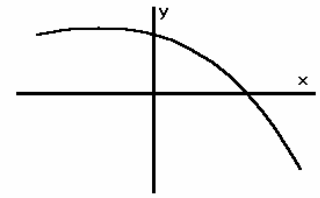
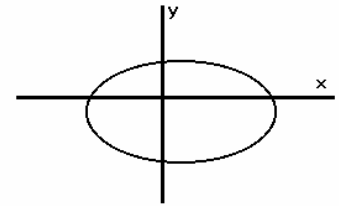
Math 100 Final Exam Review Problems
Rockville Campus
Revised : Spring 2004

1. For each graph below, explain why it is or is not a function.

a.



b.



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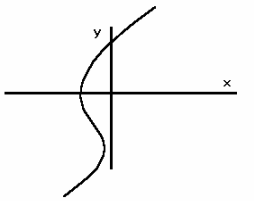
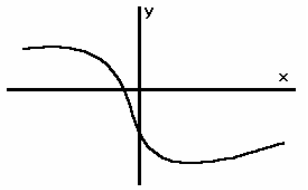


d.



2. Given $2x+3y=18$
   a. Find the *x*-intercept.
   b. Find the *y*-intercept.
   c. Find the slope.
   d. Sketch the graph.
   e. Find the equation of a line parallel to this line passing through (-3,-2)

3. In 1970 the enrollment at a certain college was 2000 students; in 1990 there were 2800 students. Let $f(t)$ represent the enrollment *t* years after 1970. Assume $f(t)$ is a linear function.
   a. Write an equation for $f(t)$ expressing enrollment in terms of *t*.
   b. Predict the enrollment for the year 2000.
   c. Determine when the enrollment will reach 4000.

4. Given

$f(x)=3x^2-6x+1$ $\quad$ $g(x)=-9x-2x^2$

$h(x)=-x^3+8x^2-17x+6$ $\quad$ $j(x)=x^3-8$

For each function above, use any suitable method to answer the following questions:

   a. Find all intercepts. When it is required to estimate them, estimate to the nearest thousandth.
   b. Evaluate $f(2)$ and $f(-2)$.
   c. Sketch a graph that shows the significant features of the function.
   d. If the function is a parabola, find the exact value of the *x*-coordinate of the vertex.

5. Simplify $\sqrt{48}$

6. Let $f(x) = 3 - 4x$

a. Find and simplify $f(t+2)$

b. Find and simplify $f(-2)$

c. Find x when $f(x) = -2$

7. Simplify each expression below where possible and write your answer without using negative or fractional exponents. Assume that $x > 0$.

a. $8x^{1/3}$ b. $(-8x)^{1/3}$ c. $(8x)^{-1/3}$ d. $(3x^{3/4})(16x)^{1/4}$ e. $\dfrac{x^{1/2}}{x^{5/2}}$

8. Simplify $\dfrac{8-\sqrt{-36}}{4}$ and express the answer in $a+bi$ form.

9. Find all real and non-real solutions to the following:

a. $(x+3)^2 - 81 = 0$ b. $3x^2 + 5x = 2$ c. $(x+1)(x-2) = 18$

d. $2x^2 - 4x + 5 = 0$ e. $x(x^2 + 3x - 1) = 3$

10. Your calculator gives an answer of $x = 1.1578074156$, $y = 2\text{E-}13$ when using **ROOT** or **ZERO** in the graphing mode. Give the coordinates of the x-intercept your calculator has given you to the nearest thousandth.

11. A company started business in 1995. Assume that a company has a profit per year shown in the table below. (Negative profits are losses.)

	Profits in millions of dollars
1995	-2.4
1996	0
1997	2.0
1998	3.6
1999	4.8
2000	5.7
2001	6.1

Let $f(t)$ represent the amount of profit in millions of dollars, where t is the number of years since 1995. A possible formula for $f(t)$ is $f(t) = -0.19t^2 + 2.56t - 2.37$

a. Use f to predict the amount of profit in 2003.

b. Find t when $f(t) = 4.23$. What do your results mean in terms of the company's profits.

c. For what years is there likely model breakdown? (Hint: consider the past and future.)

d. When does $f(t) = 0$? What does this mean in terms of the situation?

e. When will the company have maximum profits?

12. Solve for x

a. $\dfrac{x}{x+1}+\dfrac{2x-2}{x}=\dfrac{-1}{x+1}$ b. $2x+1=\sqrt{7-x}$

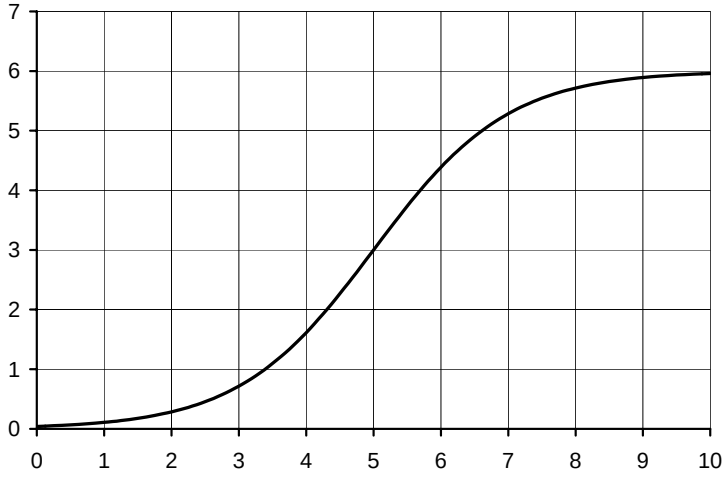
13. The equation of the height of an object thrown upwards starting at 6 feet above the ground is $h(t)=6+95t-16t^2$ where t is time in seconds and h is height in feet.

a. When will the object hit the ground?
b. When will it reach its maximum height?
c. What is its maximum height?

14. Let $f(x)=-\dfrac{\sqrt{x+3}}{1-x^2}$ Evaluate each of the following. If you cannot evaluate, explain why.

$f(-4),\ f(-3),\ f(0),\ f(1),\ f(2),\ f(6)$

15. What is the domain of f if $f(x)=\dfrac{2x-3}{4x+5}$?



16. Bacteria are grown in a dish. The chart above is a graph of the number of bacteria in the dish. Food is added as needed. The vertical axis is the number of bacteria in thousands. The horizontal axis is the time in hours.

a. Approximately how many bacteria are in the dish after five hours?
b. After 10 hours the number of bacteria appears to be leveling off at what number? Explain why this might be the case.

17. Evaluate

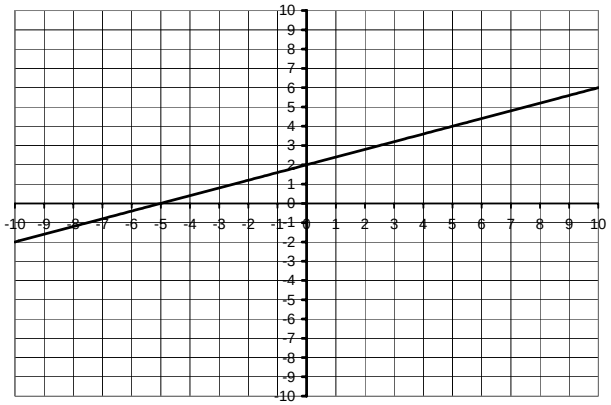
a. $\log(1)$ b. $\ln\left(e^5\right)$ c. $\ln(5)$ d. $\log_2 16$

18. Which part of problem 17 is difficult to do without a calculator? Explain.

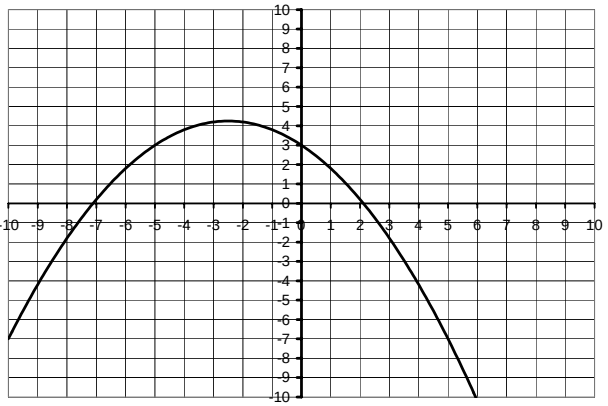
19. Label all intercepts and sketch the graph of

a. $f(x)=2^x$
b. $f(x)=\ln(x)$

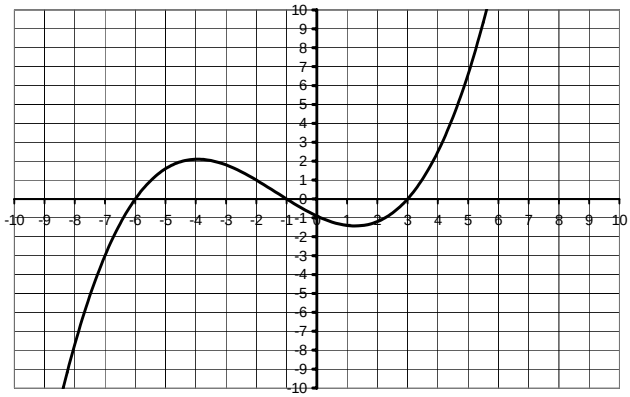
20. Function I



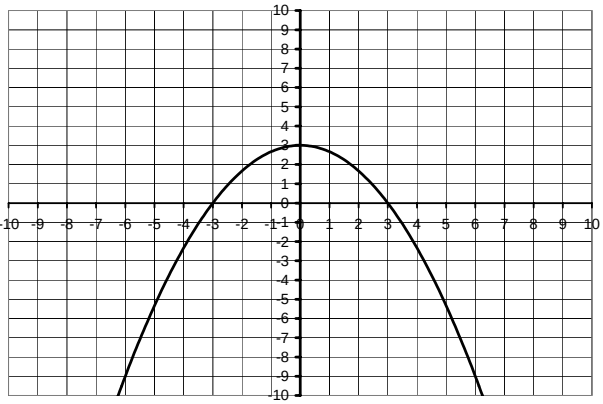
Function II



Function III



Function IV



a. List all the functions for which $f(3) = 0$.

b. List all the functions for which $f(0) = 3$.

c. What are the zeros of function III?

d. Approximate the equation of the line in function I

21. $100 is deposited in a savings account that pays 4% interest compounded annually.

a. How much is in the account after 5 years?

b. Use logarithms to determine how many years it will take for the account to grow to $1000. Confirm your results by graphing $y = A(t)$ in an appropriate window and using the TRACE feature on your calculator.

22. Sketch the graph of a line with the following characteristics:

a. $m > 0$ and $b < 0$ b. $m < 0$ and $b < 0$ c. $m = 0$ and $b > 0$

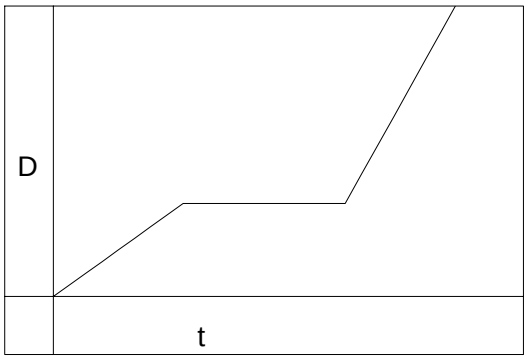
23. A retired man looks at the his net worth since he retired. The data is in the table below.

Years since retirement	Net worth in thousands
0	400
1	366
2	339
3	307
4	275
5	244

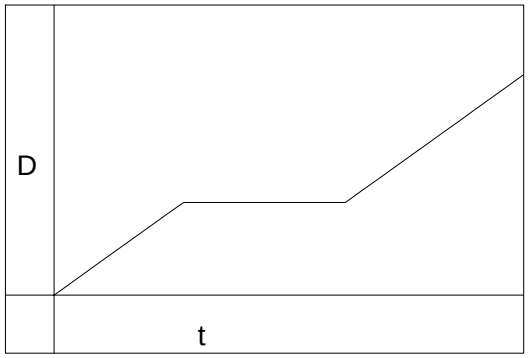
a. Create a scattergram of the data.
b. Sketch a line that approximates the data.
c. Find the equation of that line.
d. Let $y = f(x)$ be the function that models the data. Find $f(10)$
e. What is the x-intercept? What does it mean in terms of the model?
f. What is the y-intercept? What does it mean in terms of the model?

24. Match each graph with each scenario. Let D represent the distance traveled in t minutes.
___ i. A person starts running quickly and gradually slows down.
___ ii. A person starts walking at a steady pace, stops to talk to someone and continues walking at the same pace.
___ iii. A person starts walking at a steady pace, stops to talk to someone and then walks faster.
___ iv. A person walks at a steady pace with no stops.

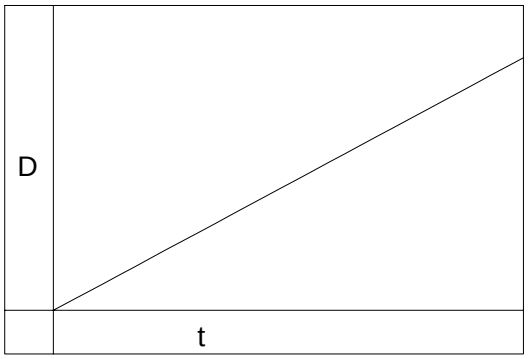
a.



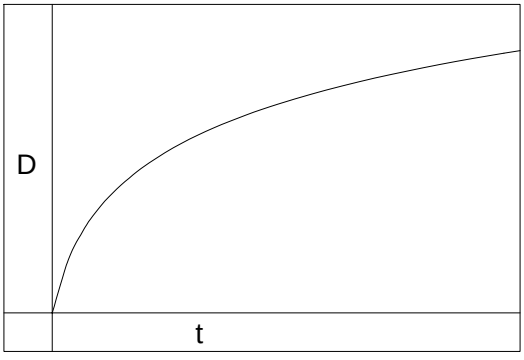
b.



c.



d.



25. Solve the following systems of equations.

a. $\begin{cases} 3s - 4t = 8 \\ 2s + 3t = -6 \end{cases}$ b. $\begin{cases} x^2 + y^2 = 26 \\ x^2 - 2y = 23 \end{cases}$

26. Solve $3 \cdot 5^x = 10$ algebraically.

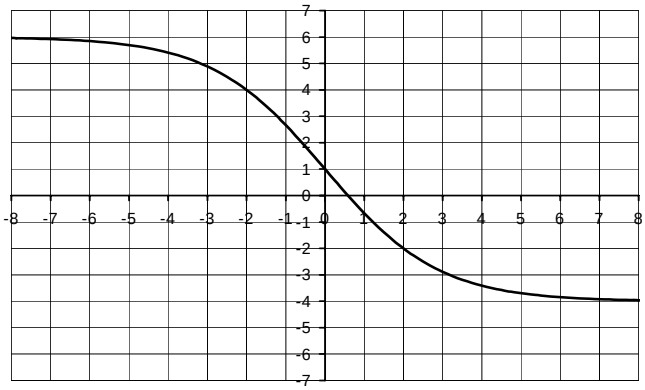
27. Find the domain of each function below. Which of these functions is a polynomial?

a. $f(x) = 3x^2 - 5x + 9$ b. $g(x) = \dfrac{1}{x^3 - 4x}$ c. $k(x) = \sqrt{3 - x}$

28. a. Convert $3^4 = 81$ to logarithmic notation.
b. Convert $\log_{25} 5 = \frac{1}{2}$ to exponential notation.

29. a. If $f(x) = \dfrac{x}{7} - 3$ find $f^{-1}(x)$

b. Below is the graph of $h(x)$. Approximate $h^{-1}(3)$



30. Find the equation for the exponential function of the form $y = ab^x$ that passes through the points $(0, 2)$ and $(3, 250)$.

31. Given $f(x) = ax^2 + c$.

a. How does the sign of a affect the graph of f?
b. How does the sign of c affect the graph of f?

32. Four tables are given below -- three are functions, and one is not. One of the functions is linear, and one is exponential.
   a. Which one is the linear function? What is the slope of the line?
   b. Which one is the exponential function? Explain how you recognize the exponential function.
   c. Which one is not a function? Explain how you know this relation isn't a function.

Table A	
x	*y*
-1	4
0	0
1	0
2	4

Table B	
x	*y*
-1	-2
1	-1
3	0
5	1

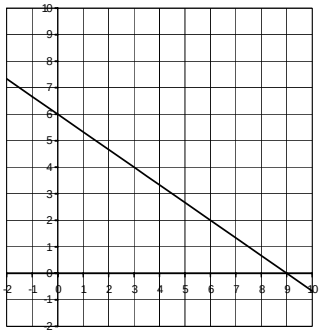
Table C	
x	*y*
0	-5
1	-3
1	3
0	5

Table D	
x	*y*
0	2
1	10
2	50
3	250

Answers. Note: answers marked by an asterisk (*) are difficult to find without a *graphing* calculator.

1. a and d are functions because it is impossible to draw a vertical line that intersects the graph more than once. b and c are not functions because in each case, it *is* possible to draw a vertical line that intersects both these graphs twice.

2. a. (9,0) b. (0,6) c. $-\frac{2}{3}$ d. See the graph to the right.
e. $y = -\frac{2}{3}x - 4$



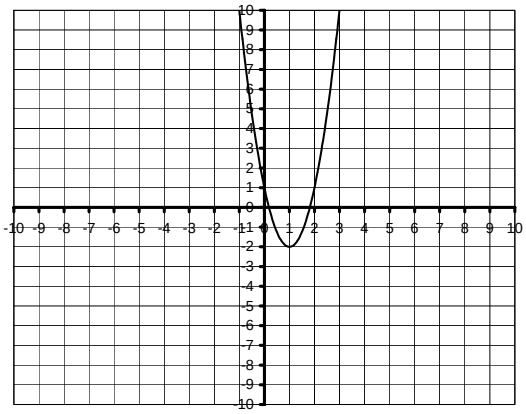
3. a. $f(t) = 2000 + 40t$ b. 3200 c. In the year 2020

4.

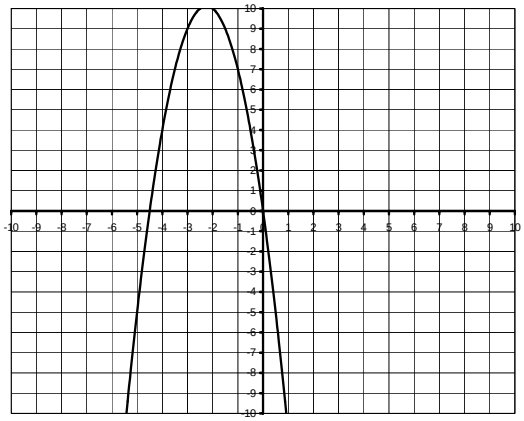
	f	*g*	*h*	*j*
*a	(0,1), (.184,0), (1.816,0)	$(0,0),(-4.5,0)$	$(0,6),(.438,0),$ $(3,0),(4.562,0)$	$(0,-8),(2,0)$
b	$f(2)=1,\ f(-2)=25$	$g(2)=-26$ $g(-2)=10$	$h(2)=-4,$ $h(-2)=80$	$j(2)=0$ $j(-2)=-16$
d	1	-2.25	NA	NA

*Graphs:

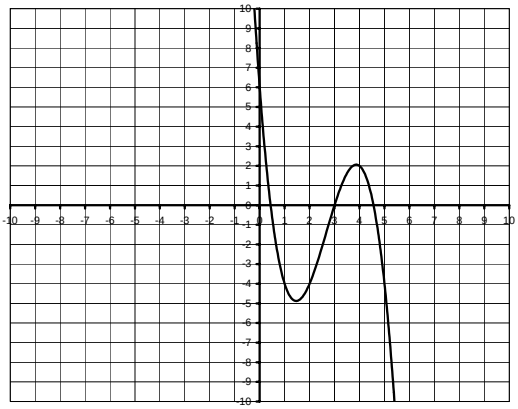
$f(x) = 3x^2 - 6x + 1$



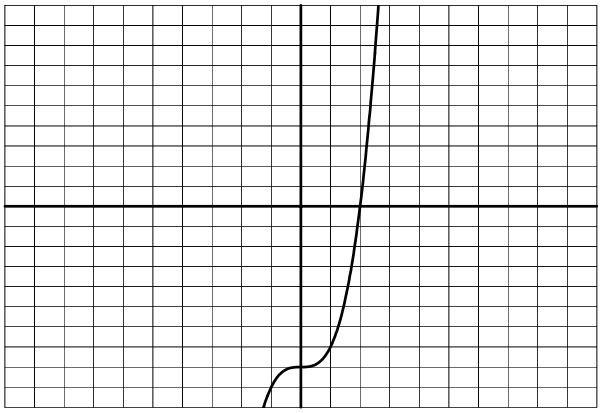
$g(x) = -9x - 2x^2$



$h(x) = -x^3 + 8x^2 - 17x + 6$



$j(x) = x^3 - 8$



5. $4\sqrt{3}$

6. a. $-4t - 5$ b. 11 c. 5/4

7. a. $8\sqrt[3]{x}$ b. $-2\sqrt[3]{x}$ c. $\dfrac{1}{2\sqrt[3]{x}}$ d. $6x$ e. $\dfrac{1}{x^2}$

8. $2 - \dfrac{3}{2}i$

9. a. 6, -12 b. $\frac{1}{3}$, -2 c. 5, -4 d. $1 \pm \dfrac{\sqrt{6}}{2}i$ e. $-3, \pm 1$

10. (1.158,0)

11. a. 5.95 million
b. 10 or about 3.47. The company will make a profit of 4.23 million in 2005 and which is close to what it did in1998.
c.When t is negative the company did not exist. Also the model predicts losses after 2008.
d. 1996, about 2008. The company would break even then.
e. About 2002

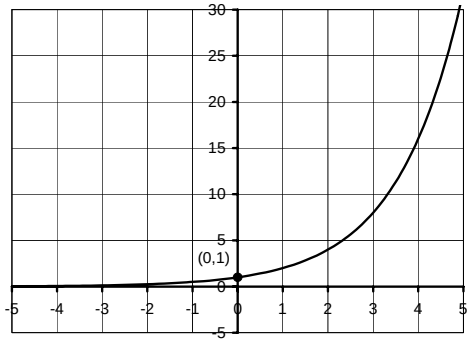
12. a. $\frac{2}{3}$, (-1 is extraneous) b. $\frac{3}{4}$, (-2 is extraneous)

13. a. 6 seconds b. 95/32 seconds c. 9409/64 feet

14. $f(-4)$ is undefined because $\sqrt{-1}$ is undefined on the reals, $f(-3)=0$, $f(0)\approx -1.732$, $f(1)$ is undefined because division by zero is undefined, $f(2)\approx 0.745$, $f(6)=\frac{3}{35}$

15. $x\neq -\frac{5}{4}$.

16. a. 3000 b. 6000. They've run out of space.
17. a. 0 b. 5 c. 1.609 d. 4
18. c. The others can be evaluated using the properties of logarithms.
19. a.



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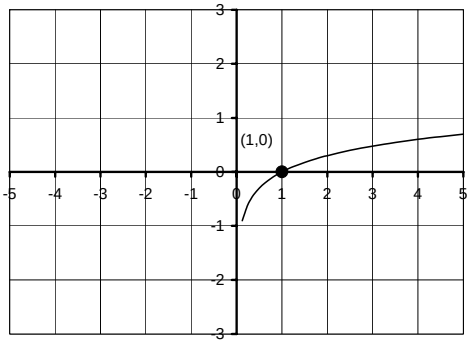
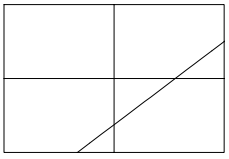


20. a. III and IV b. II and IV c. $-6, -1, 3$ d. $y=0.4x+2$

21. a. \$121.67 b. 58.7 years

22. a.



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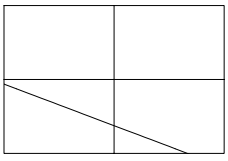
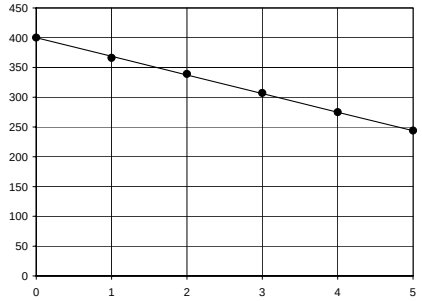


c.

23 a. & b Graph to the right.



c. $y=-31.25x+400$ (Other answers possible.)
d. 87.5
e. 12.8 or about 13 years. He will run out of money in 12.8 years.
f. 400. He started with a net worth of \$400,000 when he retired.

24. i d, ii b, iii a, iv c

25. a. $(0,-2)$ b. $(5,1),\ (-5,1),\ \left(\sqrt{17},-3\right),\ \left(-\sqrt{17},-3\right)$

26. 0.748, or $\dfrac{\ln\left(\frac{10}{3}\right)}{\ln(5)}$

27. a. All reals b. $x \neq 0$ and $x \neq \pm 2$ c. $x \leq 3$

28. a. $\log_3 81 = 4$ b. $25^{1/2} = 5$

29. a. $y = 7x + 21$

b. -1.2 or -1.3

30. $y = 2(5)^x$

31. a. If $a > 0$ the parabola opens upwards. If $a < 0$ the parabola opens downwards.
b. If $c > 0$, the vertex and y-intercept are above the *x*-axis. If $c < 0$, the vertex and *y*-intercept are below the x-axis.

32. a. The linear function is Table B. The slope of the line is 1/2.
b. Table D is the exponential function. For each 1-unit increase in *x*, the *y*-coordinate increases by a multiplicative factor of 5.
c. Table C is not a function because there are two output values (*y*) for same input number (*x*).