

Simplifying Algebraic Expressions with Exponents:

How can we simplify algebraic expressions with exponents? That is, something like

$$\frac{8a^6b^{-4}c^8}{32a^{-4}b^5c^9}$$

One approach* would be to break the problem into the following steps:

1. Break the expression into separate fractions, one containing only numbers, and one for each variable:

$$\begin{array}{c} \frac{8a^6b^{-4}c^8}{32a^{-4}b^5c^9} \\ \swarrow \quad \downarrow \quad \downarrow \quad \searrow \\ \frac{8}{32} \quad \frac{a^6}{a^{-4}} \quad \frac{b^{-4}}{b^5} \quad \frac{c^8}{c^9} \end{array}$$

2. Reduce the fraction containing only numbers: $\frac{8}{32} = \frac{1}{4}$

For each fraction containing a variable:

3. Move all terms in the numerator (denominator) with negative exponents to the denominator (numerator), and make the exponent positive. Leave a "1" if moving all the terms:

$$\begin{array}{ccc} \frac{a^6a^4}{1} & \frac{b^{-4}}{b^5} & \frac{c^8}{c^9} \text{ stays the same} \\ \frac{a^6}{a^{-4}} & \frac{1}{b^4b^5} & \end{array}$$

4. Multiply all terms in the numerator, and all terms in the denominator:

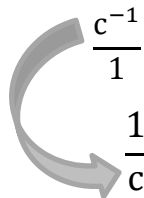
$$\frac{a^6a^4}{1} = \frac{a^{6+4}}{1} = \frac{a^{10}}{1} \qquad \frac{1}{b^4b^5} = \frac{1}{b^{4+5}} = \frac{1}{b^9}$$

*If you already have an approach that works for you, that's great! Remember that there's almost always more than one right way to approach a math problem.

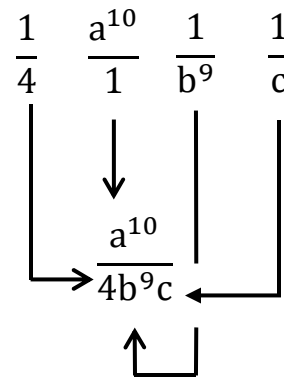
4. Divide the numerator by the denominator:

$$\frac{c^8}{c^9} = \frac{c^{8-9}}{1} = \frac{c^{-1}}{1}$$

5. If the exponent is negative, then move the term and make it positive:



6. Combine all the simplified fractions into one:



And the result is:
$$\frac{8a^6b^{-4}c^8}{32a^{-4}b^5c^9} = \frac{a^{10}}{4b^9c}$$

Exercises:

Simplify the following expressions:

1. $3a^8b^{-6}$
2. $5a^{-7}b^4$
3. $\frac{r^{-4}}{3s^5}$
4. $\frac{y^{-5}}{x^{-3}}$
5. $\frac{y^4z^{-3}}{x^{-2}}$
6. $\frac{24a^5b^6}{-8a^4b^3}$
7. $\frac{-6x^{-2}y^4z^8}{24x^{-5}y^6z^{-3}}$
8. $\frac{-4x^4y^{-2}}{5x^{-1}y^4}$
9. $\frac{(3x^3y^4)^3}{6xy^3}$
10. $\left(\frac{4a^3b^{-9}}{6a^{-2}b^5}\right)^0$

- Answers: 1. $\frac{3a^8}{b^6}$ 2. $\frac{b^6}{3a^8}$ 3. $\frac{a^7}{5b^4}$ 4. $\frac{r^4s^5}{x^3}$ 5. $\frac{x^2y^4}{z^3}$ 6. $-3ab^3$ 7. $\frac{-x^3z^{11}}{4y^2}$ 8. $\frac{5y^6}{-4x^5}$ 9. $\frac{2}{9x^8y^9}$ 10. 1