'(x) = \dfrac{12x^2}{x^3-7}$
(e) $f'(x) = 21x^5(3x-1)^6 + 5x^4(3x-1)^7 = x^4(3x-1)^6(36x-5)$	(k) $f'(x) = [\ln(x^3-7)]^3 \dfrac{12x^2}{x^3-7}$
(f) $f'(x) = -\dfrac{28x}{(3x^2-5)^2}$	

5. $y = \dfrac{3}{5}x + \dfrac{16}{5}$

6. $f''(x) = \dfrac{90}{(3x+4)^3}$

7. Given $f\left(x\right)=x^4-12x^3$

(a) $f'\left(x\right)=4x^3-36x^2$, $f''(x)=12x^2-72x$

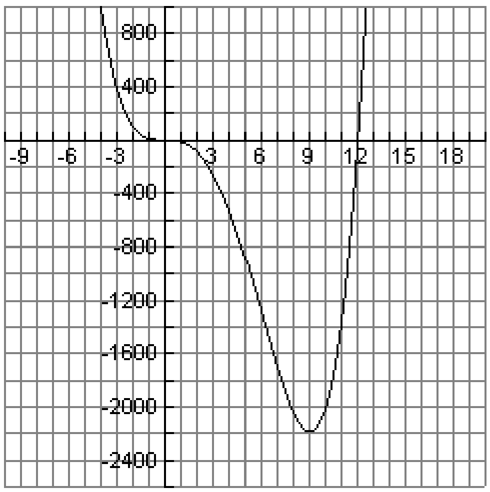
(b) Increasing over $(9,\infty)$, Decreasing over $(-\infty,9)$

(c) No relative maximum; relative minimum at (9, -2187)

(d) Concave up over $(-\infty,0)$ *and* $(6,\infty)$; concave down over (0, 6).

(e) Points of inflection at (0, 0) and (6, -1296).

(f)



8. (c) is true: f has a relative minimum at (3, 2).

9. (a) (-2, 4) (b) $(-\infty,-2)$ *and* $(4,\infty)$ (c) x = 4 (d) x = -2

10. (a) $R'\left(64\right)=\$900$ per item (b) $x=100$ items

11. (a) $R\left(x\right)=500x-x^2$ (b) $P\left(x\right)=-2x^2+496x-150$ (c) $x=124$

12. Revenue will be maximized if the rental price is \$40. .

13. The distributor should order 25 times per year with a lot size of 400 cases per order.

14. The side with fencing and the side parallel to it should be 10 feet, and the other two sides should each be 7.5 feet.

15. Base: 2 ft by 2 ft, height: 6 feet

16. $\left(-\frac{1}{12},\ln\left(\frac{3}{4}\right)\right)$

17. (a) $\frac{dy}{dx}=2e^{2x}(x^2+x-6)$ (b) x = -3, x = 2

(c) Increasing over $(-\infty,-3)$ *and* $(2,\infty)$; decreasing over (-3, 2).

(d) Local/relative minimum is $f(2)=-2e^4$.

(e) Local/relative maximum is $f(-3)=3e^{-6}$.

18. Absolute maximum of 8.5 at $x = 0.5$; absolute minimum of 4 at $x = 2$

19. (a) 100 million (b) about 120 million (c) about 13.5 years (d) about 23 years

20. (a) -.0866 (b) about 2.1 mg

21. $170,429

22. $x \approx 250$ units

23. (a) $\frac{1}{4}x^4 - 2x^3 + x^2 - x + C$ (b) $5\ln|x| + C$

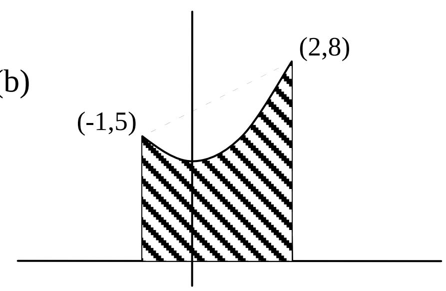
(c) $4t + e^{-5t} + \frac{e^{2t}}{6} + C$

24. $f(x) = x^3 + \ln|x| - 4x + 5$

25. (a) $6\frac{1}{6} = \frac{37}{6}$ (b) $\frac{1}{4}(e^4 - 1)$ (c) $\frac{14}{3}$

26. 0

27. (a) 15 (b)



(c) 5

28. 132 units

29. (a) $k = .0503$; $P(t) = 4.5e^{.0503t}$

(b) $P(13) \approx 8.65$; In 2000, the population of the country was approximately 8.65 million people.

(c) $P'(13) \approx .44$; In 2000, the population of the country was growing at a rate of approximately .44 million (440,000) people per year.

(d) The average population between 1987 and 2007 was approximately 7.76 million people.

30. 36 sq. units

31. (a) (2, $16) (b) $18.67 (c) $17.33

32. $12,946.74

33. (a) $4y^2 - 9x^2y$ (b) $8xy - 3x^3 + 5y^4$

Revised Nov 2015