MODULE 2

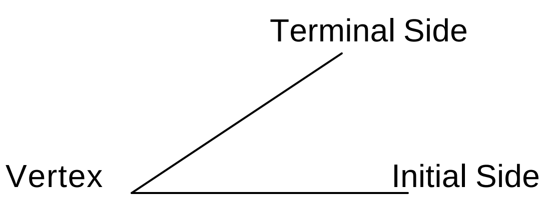
Margaret Latimer
HT 128

Angle Measure

"Trigonometry" means measure of trigons (3-sided polygons or triangles). Originally, the early Greeks developed trigonometry for use in astronomy, navigation, and surveying. Measurements that were impossible to make physically, could be calculated by employing an understanding of the relationships between the sides and angles of triangles. You have already learned how the ratios of sides of right triangles relate to interior angles.

By the 17th century, developments in physics and engineering lead mathematicians and scientists to understand these relationship as functions where the domain (input or x-values) could be any real number. (Recall that the set of Real Numbers includes integers, fractions, decimals and irrational numbers like p or $\sqrt{3}$.) By realizing that a real number can correspond to an angle-measure, trigonometry could be used to model wave motion (sound waves, light waves, etc.), vibrations, and orbits of planets as well as of subatomic particles.

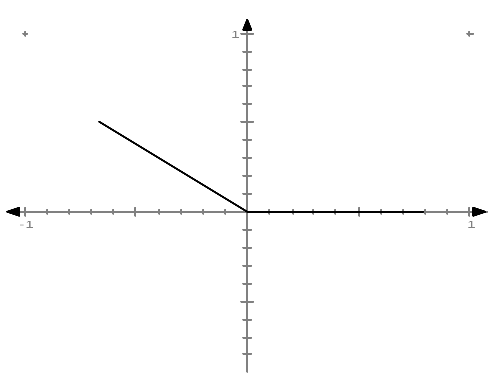
Let's define a few terms:
Rotating a **ray (**a half-line) about an endpoint forms an angle.



The **angle** is the measure of how much rotation occurs. The original position of the ray is **the initial side**. The final position is the **terminal side**. The common endpoint (where the initial and terminal sides intersect) is the **vertex**.

If an angle is drawn in a rectangular (Cartesian) coordinate system so that its initial side is drawn from the origin along the positive x-axis, then the angle is said to be in **standard position**. The vertex of an angle in standard position is __________________, and has coordinates (__,__).

(the origin) (0, 0)



?? If an angle has a vertex at the origin, is the angle necessarily in standard position? ________________ (No)

If the ray is rotated in a **counterclockwise** direction, the angle and the rotation are said to be **positive**.

If the ray is rotated in a **clockwise** direction, the angle and the rotation are said to be **negative**.

If the initial and terminal sides of two or more angle are the same, then the angles are said to be **coterminal.**

?? Sketch a positive angle, a negative angle, and two coterminal angles.

?? Does the angle measure of two coterminal angles have to be the same? (No)

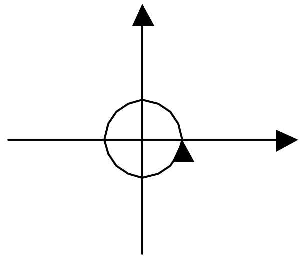
Review a little geometry.

- A right angle measures _________ degrees.
- A straight angle measures _________ degrees.
- An angle, a (alpha), is defined as___________ if its measure is $0° < a < 90°$.
- If an acute angle is drawn in standard position, the terminal side lies in the _______________ quadrant.
- An angle, b (beta), is defined as ______________________if its measure is $90° < b < 180°$.
- If an obtuse angle is drawn in standard position, the terminal side lies in the ________________quadrant.
- Two angles are ________________if the sum of their angles is 90°.
- Two angles are ________________if the sum of their angles is 180°.
- The acute angles of a right triangle are _________________.

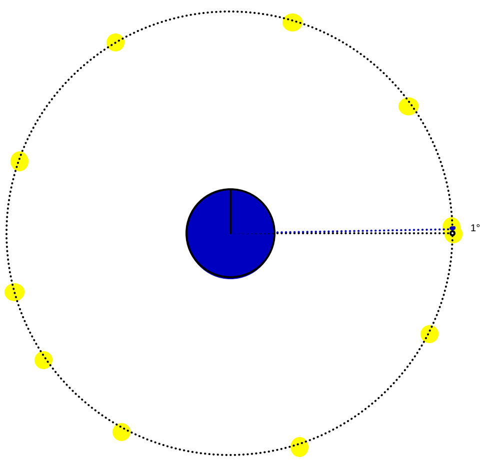
(Answers: 90°, 180°, acute, first, obtuse, second, complementary, supplementary, complementary)

Degrees and Radians

If the initial side of an angle is rotated counterclockwise until it coincides with itself, the measure of the angle is 360°. This is also referred to as 1 revolution. (Consider a spinner in a game or one hand of a clock. Imagine that the spinner or hand on the clock moves around a circle in a counterclockwise direction, exactly one time, ending exactly where it started. The angle defined by this motion is 360°).



We have inherited the notion of 360° in one revolution from the Babylonians, who defined a year as 360 days. They believed that each day the sun was moving 1/360 of the distance that it would eventually move in one year. We refer to the angle measure defined by this motion as 1 degree or 1°. It was discovered much later that the orbit is



not exactly circular.

Fractions of a degree can be defined in one of three ways. Common fractions and decimal notation are familiar to you. For example, we can write 25 ½° or 137.204°. The third system, referred to as DMS or degree-minute-second, subdivides each degree into 60 minutes and subdivides each minutes into seconds. DMS measurements are written using the symbols ° (degree),
’ (minutes), and “(seconds). For example: 52°36’47”. This is equivalent to

$\left(52+\frac{36}{60}+\frac{47}{3600}\right)$ degrees or approximately 52.613°

?? Can you explain why the denominators are 60 and 3600?

Equivalent angles can be written using any method: 10 ½° = 10.5° = 10° 30'
Most graphing calculators will allow you to do calculations using the DMS system, as well as convert from DMS to decimals and from decimals to DMS.

Convert 52°36'47" to a decimal using a TI-83 or TI-83Plus by entering the following key strokes.

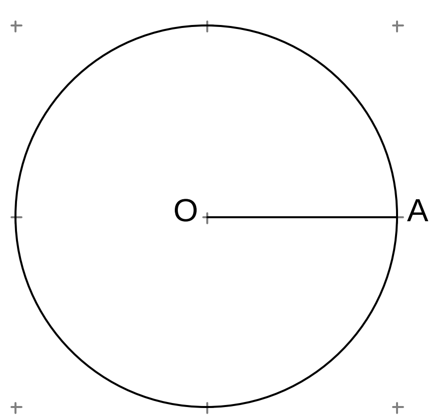
5	2	2nd	ANGL E	1	3	6	2nd	ANGLE	2	4	7	ALPH A	"	ENTE R

The decimal equivalent is 52.613, rounded to three places. Since 36' is more than half of a degree, does it make sense that the answer is more than 52.5° ?

Notice that the degree and minute symbols require the ANGLE menu, but the second symbol ALPH "

To convert from decimal notation to DMS is a little easier on your fingers.
Enter 52.613. You do not need to hit ENTER, but it doesn't matter is you do.
After keying in the number that you want to convert, 2nd ANGL 4

Just as some people measure the temperature using the Fahrenheit scale (°F) and others use the Celsius or centigrade scale (°C), angle measure has a second scale of measure. Consider the circle shown below, with the radius indicated.



Take a piece of string, or anything that you can curve slightly, and measure the length of the radius. Don't worry about standard units of measure like inches or centimeters. Call your radius length 1 unit. Starting where the radius intersects the circle, A, measure, in a counterclockwise direction, 1 unit along the circumference of the circle. (This is why you need something flexible. You need to measure a curved line). Mark the point on the circle that indicates a distance along the circumference of 1 unit and label the point B. Draw a line segment that connects point B and point O (the center of the circle). There are several things worth noting and learning.

- The angle that you have just created, $\angle AOB$, with a vertex at the center of the circle, is called a **central angle.** In just a minute we'll talk about how to define the measure of that angle.

- The part of the circumference of the circle that lies between A and B is called an **arc.** It can be written using this notation: $\overset{\frown}{AB}$, which is read as "arc length AB" or simple "arc AB". You can say that $\overset{\frown}{AB}$ **subtends** the central angle.

- The measure of the central angle that you have drawn is **1 radian**. That is, a central angle that is subtended by an arc whose length is the same as the length of the radius is defined as 1 radian. (You could take a protractor and measure the angle in degrees, but we have just defined a new unit of measure. You know what 1 degree is, you know what 1 foot is, or 1 gallon. Now you know what 1 radian is!) **??** If you were to draw a larger or smaller circle, measure the radius, measure an arc length equal to the radius, and draw the central angle subtended by that arc, would the measure of the central angle vary? Why **not**?

- Any angle whose vertex is at the center of a circle, and whose initial and terminal sides intersect the circumference of the circle, is a central angle. The arc (part of the circumference) that lies between the intersections, and associated with the measure of the central angle is said to subtend that angle.

Fix highlighted arc

On your circle, starting at point B, measure another arc that has a length of 1 unit (the length of the radius). Label the point on the circumference, D. What is the measure of

$\angle$AOD? ___________ . Keep measuring and marking 1 unit lengths on your circle. Approximately how many radius-length units are there in a circle? _________________. (Another way to think of this is to imagine cutting the circle, straightening it out, and thinking of the line as a number line. To number, or calibrate, the line, measure units of length that are equal to the radius). If you divide the circumference of a circle by the radius, what number do you get?

If you could have measured accurately, you would have found that there are approximately 6.28 radius lengths in a circle. We know this to be 2 p .
Recall the formula for the circumference of a circle: $C = 2pr$
Divide both sides by the radius, r, and the result is C/r = $2p$
Above, you divided your circumference by r. You found approximately 6.28 radiuses in a circumference. What is 2 x p ? ___________

The radian measure, q, of any angle is the ratio of the arc length,s, to the radius r.

$$q = \frac{s}{r}$$

Solve for the arc length, s. ___________________
When using either of these equations, consistent units must be used. That is, the lengths, (arc length and radius) must be measured in the same units (feet, mm, miles, etc.). When you divide one number by another and they have the same units, what are the units of the resulting quotient? _____________.
So, do radians have units? ______________

Above, we defined the length of the radius as 1 unit and you measured an arc length, s, equal to 1 unit. What is the central angle defined by the arc if s = r?

$$\theta = \frac{s}{r} = \frac{\square}{\square} = \_\_\_\_\_$$

Recall that we defined 1 revolution as 360°. If we do the same thing, but make the measurement in radians, one revolution is defined as ______________.

Using the fact that 360° = $2p$, conversion factors can be derived that allow us to convert radian measures to degrees and degree measures to radians.

Start with what we know: 360° = $2p$ Both are expressions for 1 revolution.
Divide both sides by 360 (not 360°, just 360. Remember that the units have to work out, too!)

$$\frac{360°}{360} = \frac{2p}{360}$$

Simplify by reducing. $1° = \frac{p}{180}$.

To convert from degrees to radians, use the fact that 1° = p /180 or 0.001745 radians and multiply degrees by p /180.

For example, 90° = 90° x $\frac{p}{180}$ = $\frac{p}{2}$ radians. Because $\frac{p}{2}$ is an exact value, numbers are often expressed using p . You can substitute 3.14 and find that 90° is approximately equal to 1.57 radians, or simply 1.57. Keep in mind that this is an approximation, so we have introduced some error. In trigonometry, as well as in many science and engineering applications, symbols that represent exact values are often used, even in a final answer. You've seen this with radicals. $\sqrt{2}$ represents an exact value. 1.414 is an approximation. If you square $\sqrt{2}$, you get exactly 2. If you square 1.414, you get ____________. So make friends with notation that include p . We've already seen an example above. What decimal approximation did we use for $2p$?

____