

**MONTGOMERY COLLEGE
GERMANTOWN CAMPUS**

MA 103 Final Exam Review Sheet

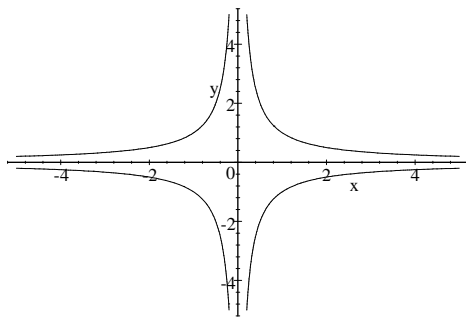
(Revised Spring 2009)

This is a review sheet. The actual final examination will be different.

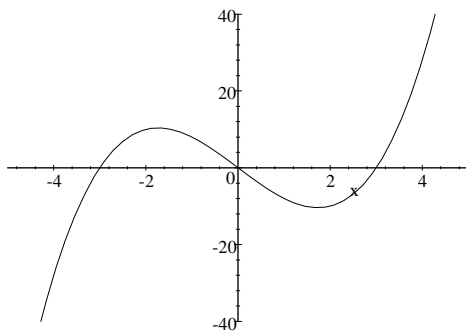
1. Which of the following graphs

- i) Represents a function? Give a reason for your answer.
- ii) Represents a 1-1 function? Give a reason for your answer.

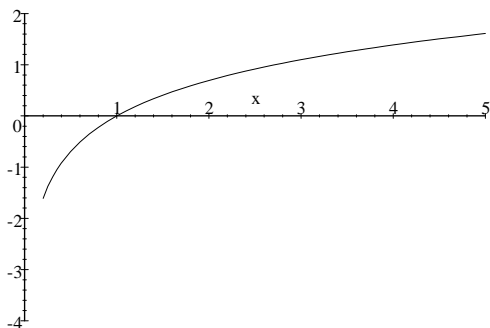
a)



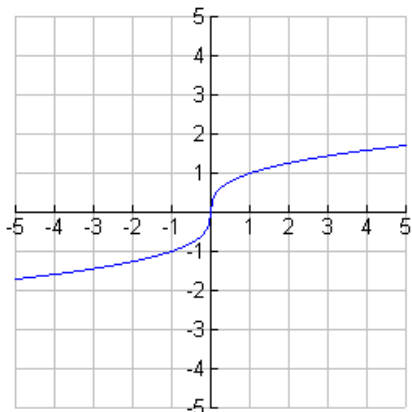
b)



c)



2. Let the following be a graph of a function f . Sketch a graph of the line $y = x$ and a graph f^{-1} on the same set of axes.



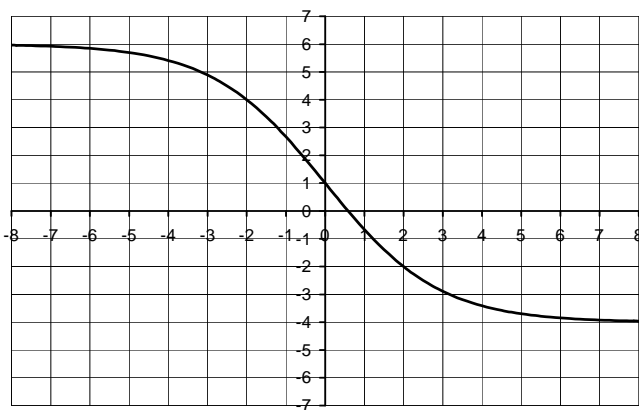
3. Find an equation of the inverse of the function.

$$f(x) = \sqrt[3]{x+1}$$

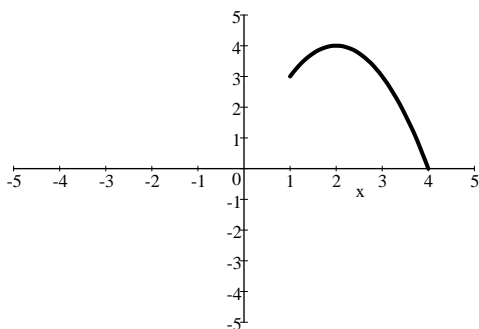
4. a) If $f(x) = \frac{x}{7} - 3$ find $f^{-1}(x)$

b) If $g(x) = 2^x$ find $g^{-1}(x)$

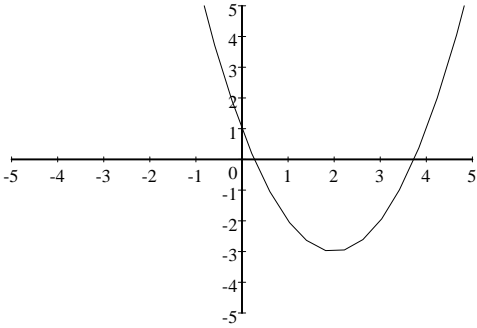
c) Below is the graph of $h(x)$. Approximate $h^{-1}(4)$



5. What are the domain and the range of the following function if the entire graph is shown? Express your answers using interval notation.



6. Find h and k such that $y = (x-h)^2 + k$ is the equation whose graph is shown below.



7. Graph the following equation and write the exact coordinates of the vertex and x-intercepts.

$$f(x) = 2x^2 - 7x - 4$$

8. Find the vertex of the quadratic function by using the following three methods.

$$f(x) = x^2 + 2x + 3$$

- (a) Graphing (b) $x = \frac{-b}{2a}$ (c) Completing the square

9. Which of the following ordered pairs is a solution of the equation $y = 1 - 2x - x^2$, and which are not. Explain your answer.

$(-1, 2)$ $(2, -7)$ $(0, 1)$ $(1, 3)$

10. Which of the following relations represents a function, and which does not. Give a reason for your answer.

$$A = \{(2, 5), (4, 7), (7, 9), (8, 11)\}$$

$$B = \{(1, 3), (2, 3), (2, 6), (3, 8)\}$$

11. For $f(x) = x^2 - 4$ find and simplify $f(t+2)$.

12. Solve each equation for x .

a) $x^2 - 5x = 14$

b) $5x^2 + 7x - 66 = 0$

c) $2x^3 - x^2 - 3x = 0$

d) $16x^3 - x = 0$

e) $x^{2/3} - 2x^{1/3} - 8 = 0$

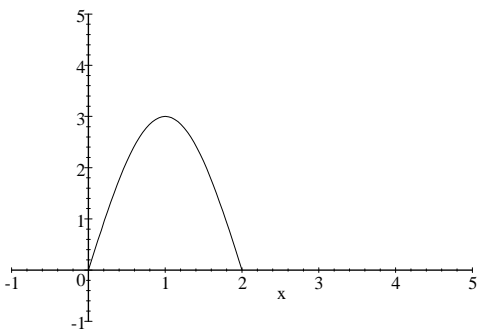
f) $2x^{2/3} + 7x^{1/3} = -6$

13. Use the discriminant to determine the number and type of solutions of the equation.

$$3x = -2x^2 + 8$$

14. Solve the equation by completing the square. $x^2 - 4 = 10x$

15. The equation giving the height of an object thrown upwards starting at 12 feet above the ground is $h(t) = 12 + 40t - 16t^2$.
- When will the object reach its maximum height?
 - What is its maximum height?
 - When will the object hit the ground?
16. A company started business in 1995. Assume that the company has a profit per year given by the equation $f(t) = -0.19t^2 + 2.56t - 2.37$ where $f(t)$ represents the amount of profit in millions of dollars and t is the number of years since 1995.
- Use f to predict the amount of profit in 2003.
 - Find t when $f(t) = 4.23$. What do your results mean in terms of this company's profits?
 - When does $f(t) = 0$? What does this mean in terms of the situation?
 - When will the company have maximum profits?
17. The following is a graph of f . Sketch a graph of the equation $y = f(x-1) + 2$ on the same set of axes.



18. Simplify the following.

a) $\frac{x^2 - 2x - 24}{x^2 - 16}$

b) $\frac{x^3 - 125}{5 - x}$

19. Divide and simplify. $\frac{4a + 8}{a^2 - 7a - 18} \div \frac{a^2 - a - 6}{a^2 - 81}$

20. (Optional) Divide the polynomial $x^4 + 6x^3 + 6x^2 - 10x - 3$ by $x^2 + 2x - 3$.

21. (Optional) Use synthetic division to divide the polynomial $f(x) = 3x^3 + 8x^2 + 5x - 7$ by $x + 2$.

22. Simplify each complex fraction.

a) $\frac{4 - \frac{5}{x}}{x + \frac{7}{9x}}$

b) $\frac{\frac{5}{x} + \frac{6}{x^2}}{\frac{y}{x^2}}$

23. Solve the following equations for x **algebraically**, and verify your answer by substitution.

a) $\frac{2}{x} = \frac{3}{x-2} - 1$ b) $5 + \frac{10}{x} = x + 8$ c) $\frac{19}{x+8} = \frac{12x+5}{x^2+15x+56} - \frac{17}{x+7}$

24. Solve the following equation for x **algebraically**. Verify your solution, by substitution as well as by sketching a graph.

$$2x+1 = \sqrt{13-4x}$$

25. Write in terms of i and simplify if possible. a) $\sqrt{-16}$ b) $\sqrt{-31}$

26. Multiply the complex numbers. Write the product in the form $a + bi$. $(3+2i)^2$

27. Simplify $\frac{4+5i}{1-3i}$ and express the answer in $a + bi$ form.

28. Write the domains of the following functions in set builder notation.

a) $f(x) = \frac{x}{x^2-1}$ b) $g(x) = \frac{\sqrt{x-2}}{6}$

29. Expand the following expression by writing it as a sum or difference of multiples of logarithms:

$$\log\left(\frac{x^2}{yz^2}\right)$$

30. Solve the following system of equations algebraically and sketch a graph of the two equations to verify your solution.

$$\begin{cases} y = x^2 - 2x - 3 \\ y = x + 1 \end{cases}$$

31. Solve the following systems of equations (problem (c) is optional).

a) $\begin{cases} \frac{1}{2}x - y = 3 \\ x = 6 + 2y \end{cases}$ b) $\begin{cases} 6x + 12y = 5 \\ -4x - 8y = 0 \end{cases}$ c) (optional) $\begin{cases} x + y + z = 3 \\ -x + y + z = 5 \\ x + 2y - 3z = 2 \end{cases}$

32. A new machine can make 10,000 aluminum cans three times faster than an older machine. With both the machines working, 10,000 cans can be made in 9 hours. Find how long it will take the new machine, working alone, to make 10,000 cans.

33. The speed of a boat in still water is 24 mph. If the boat travels 54 miles upstream in the same time that it takes to travel 90 miles down stream, find the speed of the stream.

34. Jim can finish a paint job in 12 hours while Jacob can finish the same paint work in 15 hours. How long will it take for them to finish the paint job if they work together?
35. Solve the equation $3x^2 - x + 1 = 0$ and write the solutions in $a \pm bi$ form.
36. Linda invests \$15,000 in a retirement account that pays 9% interest compounded monthly.
- How much will she have in this account after 5 years?
 - Use logarithms to find the number of years that it will take for the account to have a final amount of \$50,000.
 - How much will she have in this account after 5 years if the interest rate is changed to 9% compounded continuously?
37. Evaluate
- $\log(7)$
 - $\ln(e^4)$
 - $\ln(1)$
 - $\log_3 27$
38. a) Convert $4^6 = 4096$ to logarithmic notation.
b) Convert $\log_{25} 5 = \frac{1}{2}$ to exponential notation.
39. a) Find x correct to 4 decimal places so that $(3.4)^{(2x-1)} = 5$.
b) Solve $\log_2 x + \log_2(x-1) = 1$ for x .
40. Write $\log_7(x-1) - \log_7(x^2+1) + 4\log_7 x$ as a logarithm of a single expression.
41. Use your calculator to find $\log_5 8$.
42. Perform the indicated operation and simplify.
- $\frac{3x^2 - 27}{x^2 + 5x + 6} \cdot \frac{2x + 6}{x^2 + 6x + 9}$
 - $\frac{x^2 + 2x - 8}{9x^7} \div \frac{2x - 4}{18x^8}$
43. Add/Subtract: $3x\sqrt{32x} + 5\sqrt{8x^3} - x\sqrt{50x}$
44. Multiply and simplify. $(6\sqrt{5} - 2\sqrt{2})(\sqrt{5} - \sqrt{2})$
45. Rationalize the denominators of each of the following:
- $\frac{3}{\sqrt{2}}$
 - $\frac{2\sqrt{2}}{1 + \sqrt{5}}$
 - $\frac{3}{\sqrt[3]{4}}$
46. Solve each equation for x .
- $\sqrt{3x+7} = -11$
 - $\sqrt[3]{4x-7} + 5 = 3$
 - $\sqrt{x+26} + \sqrt{x-51} = 11$
 - $\sqrt{9-x} = x+11$
47. Solve for x using a quadratic method: $x^4 + 5x^2 + 6 = 0$

48. Pete is watching his ant farm one afternoon and observes that two ants walk away from each other at a 90 degree angle. After a few seconds, the first ant has traveled 3 inches and the second ant has traveled 4 inches. How far apart are the two ants at that point?

49. Solve for x using the square root property. $5x^2 - 3 = 122$

50. Solve the equation for B . $\frac{A}{BH} = \frac{1}{2}$

51. Perform the indicated operation and simplify.

a) $\frac{x}{4x-3} - \frac{3}{8x-6}$

b) $\frac{x+4}{x^2+2x-15} + \frac{x+8}{x^2-12x+27}$

52. Factor by grouping. $xy - 2yz + 5x - 10z$

53. Factor each of the following completely.

a) $x^2 - 13xy + 42y^2$

b) $4x^3 + 28x^2 + 40x$

c) $4x^2y + 22xy - 42y$

d) $16x^2 + 56x + 49$

e) $49y^2 - 4$

f) $x^3 - 125y^3$

g) $7x^3 + 56$

h) $8x^3 + y^3$

i) $6r^3 - 162$

54. Graph $f(x) = x^3 - 2$.

55. Determine whether the lines, when graphed, are parallel, perpendicular, or neither.

$$f(x) = 2x - 12 \text{ and } g(x) = \frac{1}{2}x - 6$$

56. Find an equation of the line through the point (1, 5) and parallel to the given line.

$$f(x) = 3x - 4.$$

57. Find an equation of the line through the point (1, 5) and perpendicular to the given line.

$$f(x) = 3x - 4.$$

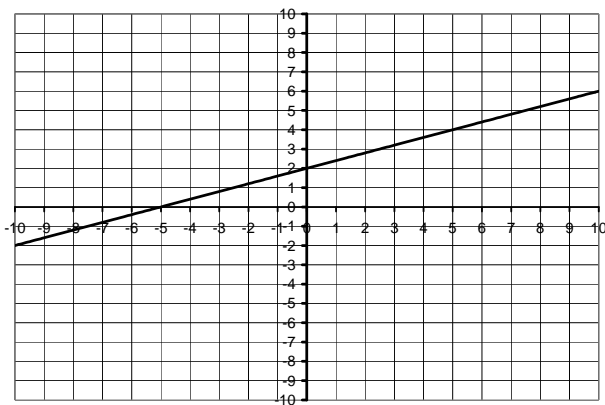
58. Sketch the graph of the line with the following characteristics.

a) $m > 0$ and $b < 0$

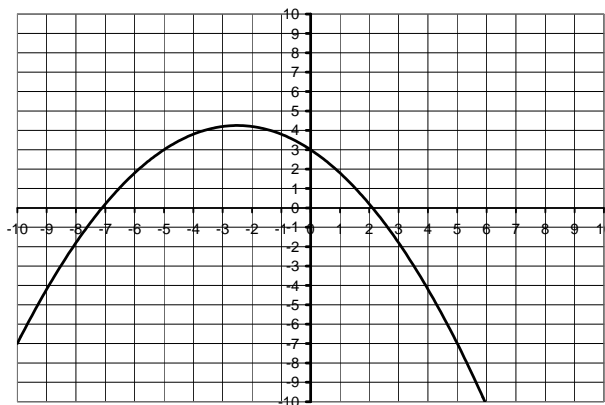
b) $m < 0$ and $b < 0$

c) $m = 0$ and $b > 0$

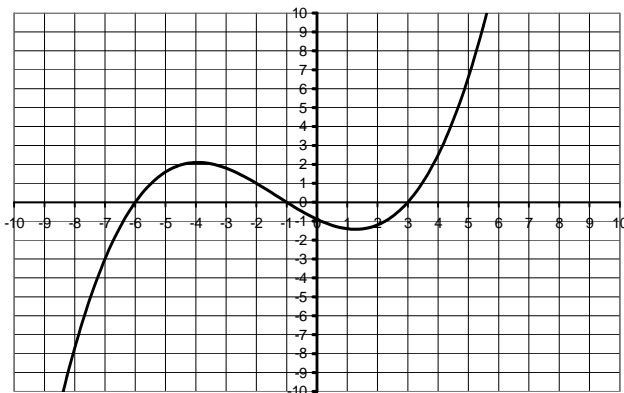
59. Function I



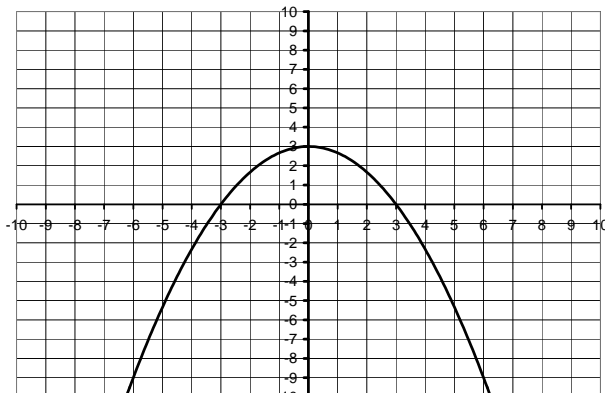
Function II



Function III



Function IV



Refer to the charts above to answer the following.

- List all the functions for which $f(3) = 0$.
- List all the functions for which $f(0) = 3$.
- What are the zeros of function III?
- Approximate the equation of the line in function I.

60. Four tables are given below; three are functions and one is not. One of the functions is linear and one is exponential.

- Which one is the linear function? What is the slope of this line?
- Which one is the exponential function? Explain how you recognize it.
- Which one is not a function? Explain how you know this relation is not a function.
- Make a table of the inverse function of the relation in table B.
- State the domain and range of the function found in part (d).

Table A	
x	y
-1	4
0	0
1	0
2	4

Table B	
x	y
-1	-2
1	-1
3	0
5	1

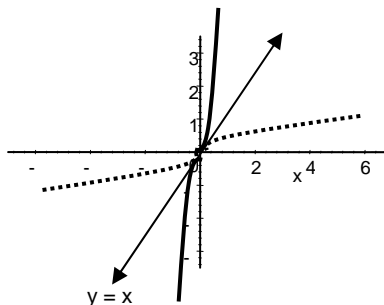
Table C	
x	y
0	-5
1	-3
1	3
0	5

Table D	
x	y
0	2
1	10
2	50
3	250

Solutions

1. a) i) no ii) no
 b) i) yes ii) no
 c) i) yes ii) yes

2.



3. $f^{-1}(x) = x^3 - 1$

4. a) $f^{-1}(x) = 7x + 21$

5. Domain: $[1, 4]$ Range: $[0, 4]$

b) $g^{-1}(x) = \log_2 x$

c) -2

6. $h = 2, k = -3$

7. vertex: $\left(\frac{7}{4}, -\frac{81}{8}\right)$

x-intercepts: $\left(-\frac{1}{2}, 0\right), (4, 0)$

8. (a) $(-1, 2)$ (b) $(-1, 2)$ (c) $(-1, 2)$

9. $(-1, 2)$: YES $(2, -7)$: YES $(0, 1)$: YES $(1, 3)$: NO

10. A: YES B: NO

11. $t^2 + 4t$

12. a) $\{x : x = 7, -2\}$

b) $\left\{x : x = 3, \frac{-22}{5}\right\}$

c) $\left\{x : x = -1, 0, \frac{3}{2}\right\}$

d) $\left\{x : x = 0, \frac{1}{4}, -\frac{1}{4}\right\}$

e) $\{x : x = -8, 64\}$

f) $\left\{x : x = -\frac{27}{8}, -8\right\}$

13. Two Real Solutions

14. $x = 5 \pm \sqrt{29}$

15. a) 1.25 second

b) 37 feet

c) 2.77 seconds

16. a) 5.95 million

b) 10 or about 3.47. The company will make a profit of 4.23 million in 2005 and which is close to what it did in 1998.

c) 1996, about 2008. The company would break even then.

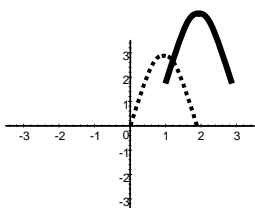
d) About 2002.

17.

18. a) $\frac{x-6}{x-4}$

19. $\frac{4a+36}{a^2-a-6}$

b) $-x^2 - 5x - 25$



20. $x^2 + 4x + 1$
21. $3x^2 + 2x + 1 - \frac{9}{x+2}$
22. a) $\frac{36x-45}{9x^2+7}$
b) $\frac{5x+6}{y}$
23. a) $x = 4, x = -1$ b) $x = -5, x = 2$ c) $x = -11$
24. $x = 1$ 25. a) $4i$ b) $i\sqrt{31}$ 26. $5 + 12i$
27. $-\frac{11}{10} + \frac{17}{10}i$ 28. a) $\{x : x \neq -1, 1\}$ b) $\{x : x \geq 2\}$
29. $2 \log x - (\log y + 2 \log z)$ 30. $(4, 5)$ and $(-1, 0)$
31. a) ∞ number of solutions b) No solution c) $(-1, 3, 1)$ (c is optional)
32. 12 hours 33. 6 mph 34. $6 \frac{2}{3}$ hours 35. $x = \frac{1}{6} \pm \frac{\sqrt{11}}{6}i$
36. a) \$23,485.22
b) 13.4 years
c) \$23,524.68
37. a) 0.845
b) 4
c) 0
d) 3
38. a) $\log_4 4096 = 6$
b) $25^{\frac{1}{2}} = 5$
39. a) 1.1576
b) $x = 2$
40. $\log_7 \left[\frac{x^5 - x^4}{x^2 + 1} \right]$
41. 1.292
42. a) $\frac{6x-18}{x^2+5x+6}$ 43. $17x\sqrt{2x}$
b) $(x+4)x$
44. $34 - 8\sqrt{10}$
45. a) $\frac{3\sqrt{2}}{2}$ b) $\frac{\sqrt{10}-\sqrt{2}}{2}$ c) $\frac{3\sqrt[3]{2}}{2}$
46. a) No Solution b) $x = -\frac{1}{4}$ 47. $x = \pm i\sqrt{2}, \pm i\sqrt{3}$ 48. 5 inches
c) $x = 55$ d) $x = -7$
49. $x = \pm 5$ 50. $B = \frac{2A}{H}$

51. a) $\frac{2x-3}{2(4x-3)}$

b) $\frac{2x^2+8x+4}{(x-3)(x+5)(x-9)}$

52. $(x-2z)(y+5)$

53. a) $(x-6y)(x-7y)$

b) $4x(x+2)(x+5)$

c) $2y(2x-3)(x+7)$

d) $(4x+7)^2$

e) $(7y+2)(7y-2)$

f) $(x-5y)(x^2+5xy+25y^2)$

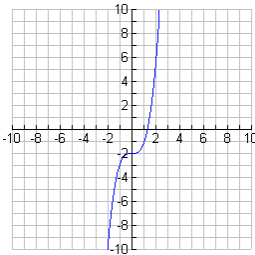
g) $7(x+2)(x^2-2x+4)$

h) $(2x+y)(4x^2-2xy+y^2)$ i) $6(r-3)(r^2+3r+9)$

54.

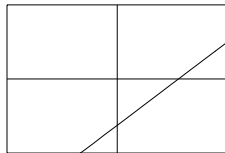
55. Neither

56. $f(x) = 3x + 2$

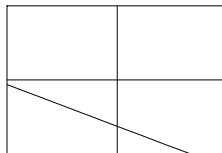


57. $f(x) = -\frac{1}{3}x + \frac{16}{3}$

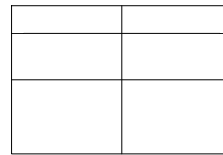
58. a)



b)



c)



59. a) III and IV

b) II and IV

c) $-6, -1, 3$

d) $y = 0.4x + 2$

60. a) Table B is the linear function. The slope of the line is $1/2$.

b) Table D is the exponential function. For each 1-unit increase in x , the y -coordinate increases by a multiplicative factor of 5.

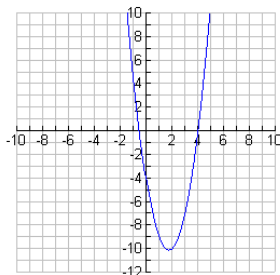
c) Table C is not a function because there are two output values (y) for same input number (x).

e) Domain: $\{-2, -1, 0, 1\}$. Range: $\{-1, 1, 3, 5\}$

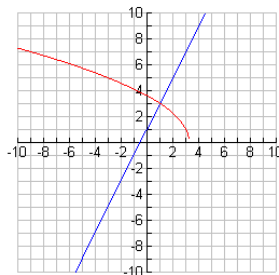
d)

Table B	
x	y
-2	-1
-1	1
0	3
1	5

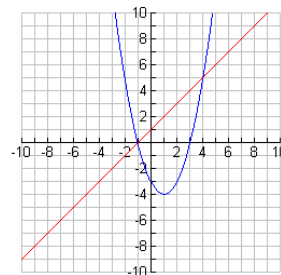
#7 Graph



#24 Graph



#30 Graph

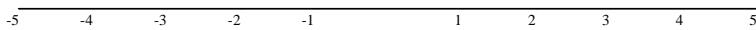


Chapter 9 Questions (Optional)

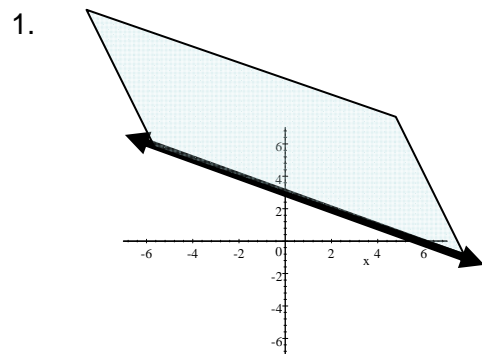
1. Graph and clearly label the set of all points that satisfy $x+2y \leq 6$.
2. a) Solve the following compound inequality and write your answer using interval notation:

$$3x - 2 < 4 \text{ and } -4x - 3 \leq 9$$

- b) Sketch a graph of the solution set on the number line



Solutions



2. a) $x < 2$ and $x \geq -3$ so the solution set is $[-3, 2)$

