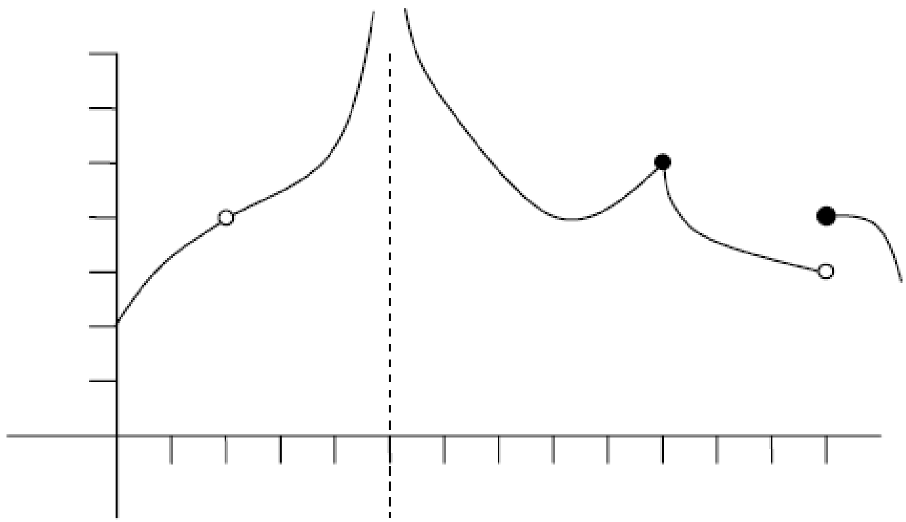
Name: ____________________________

Final Exam Review
MA160

1. Use the information given about the function $f(x)$ in the sketch below to answer the questions. Assume that each tick mark represents one unit.



	Is $f(x)$ defined for this value of x? If so estimate $f(a)$.	Find $\lim_{x\to a} f(x)$ if it exists.	Is $f(x)$ continuous at this value of x?	Is $f(x)$ differentiable at this value of x?
$a=2$				
$a=5$				
$a=8$				
$a=10$				
$a=13$				

2. Find each limit algebraically, if it exists. If a limit does not exist, state that fact.

   a. $\lim_{x\to -3}\left(x^3+x^2-5x\right)$

   b. $\lim_{x\to -3}\frac{x^2-3x-18}{x+3}$

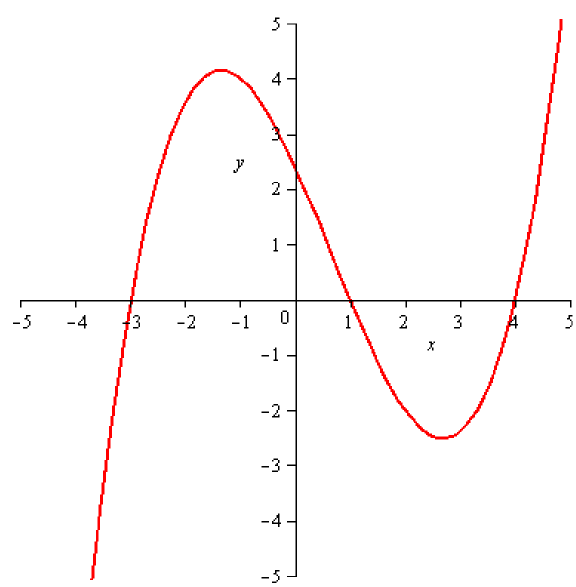
3. Use the limit definition of the derivative to find $f'(x)$ when $f(x)=5x^2-2x+7$.

4. Differentiate:
   a. $f(x) = 4x^2 + x^{4/3} + \sqrt[5]{x} + \frac{7}{x}$
   b. $g(x) = x^4(5x-3)^2$
   c. $h(x) = \sqrt[3]{x^5 + 6x}$
   d. $y = \frac{1}{(3x+8)^3}$
   e. $f(x) = \frac{3x^2 + 2x}{x^2 - 1}$
   f. $f(x) = x^5 e^{2x}$
   g. $g(x) = e^{x^2} + 4x^3$
   h. $h(x) = \ln(5x^2 + 3x - 2)$
   i. $f(x) = \left[\ln(x^2 + 3x)\right]^2$

5. Find the equation of the tangent line to the graph $f(x) = \sqrt{x^2 + 6x}$ when $x = 2$.

6. The temperature of a person during an illness is given by $F(t) = -0.1t^2 + 1.2t + 98.6$ where F is the temperature in degrees Fahrenheit at time t in days.
   a. Find the rate of change of the temperature with respect to time.
   b. Find $F(1.5)$ and write a sentence in everyday language explaining the answer in the context of this situation.
   c. Find $F'(1.5)$ and write a sentence in everyday language explaining the answer in the context of this situation.
   d. Why would the sign of $F'(t)$ be significant to a doctor?

7. The graph of $f'(x)$, **the derivative of** $f(x)$**, is shown**. Use this graph to answer the following questions about $f(x)$. Again, note that the graph of $f(x)$ is NOT shown.
   a. On what interval or intervals is the function $f(x)$ increasing?
   b. On what interval or intervals is the function $f(x)$ decreasing?
   c. At what x-values does the function $f(x)$ have a relative maximum?
   d. At what x-values does the function $f(x)$ have a relative minimum?



8. Consider the function $f(x) = -x^3 + 3x - 2$ and find the following:

(Show all work! Use curve sketching techniques from Chapter 2, which do not include staring at the graph on your calculator.)

a. Critical values
b. Relative minima (as ordered pairs):
c. Relative maxima (as ordered pairs):
d. On what intervals is this function increasing?
e. On what intervals is this function decreasing?
f. Inflection points
g. On what intervals is this function concave up?
h. On what intervals is this function concave down?

9. Determine all of the asymptotes on the following functions: (vertical, horizontal, and slant and label them as such).

a. $f(x) = \dfrac{5x-3}{4x-12}$

b. $g(x) = \dfrac{x^2-7}{x+1}$

10. Determine the absolute maximum and absolute minimum values for the function $f(x) = x^2 - 4x + 5$ over the interval $[-1,3]$. Show work and prove it as we did in class and write your answers as ordered pairs.

11. A manufacturer of cameras finds that the price at which it can sell x cameras per week is $p(x) = 500 - x$. The total cost of producing x cameras per week is $C(x) = 150 + 4x + x^2$ dollars.

a. Find the revenue function, $R(x)$.
b. Find the profit function, $P(x)$.
c. Find the production level which maximizes the profit.
d. What price per camera must be charged to maximize profit?

12. A rectangular garden of area 75 square feet is to be surrounded on three sides by a brick wall costing \$10 per foot and on one side by a fence costing \$5 per foot. Find the dimensions of the garden such that the cost of the materials is minimized.

13. A population is growing at a rate of 3% per month, that is $\dfrac{dP}{dt} = 0.03P$, where t is time in months.

a. Find the function that satisfies the equation, assuming that the initial population was 100 ($t = 0$).
b. What is the population after 70 months? After 120 months?
c. After how long will the population double from the original population of 100?

14. A homeowner wants to have \$10,000 available in 5 years to pay for new siding. Interest is 4.5%, compounded continuously. How much money should he invest now?

15. A sample of 8 grams of radioactive material is placed in a vault. Let $N(t)$ be the amount remaining after t years, and suppose that $N(t)$ satisfies $\frac{dN}{dt} = -0.021N$.

a. Find the exponential function that models the situation.
b. After 10 years, how much of the 8 g will remain? Round to the nearest tenth of a gram.
c. After how long will half of the original amount remain? Round to the nearest year.

16. Evaluate the integrals:

a. $\int \left(x^5 - \frac{9}{2}\sqrt{x} + x^{-3/2} + \frac{4}{x} \right) dx$

b. $\int \left(4 - 5e^{-5t} + \frac{e^{2t}}{3} \right) dt$

c. $\int_2^3 \left(6 + \frac{1}{x^2} \right) dx$

d. $\int_0^1 e^{4x} dx$

e. $\int_1^4 \sqrt{x} dx$

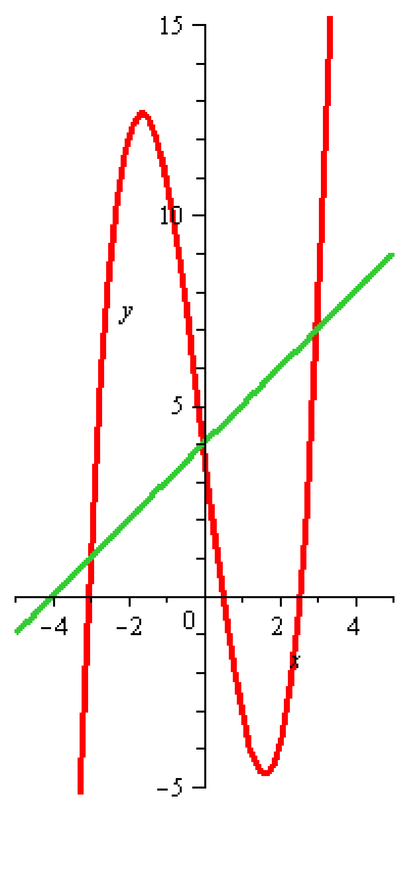
17. Given the integral $\int_{-1}^{2} (x^2 + 4) dx$

a. Calculate the area given by the integral.
b. Sketch the area represented by the integral.
c. Find the average value of the function over the interval $-1 \le x \le 2$.

18. Find the area of the region bounded by the curves $y = x + 1$ and $y = x^2 - 3x - 4$.

19. Find the area of the region bounded by the graphs of the functions below:

$$f(x) = x^3 - 8x + 4 \quad \text{and} \quad g(x) = x + 4$$



20. Find f such that $f'(x)=3x^2+\frac{1}{x}-4,\ f(1)=2$.

21. Suppose that the marginal cost function of a handbag manufacturer is $C'(x)=\frac{3}{32}x^2-x+200$ dollars per unit at production level x. Find total cost of producing 5 additional units if 2 units are currently being produced.

22. Polm, Inc. determines that its marginal revenue per day is given by $R'(t)=110e^t$, $R(0)=0$, where R is the total accumulated revenue, in dollars, on the tth day. The company's marginal cost per day is given by $C'(t)=110-0.8t$, $C(0)=0$ where $C(t)$ is the total accumulated cost, in dollars on the tth day.
a. Find the total profit for the first 10 days.
b. Find the average daily profit for the first 10 days.

23. $D(x)$ is the price, in dollars per unit, that consumers are willing to pay for x units of an item, and $S(x)$ is the price, in dollars per unit, that producers are willing to accept for x units. Given $D(x)=(x-9)^2$, $S(x)=x^2+4x+15$, find:
   a. The equilibrium point.
   b. The consumer surplus at the equilibrium point.
   c. The producer surplus at the equilibrium point.

24. Evaluate using substitution:
   a. $\int 3x^2\left(x^3+8\right)^6 dx$
   b. $\int_1^3 \frac{2x+3}{x^2+3x}dx$

25. Given $f(x,y)=x^5+9x^2y+4xy^3$, find:
   a. $\frac{\partial f}{\partial x}$
   b. $\frac{\partial f}{\partial y}$

26. Find the relative maximum or minimum value of $f(x,y)=x^2+2xy+2y^2-6y+2$.

27. A researcher's data consists of the points $(1,3)$, $(4,5)$, and $(6,7)$. Find the regression line using the least squares techniques.

**Credit is due to the Rockville Mathematics Department for problems 1, 6, 11, 12, 13, 16 c-e, 17, 18, 20, as these problems are either exact problems or slightly modified versions of problems found on their Final Exam Review.

Updated: Fall 2009
(J. Polm)

Final Exam Review Solutions
MA160

28. At $a=2$: $f(2)$ undefined, limit = 4, discontinuous, not differentiable

At $a=5$: $f(5)$ undefined, no limit, discontinuous, not differentiable

At $a=8$: $f(8)=4$, limit = 4, continuous, differentiable

At $a=10$: $f(10)=5$, limit = 5, continuous, not differentiable

At $a=13$: $f(13)=4$, no limit, discontinuous, not differentiable

29. a. -3 b. -9

30. $f'(x)=10x-2$

31.

a. $f'(x)=8x+\frac{4}{3}x^{1/3}+\frac{1}{5x^{4/5}}-\frac{7}{x^2}$

b. $g'(x)=10x^4(5x-3)+4x^3(5x-3)^2$
$=2x^3(5x-3)(15x-6)$
$=6x^3(5x-3)(5x-2)$

c. $h'(x)=\frac{5x^4+6}{3(x^5+6x)^{2/3}}$

d. $y'=\frac{-9}{(3x+8)^4}$

e. $f'(x)=\frac{(x^2-1)(6x+2)-(3x^2+2x)2x}{(x^2-1)^2}$
$=\frac{-2(x^2+3x+1)}{(x^2-1)^2}$

f. $f'(x)=2x^5e^{2x}+5x^4e^{2x}$
$=x^4e^{2x}(2x+5)$

g. $g'(x)=2xe^{x^2}+12x^2$
$=2x(e^{x^2}+6x)$

h. $h'(x)=\frac{10x+3}{5x^2+3x-2}$

i. $f'(x)=\frac{2(2x+3)\ln(x^2+3x)}{x^2+3x}$

32. $y=\frac{5}{4}x+\frac{3}{2}$

33. a. $F'(t)=-0.2t+1.2$

b. About 100.2º. After one and a half days, the person's temperature is about 100.2º.

c. 0.9 º per day. After one and a half days, the person's temperature is rising at a rate of about 0.9 º per day.

34.

a. Increasing on $(-3,1)\cup(4,\infty)$
b. Decreasing on $(-\infty,-3)\cup(1,4)$
c. Relative maximum at $x = 1$.
d. Relative minimum at $x = -3, 4$.

35.

a. $x = -1, 1$.
b. Relative minimum at $(-1,-4)$.
c. Relative maximum at $(1,0)$.
d. Increasing on $(-1,1)$.
e. Decreasing on $(-\infty,-1)\cup(1,\infty)$.
f. Inflection point at $(0,-2)$.
g. Concave up on $(-\infty,0)$.
h. Concave down on $(0,\infty)$.

36. a. VA: $x = 3$, HA: $y=\frac{5}{4}$, Slant: None b. VA: $x = -1$, HA: None, Slant: $y=x-1$

37. Absolute maximum of 10 at $x = -1$, so $(-1,10)$. Absolute minimum of 1 at $x = 2$, so $(2,1)$.

38. a. $R(x)=500x-x^2$ b. $P(x)=496x-2x^2-150$ c. $x=124$ d. \$376

39. Dimensions: 7.5 feet x 10 feet. Minimum cost is \$300.

40. a. $P(t)=100e^{.03t}$ b. 817 and 3660 c. 23 months

41. \$7985.16

42. a. $N(t)=8e^{-.021t}$ b. 6.5 g c. 33 years

43.

f. $\frac{1}{6}x^6-3x^{3/2}-2x^{-1/2}+4\ln x+C$

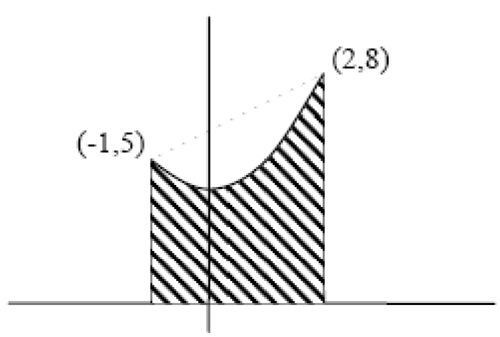
g. $4t+e^{-5t}+\frac{e^{2t}}{6}+C$

h. $6\frac{1}{6}=\frac{37}{6}$

i. $\frac{1}{4}e^{4x}-\frac{1}{4}$

j. $\frac{14}{3}$

44. a. 15 b. c. 5



45. Area = 36 units^2

46. Area = $\frac{81}{2}$ units^2

47. $f\left(x\right)=x^3+\ln x-4x+5$

48. $987.97

49. a. $2,421,741.24 b. $242,174.12

50. a. (3, 36) b. $63 c. $36

51. a. $\frac{1}{7}\left(x^3+8\right)^7$ b. $\ln\frac{9}{2}$

52. a. $\frac{\partial f}{\partial x}=5x^4+18xy+4y^3$ b. $\frac{\partial f}{\partial y}=9x^2+12xy^2$

53. Relative minimum of -7 at $\left(-3,3\right)$.

54. $y=\frac{15}{19}x+\frac{40}{19}$

Updated: Fall 2009
(J. Polm)