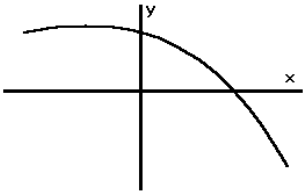
Math 103 Final Exam Review Problems
Rockville Campus
Spring 2010

1. Define a. relation b. function

2. For each example below, explain why it is or is not a function.

a.



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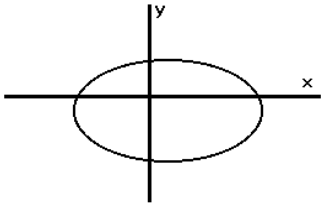

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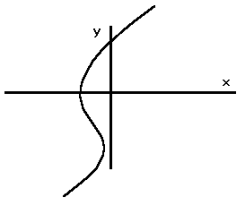

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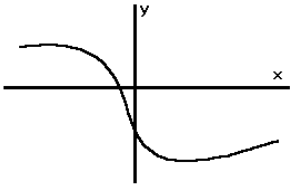

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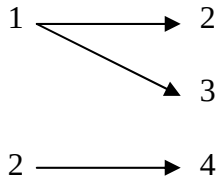
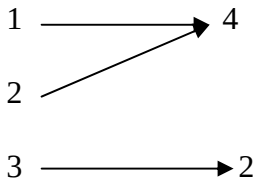


f.



3. Given $2x+3y=18$
   a. Find the *x*-intercept.
   b. Find the *y*-intercept.
   c. Find the slope.
   d. Sketch the graph.
   e. Find the equation of a line parallel to this line passing through (-3,-2)

4. Given $f(x)=3x^2-12x+4$

a. Find the vertex $f(x)$ of by completing the square.

b. Find the vertex of $f(x)$ by using the formula for the x-coordinate of the vertex.

c. Use your graphing calculator to find the x-intercepts of $f(x)$

d. What is the y-intercept of $f(x)$?

5. Simplify a. $\sqrt{48}$ b. $\dfrac{5}{\sqrt{15}}$ c. $3\sqrt{20}-7\sqrt{45}+\sqrt{5}$

6. Let $f(x)=3-4x$

a. Find and simplify $f(-2)$

b. Find x when $f(x)=-2$

7. Given

$f(x)=3x^2-6x+1$ $\qquad$ $g(x)=-9x-2x^2$

$h(x)=x^3-2x-6$

For each function above, use any suitable method to answer the following questions:

a. Find all intercepts. When it is required to estimate them, estimate to three significant digits.

b. Evaluate the functions at $x=2$ and $x=-2$.

c. Sketch a graph that shows the significant features of the function.

d. If the function is a parabola, find the exact value of the x-coordinate of the vertex.

8. Simplify each expression below where possible and write your answer without using negative or fractional exponents. Assume that $x>0$.

a. $8x^{1/3}$ b. $(-8x)^{1/3}$ c. $(8x)^{-1/3}$ d. $\left(3x^{3/4}\right)(16x)^{1/4}$ e. $\dfrac{x^{1/2}}{x^{5/2}}$

9. Simplify $\dfrac{8-\sqrt{-36}}{4}$ and express the answer in $a+bi$ form.

10. Simplify a. $\sqrt{x}\cdot\sqrt[3]{x}$ b. $5\sqrt{3}-\sqrt{3}$

11. Find all real and non-real solutions to the following:

a. $(x+3)^2-81=0$ b. $3x^2+5x=2$

c. $(x+1)(x-2)=18$ d. $2x^2-4x+5=0$

12. A company started business in 1995. Assume that a company has a profit per year given by the equation $f(t)=-0.19t^2+2.56t-2.37$ where $f(t)$ represents the amount of profit in millions of dollars, where t is the number of years since 1995.

a. Use f to predict the amount of profit in 2003.

b. Find t when $f(t)=4.23$. What do your results mean in terms of the company's profits.

c. For what years is there likely model breakdown? That is, what years is it unlikely that the model would apply to the situation described. (Hint: consider the past and future.)

d. When does $f(t)=0$? What does this mean in terms of the situation?

e. When will the company have maximum profits?

13. Your calculator gives an answer of $x = 1.1578074156,\ y = 2\text{E-}13$ when using **ROOT** or **ZERO** in the graphing mode. Give the coordinates of the x-intercept your calculator has given you to the nearest thousandth.

14. Solve for *x* algebraically.

a. $\dfrac{x}{x+1} + \dfrac{2x-2}{x} = \dfrac{-1}{x+1}$

b. $2x+1 = \sqrt{7-x}$

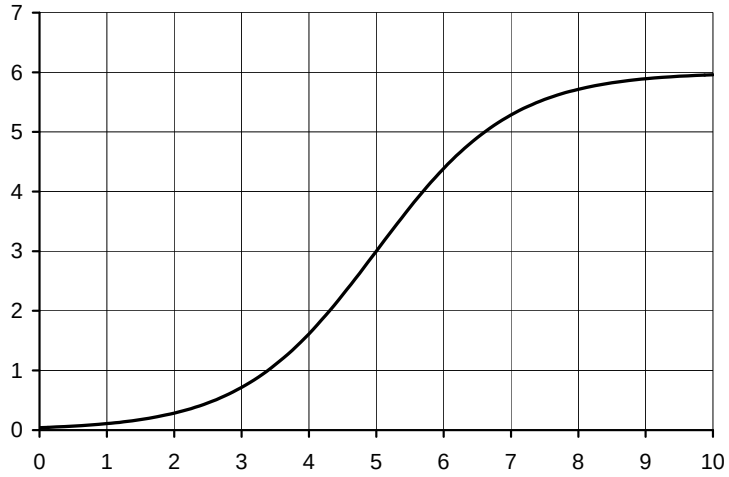
15. The equation of the height of an object thrown upwards starting at 6 feet above the ground is $h(t) = 6 + 95t - 16t^2$ where *t* is time in seconds and *h* is height in feet.

a. When will the object hit the ground?
b. When will it reach its maximum height?
c. What is its maximum height?

16. Let $f(x) = -\dfrac{\sqrt{x+3}}{1-x^2}$ Evaluate each of the following. Give exact answers. Also approximate irrational answers to three significant digits. If you cannot evaluate, explain why.

$f(-4),\ f(-3),\ f(0),\ f(1),\ f(2),\ f(6).$

17. What is the domain of *f* if $f(x) = \dfrac{2x-3}{4x+5}$?



18. Bacteria are grown in a dish. The chart above is a graph of the number of bacteria in the dish. Food is added as needed. The vertical axis is the number of bacteria in thousands. The horizontal axis is the time in hours.

a. Approximately how many bacteria are in the dish after five hours?
b. After 10 hours the number of bacteria appears to be leveling off at what number? Explain why this might be the case.

19. Evaluate

a. $\log(1)$ b. $\ln\left(e^5\right)$ c. $\ln(5)$ d. $\log_2 16$

20. Which part of the previous problem is difficult to do without a calculator? Explain.

21. Label all intercepts and sketch the graph of

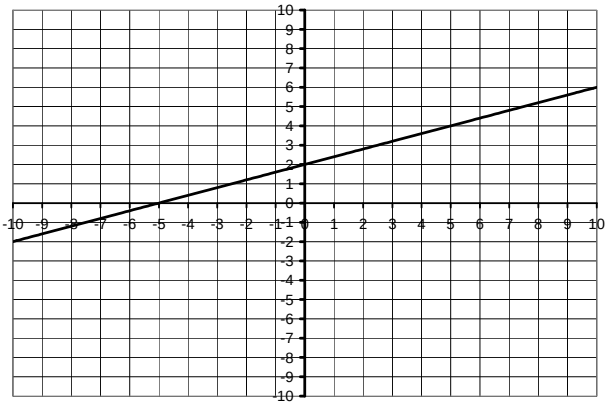
a. $f(x) = 2^x$

b. $f(x) = \ln(x)$

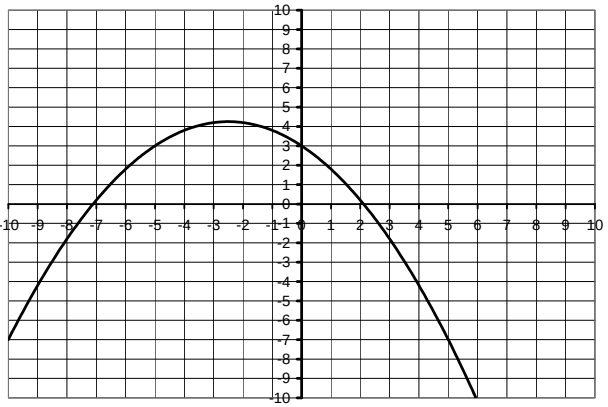
c. $f(x) = \left(\frac{2}{3}\right)^x$

22.

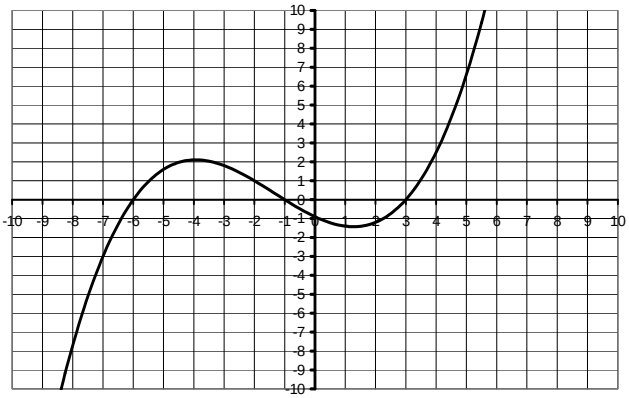
Function I



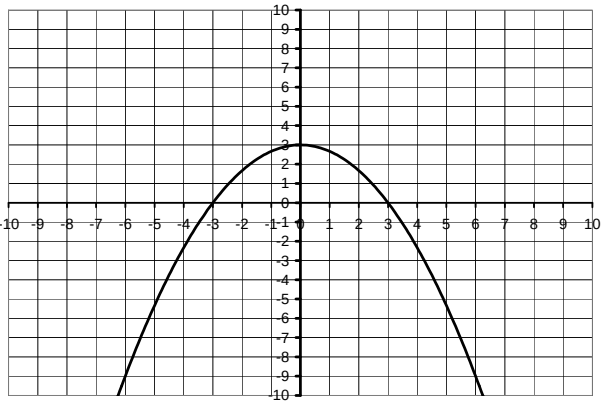
Function II



Function III



Function IV



Refer to the charts above to answer the following.

a. List all the functions for which $f(3) = 0$.

b. List all the functions for which $f(0) = 3$.

c. What are the zeros of function III?

d. Approximate the equation of the line in function I

23. $100 is deposited in a savings account that pays 4% interest compounded annually.

a. How much is in the account after 5 years?

b. Use logarithms to determine how many years it will take for the account to grow to $1000. Confirm your results by graphing $y = A(t)$ in an appropriate window.

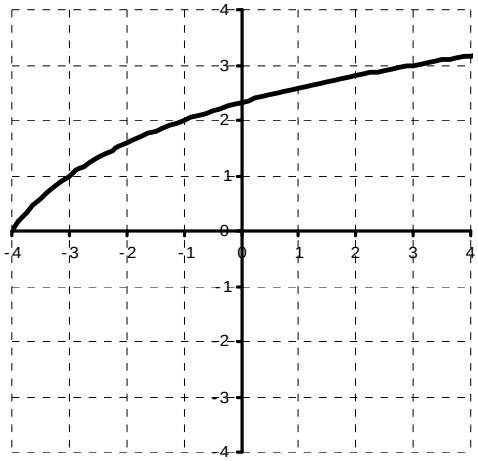
24. Sketch the graph of a line with the following characteristics:

a. $m > 0$ and $b < 0$ b. $m < 0$ and $b < 0$ c. $m = 0$ and $b > 0$

25. Solve the following systems of equations algebraically.

a. $\begin{cases} 3s - 4t = 8 \\ 2s + 3t = -6 \end{cases}$ b. $\begin{cases} x^2 + y^2 = 26 \\ x^2 - 2y = 23 \end{cases}$

26. Draw the inverse of the function shown.



27. A retired man looks at the his net worth since he retired. The data is in the table below.

Years since retirement	Net worth in thousands
0	400
1	366
2	339
3	307
4	275
5	244

a. Create a scattergram of the data.

b. Sketch a line that approximates the data.

c. Find the equation of that line.

d. Let $y = f(x)$ be the function that models the data. Find $f(10)$

e. What is the x-intercept? What does it mean in terms of the model?

f. What is the y-intercept? What does it mean in terms of the model?

g. What is the slope of the line. What does it mean in terms of the model?

28. Use a graphing calculator to solve the following systems. Give your answers to the nearest thousandth.

a. $\begin{cases} 3x-5y=8 \\ 4x+2y=9 \end{cases}$

b. $\begin{cases} y=3x^2-5x+2 \\ 2\left(y+x^2\right)=4x+9 \end{cases}$

29. Find the domain of each function below. Give your answer using both set builder notation and interval notation.

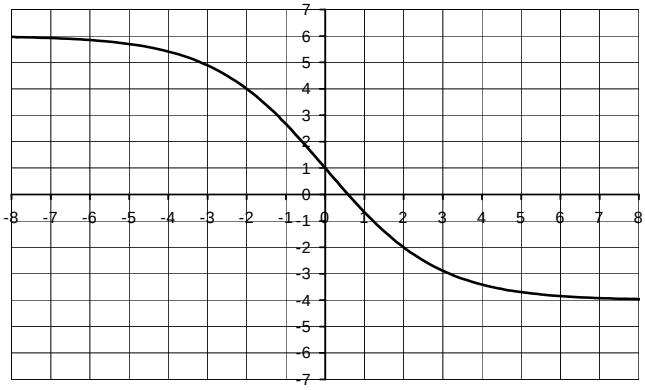
a. $f\left(x\right)=\ln\left(7x-3\right)$ b. $g\left(x\right)=\dfrac{1}{x^3-4x}$ c. $k\left(x\right)=\sqrt{5-7x}$

30. a. Convert $3^4=81$ to logarithmic notation.
b. Convert $\log_{25} 5=\frac{1}{2}$ to exponential notation.

31. Find the equation for the exponential function of the form $y=ab^x$ that passes through the points $\left(0,2\right)$ and $\left(1,10\right)$.

32. Given $f\left(x\right)=ax^2+c$.
a. How does the sign of a affect the graph of f?
b. How does the sign of c affect the graph of f?

33. a. If $f\left(x\right)=\dfrac{x}{7}-3$ find $f^{-1}\left(x\right)$
b. If $g\left(x\right)=3^x$ find $g^{-1}\left(x\right)$
c. Below is the graph of $h\left(x\right)$. Approximate $h^{-1}\left(3\right)$



34. Four tables are given below; three are functions, and one is not. One of the functions is linear, and one is exponential.
a. Which one is the linear function? What is the slope of the line?
b. Which one is the exponential function? Explain how you recognize it.
c. Which one is not a function? Explain how you know this relation isn't a function.

d. Make a table of the inverse function of the relation in table B
e. State the domain and range of the function found in part d.

Table A	
x	*y*
-1	4
0	0
1	0
2	4

Table B	
x	*y*
-1	-2
1	-1
3	0
5	1

Table C	
x	*y*
0	-5
1	-3
1	3
0	5

Table D	
x	*y*
0	2
1	10
2	50
3	250

35. Combine and simplify

a. $\dfrac{5x}{x^2+x-6}-\dfrac{6}{x^2-x-2}$ b. $\dfrac{4x^2-9}{x^2-3x+2}\div\dfrac{2x^2-x-3}{x^2-1}$

36. Solve the following by factoring.

a. $4x^3-49x=0$ b. $6x^2+11x=10$ c. $9x^2-30x+25=0$

37. Solve by completing the square.

a. $6x^2-7x=3$ b. $2x(x-4)=3(x-1)$

38. Solve the following symbolically.

a. $\log_{17}17^{x+2}=\log_7 1$ b. $7\left(e^{2x}\right)^3=35$ c. $7^{x^2-x}=7^6$ d. $\log_3 5x=4$

e. $3\cdot5^x=10$

39. a. Convert $7x^{\frac{2}{3}}$ to radical notation.

b. Convert $\left(7x^3\right)^{\frac{1}{4}}$ to radical notation.

c. Convert $2\sqrt{x}$ to exponential notation.

d. Convert $\sqrt[7]{5y^2}$ to exponential notation.

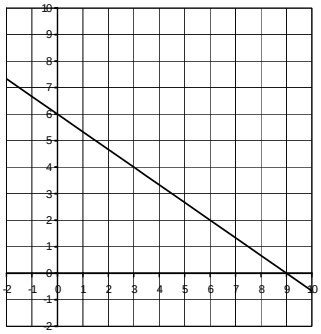
Answers. Note: answers marked by an asterisk (*) are difficult to find without a *graphing* calculator.

1. a. A relation is a set of ordered pairs. b. A function is a relation in such that if (a,b) and (a,c) are ordered pairs in the function, then $b=c$. Alternatively: A correspondence between a first set, called the domain, and a second set, called the range, such that each member of the domain corresponds to exactly one member of the range.

2. a and d are functions because it is impossible to draw a vertical line that intersects the graph more than once. b and c are not functions because in each case, it *is* possible to draw a vertical line that intersects both these graphs twice (or more). e. is not a function because two values are assigned to 1. f. is a function because each element in the domain has a unique element in the range.

3. a. (9,0) b. (0,6) c. $-\frac{2}{3}$

d. See the graph to the right.
e. $y = -\frac{2}{3}x - 4$



4. a. and b. $(2,-8)$ *c. 3.633, .367 d. 4

5. a. $4\sqrt{3}$ b. $\frac{\sqrt{15}}{3}$ c. $-14\sqrt{5}$

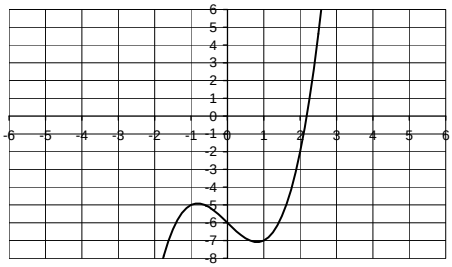
6. a. 11 c. 5/4

7.

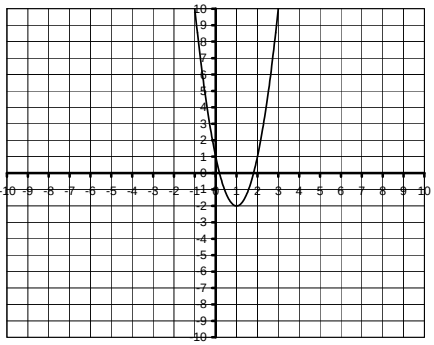
	f	g	h
*a	(0,1), (.184,0), (1.816,0)	$(0,0),(-4.5,0)$	$(0,-6),(2.18,0)$
b	$f(2)=1, f(-2)=25$	$g(2)=-26$ $g(-2)=10$	$h(2)=-2,$ $h(-2)=-10$
d	1	-2.25	NA

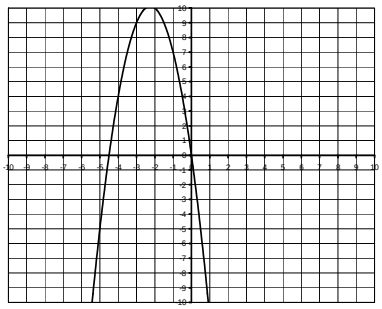
Graphs: $f(x)=3x^2-6x+1$ $g(x)=-9x-2x^2$

*$h(x)=x^3-2x-6$ (The graph is to the right.)



8. a. $8\sqrt[3]{x}$ b. $-2\sqrt[3]{x}$ c. $\frac{1}{2\sqrt[3]{x}}$





d. $6x$ e. $\frac{1}{x^2}$

9. $2-\frac{3}{2}i$

10. a. $x^{5/6}$ b. $4\sqrt{3}$

11. a. 6, -12 b. $1/3$, -2 c. 5, -4 d. $1\pm\frac{\sqrt{6}}{2}i$.

12. a. 5.95 million

b. 10 or about 3.47. The company will make a profit of 4.23 million in 2005 and which is close to what it did in1998.

c. When t is negative the company did not exist. Also the model predicts losses after 2008.

d. 1996, about 2008. The company would break even then.

e. About 2002

13. (1.158,0)

14. a. $2/3$, (-1 is extraneous) b. $3/4$, (-2 is extraneous)

15. a. 6 seconds b. 95/32 seconds c. 9409/64 feet

16. $f(-4)$ is undefined because $\sqrt{-1}$ is undefined on the reals, $f(-3)=0$, $f(1)$ is undefined because division by zero is undefined, $f(0)=-\sqrt{3}\approx -1.73$, $f(2)=\frac{\sqrt{5}}{3}\approx 0.745$, $f(6)=3/35$.

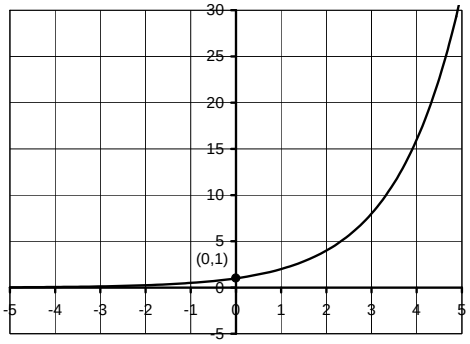
17. $x\neq -\frac{5}{4}$.

18. a. 3000 b. 6000. They've run out of space.

19. a. 0 b. 5 c. 1.609 d. 4

20. c. The others can be evaluated using the properties of logarithms.

21. a.



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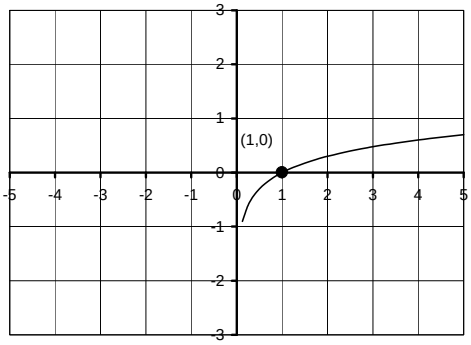

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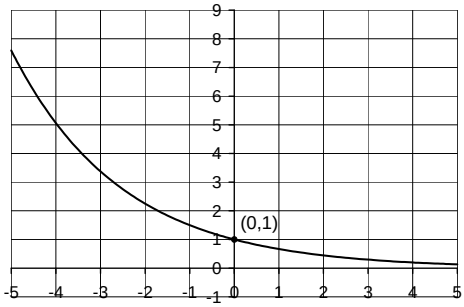
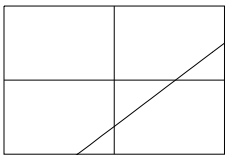
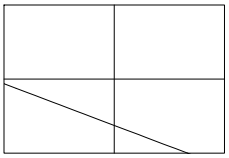
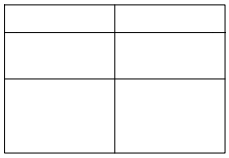


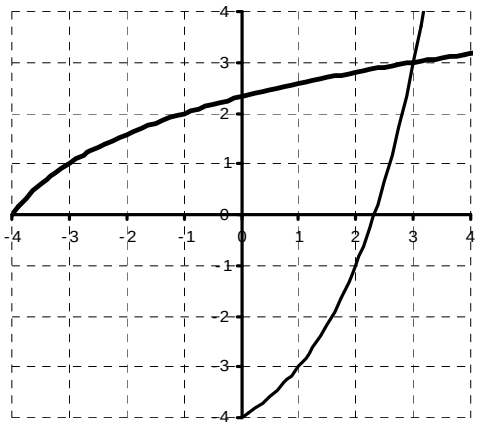
22. a. III and IV b. II and IV c. $-6, -1, 3$ d. $y = 0.4x + 2$

23. a. \$121.67 b. 58.7 years

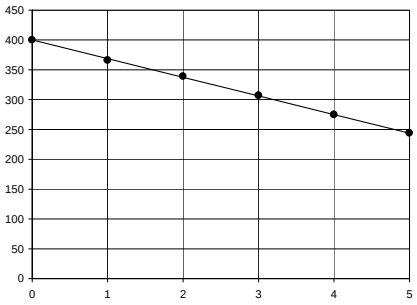
24. a.  b.  c. 

25. a. $(0,-2)$ b. $(5,1), (-5,1), \left(\sqrt{17},-3\right), \left(-\sqrt{17},-3\right)$

26.



27 a. & b Graph to the right.



c. $y = -31.25x + 400$ (Other answers are possible.)

d. 87.5

e. 12.8 or about 13 years. He will run out of money in 12.8 years.

f. 400. He started with a net worth of \$400,000 when he retired.

g. -31.25. His net worth decreased by \$31,250 per year.

28. a. $(2.346, -.192)$ b. $(-.304, 3.799)$ and $(2.054, 4.389)$

29. a. $\left\{x \middle| x > \frac{3}{7}\right\}$ or $\left(\frac{3}{7}, \infty\right)$ b. $\{x \mid x \neq 0 \text{ and } x \neq \pm 2\}$ or $(-\infty,-2) \cup (-2,0) \cup (0,2) \cup (2,\infty)$

c. $\left\{x \middle| x \leq \frac{5}{7}\right\}$ or $\left(-\infty, \frac{5}{7}\right]$.

30. a. $\log_3 81 = 4$ b. $25^{1/2} = 5$

31. $y = 2(5)^x$ or $y = 2(e)^{x \ln 5}$

32. a. If $a > 0$ the parabola opens upwards. If $a < 0$ the parabola opens downwards.
b. If $c > 0$, the vertex and y-intercept are above the *x*-axis. If $c < 0$, the vertex and *y*-intercept are below the x-axis.

33. a. $f^{-1}(x) = 7x + 21$

b. $g^{-1}(x) = \log_3 x$

c. -1.2 or -1.3

34. a. The linear function is Table B. The slope of the line is 1/2.

b. Table D is the exponential function. For each 1-unit increase in x, the y-coordinate increases by a multiplicative factor of 5.

c. Table C is not a function because there are two output values (y) for same input number (x).

e. Domain: {-2,-1,0,1} Range: {-1,1,3,5}

d.

Table B	
x	y
-2	-1
-1	1
0	3
1	5

35. a. $\dfrac{5x+9}{(x+1)(x+3)}$ b. $\dfrac{2x+3}{x-2}$

36. a. $0, \pm\frac{7}{2}$ b. $\frac{2}{3}, -\frac{5}{2}$ c. $\frac{5}{3}$

37. a. $\frac{3}{2}$, $-\frac{1}{3}$ b. $\dfrac{11 \pm \sqrt{97}}{4}$

38. a. -2 b. $\frac{1}{6}\ln 5$ c. $3, -2$ d. $\frac{81}{5}$ e. 0.748, or $\dfrac{\ln\left(\frac{10}{3}\right)}{\ln(5)}$

39. a. $7\sqrt[3]{x^2}$ b. $\sqrt[4]{7x^3}$ c. $2x^{1/2}$ d. $\left(5y^2\right)^{1/7}$ or $5^{1/7}y^{2/7}$