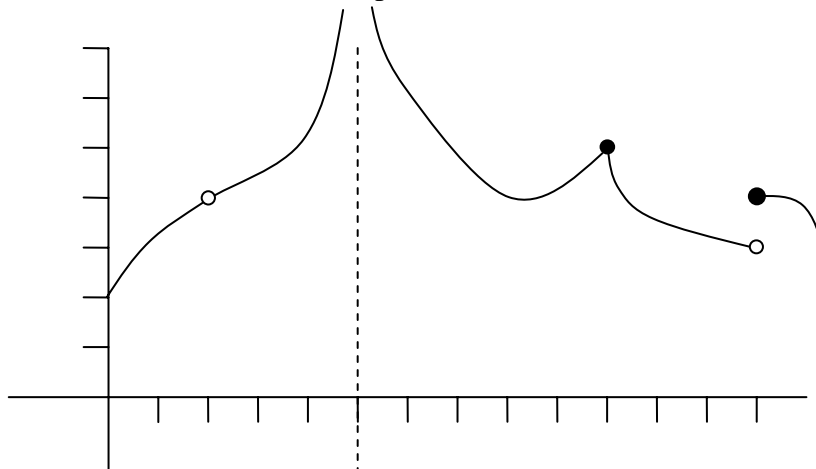


1. Use the information given about the function $f(x)$ in the sketch below to answer the questions. Assume that each tick mark represents one unit.



	Is $f(x)$ defined for this value of x ? If so estimate $f(a)$.	Find $\lim_{x \rightarrow a} f(x)$ if it exists.	Is $f(x)$ continuous at this value of x ?	Is $f(x)$ differentiable at this value of x ?
$a = 2$				
$a = 5$				
$a = 8$				
$a = 10$				
$a = 13$				

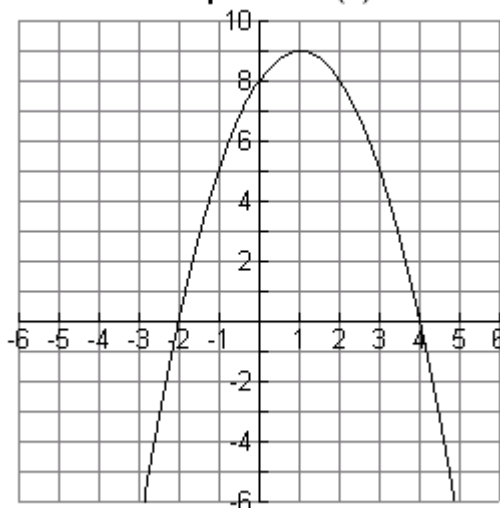
2. Use the definition of the derivative to find $f'(x)$ given that $f(x) = 3x^2 - x + 5$.
3. Find the equation of the line tangent to the curve $y = \sqrt{x^2 + 16}$ when $x = 3$ on the curve.
4. Given $f(x) = \frac{x^2}{x-3}$
- Find $f'(x)$
 - Find all critical values for the function
 - Determine whether the function has a relative maximum, a relative minimum, or neither at each critical value.

5. Differentiate the given functions:

(a) $f(x) = 9 - 4x^2 + 10x^{2/3}$	(f) $f(x) = \ln(x^2 - 2x + 5)^3$
(b) $f(x) = x^5(3x - 2)^7$	(g) $f(x) = e^{x^2} - \sqrt{x}$
(c) $f(x) = \sqrt[3]{x^2 - 1}$	(h) $f(x) = \frac{e^x}{1 + x^2}$
(d) $f(x) = \frac{1}{(x^2 + x + 1)^6}$	(i) $f(x) = \ln \left[\frac{(3x - 2)(4x^3 + 5)^6}{\sqrt{x^2 + 9}} \right]$
(e) $f(x) = \left[\ln(x^2 - 2x + 5) \right]^3$	(j) $f(x) = \frac{x^2 + 3}{3x^2 - 5}$

6. The graph of $f'(x)$, the **derivative** of a function $f(x)$, is shown. Note that the graph of $f(x)$ is not given. Based on the graph of $f'(x)$, answer the following questions about $f(x)$.

Graph of $f'(x)$



- On what interval(s) is $f(x)$ increasing?
- On what interval(s) is $f(x)$ decreasing?
- At what value(s) of x does $f(x)$ have a relative maximum?
- At what value(s) of x does $f(x)$ have a relative minimum?
- On what interval(s) is $f(x)$ concave up?
- On what interval(s) is $f(x)$ concave down?
- At what value(s) of x does $f(x)$ have a point of inflection?

7. The temperature of a person during an illness is given by

$F(t) = -0.1t^2 + 1.2t + 98.6$ where F is the temperature in degrees Fahrenheit at time t in days.

- Find the rate of change of the temperature with respect to time.
- Find $F(1.5)$ and write a sentence in everyday language explaining the meaning of your answer in the context of this situation.
- Find $F'(1.5)$ and write a sentence in everyday language explaining the meaning of your answer in the context of this situation.
- Why would the sign of $F'(t)$ be significant to a doctor?

8. The demand function for a certain product is $p(x) = 300(15 - \sqrt{x})$.

- Find the revenue function.
- Find the marginal revenue when 64 items are sold.
- For what value of x is the revenue a maximum?

9. A manufacturer of cameras finds that the price at which it can sell x cameras per week is $p(x) = 500 - x$ dollars. The total cost of producing x cameras per week is $C(x) = 150 + 4x + x^2$ dollars.
- Find the revenue function $R(x)$.
 - Find the profit $P(x)$.
 - Find the production level which maximizes the profit.
10. A tool rental company determines that it can achieve 500 daily rentals of jackhammers per year at a daily rental fee of \$30. For each \$1 increase in rental price, 10 fewer jackhammers will be rented. What rental price maximizes revenue?
11. A distributor of sporting equipment expects to sell 10,000 cases of tennis balls during the coming year at a steady rate. Yearly carrying costs (to be computed on the average number of cases in stock during the year), are \$10 per case and the cost of placing an order with the manufacturers is \$80. Determine the economic order quantity, that is, the order quantity that minimizes the inventory cost.
12. Solve the given equations for x :
- $3^{x-1} = 9$
 - $3e^{2x} = 21$
 - $\ln(x - 3) = 2$
13. Find the present value of \$20,000 payable in five years if money is invested at 6% compounded continuously.
14. For what values of x is the function $y = e^{2x}(x^2 - 6)$ increasing? For what values of x is this function decreasing?
15. If the equation $y = 100e^{0.03t}$ describes a certain population, where t is time in months, find
- The population when $t = 0$
 - The time it takes the population to grow to twice its original number [this is the “doubling time”]
 - The predicted population when $t = 70$ months and when $t = 120$ months.
16. The radioactive isotope iodine-131 has a half-life of 8 days.
- Find its decay constant.
 - Find the amount remaining after 10 days if initially there is 5 mg.
17. Given $f(x) = 2x^3 - 3x^2 - 12x + 1$
- Find $f'(x)$ and $f''(x)$.
 - Determine where $f(x)$ increases and where it decreases.
 - Find any relative maximum or minimum point.
 - Determine where $f(x)$ is concave up and concave down.
 - Find any points of inflection.
 - Sketch the graph of $f(x)$.

18. Suppose that $f(2) = 7$, $f'(2) = 0$, and $f''(2) = -4$. Describe the behavior of the function $f(x)$ at $x = 2$.
19. Find the antiderivative:
- (a) $\int (x^3 - 6x^2 + 2x - 1) dx$ (b) $\int \frac{5}{x} dx$ (c) $\int (4 - 5e^{-5t} + \frac{e^{2t}}{3}) dt$
20. Find the function that has derivative $f'(x) = 3x^2 + \frac{1}{x} - 4$ and whose graph contains the point $(1, 2)$.
21. Evaluate the given definite integrals:
- (a) $\int_2^3 (6 + \frac{1}{x^2}) dx$ (b) $\int_0^1 e^{4x} dx$ (c) $\int_1^4 \sqrt{x} dx$
22. Given the integral $\int_{-1}^2 (x^2 + 4) dx$
- (a) Calculate the area represented by the integral
 (b) Sketch the area represented by the integral
 (c) Find the average value of the function over the interval $-1 \leq x \leq 2$
23. Given the demand function $p = 1200 - 0.2x - 0.0001x^2$ find the consumer's surplus at $x = 500$.
24. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.
25. Given $f(x) = x + \frac{4}{x}$
- (a) Find $f'(x)$ and $f''(x)$.
 (b) Use the results from part (a) to find the relative maximum and minimum points on the graph of $f(x)$.
 (c) Sketch a graph of $f(x)$. Be sure to label the axes and indicate the scale used.
26. A rectangular garden of area 75 square feet is to be surrounded on three sides by a brick wall costing \$10 per foot and on one side by a fence costing \$5 per foot. Find the dimensions of the garden such that the cost of the materials is minimized.
27. Find the area bounded by the curves $y = x + 1$ and $y = x^2 - 3x - 4$.
28. Given $f(x, y) = 4xy^2 - 3x^3y + y^5$, find:
- (a) $\frac{\partial f}{\partial x}(x, y)$ (b) $\frac{\partial f}{\partial y}(x, y)$

ANSWERS

1. At $a=2$: $f(2)$ undefined, limit = 4, discontinuous, not differentiable
 At $a=5$: $f(5)$ undefined, no limit, discontinuous, not differentiable
 At $a=8$: $f(8)=4$, limit = 4, continuous, differentiable
 At $a=10$: $f(10)=5$, limit = 5, continuous, not differentiable
 At $a=13$: $f(13)=4$, no limit, discontinuous, not differentiable

2. $f'(x) = 6x - 1$

3. $y = \frac{3}{5}x + \frac{16}{5}$

4. (a) $f'(x) = \frac{x(x-6)}{(x-3)^2}$

(b) critical values are $x=0$ and $x=6$

(c) rel. max. at $(0,0)$; rel. min. at $(6,12)$

5.

(a) $f'(x) = -8x + \frac{20}{3}x^{\frac{1}{3}}$	(f) $f'(x) = \frac{6(x-1)}{x^2 - 2x + 5}$
(b) $5x^4(3x-2)^7 + 21x^5(3x-2)^6$ $= 2x^4(3x-2)^6(18x-5)$	(g) $f'(x) = 2xe^{x^2} - \frac{1}{2}x^{-1/2}$
(c) $f'(x) = \frac{2x}{3(x^2-1)^{2/3}}$	(h) $f'(x) = \frac{e^x(x-1)^2}{(1+x^2)^2}$
(d) $f'(x) = \frac{-6(2x+1)}{(x^2+x+1)^7}$	(i) $f'(x) = \frac{3}{3x-2} + \frac{72x^2}{4x^3+5} - \frac{x}{x^2+9}$
(e) $f'(x) = \frac{6(x-1)}{x^2-2x+5} \cdot [\ln(x^2-2x+5)]^2$	(j) $f'(x) = -\frac{28x}{(3x^2-5)^2}$

6. (a) $(-2, 4)$ (b) $(-\infty, -2)$ and $(4, \infty)$ (c) $x=4$ (d) $x=-2$
 (e) $(-\infty, 1)$ (f) $(1, \infty)$ (g) $x=1$

7. (a) $F'(t) = -0.2t + 1.2$

(b) $\approx 100.2^\circ$ After one and a half days, the person's temperature is about 100.2° F.

(c) 0.9° per day

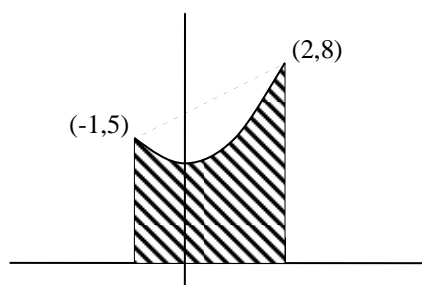
After one and a half days, the person's temperature is rising at a rate of about 0.9° per day.

8. (a) $R(x) = 300x(15 - \sqrt{x})$ (b) $R'(64) = \$900$ per item
 (c) $x=100$ items

9. (a) $R(x) = 500x - x^2$ (b) $P(x) = 496x - 2x^2 - 150$ (c) $x = 124$
10. \$40
11. 400 cases
12. (a) $x = 3$ (b) $x = \frac{\ln 7}{2}$ (c) $x = 3 + e^2$
13. $\approx \$14,815$
14. y is increasing for $x < -3$ and for $x > 2$; y is decreasing for $-3 < x < 2$
15. (a) 100 (b) 23 months (c) 817 and 3660
16. (a) .0866 (b) = 2.1 mg
17. (a) $f'(x) = 6(x-2)(x+1)$, $f''(x) = 6(2x-1)$
- (b) $x < -1$ inc. (d) $x < \frac{1}{2}$ down
 $-1 < x < 2$ dec. $x > \frac{1}{2}$ up
 $x > 2$ inc.
- (c) $(-1, 8)$ relative maximum, $(2, -19)$ relative minimum
- (e) $(\frac{1}{2}, -5\frac{1}{2})$ point of inflection
18. The function $f(x)$ has a relative maximum at the point $(2, 7)$.
19. (a) $\frac{1}{4}x^4 - 2x^3 + x^2 - x + C$ (b) $5\ln|x| + C$
- (c) $4t + e^{-5t} + \frac{e^{2t}}{6} + C$
20. $f(x) = x^3 + \ln x - 4x + 5$
21. (a) $6\frac{1}{6} = \frac{37}{6}$ (b) $\frac{1}{4}e^4 - \frac{1}{4}$ (c) $\frac{14}{3}$

22. (a) 15
(c) 5

(b)

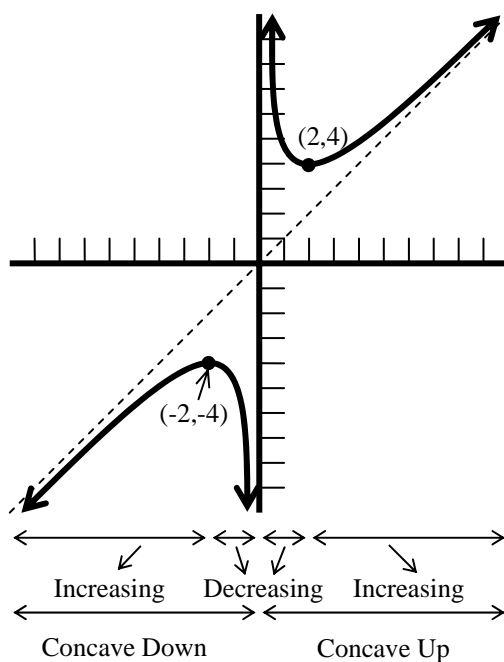


23. \$33,333.33

24. 1800 liters

25. (a) $f'(x) = 1 - \frac{4}{x^2}$ (b) rel max $(-2, -4)$
 $f''(x) = \frac{8}{x^3}$ rel min at $(2, 4)$

(c)



26. 7.5 ft. by 10 ft.

27. 36 sq. units

28. (a) $4y^2 - 9x^2y$

- (b) $8xy - 3x^3 + 5y^4$