TRIGONOMETRY: MODULE ONE

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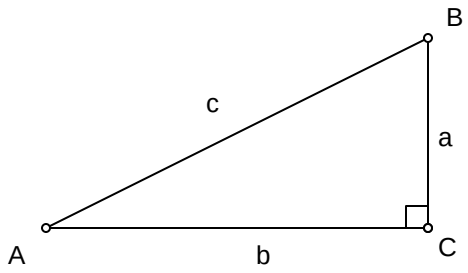
HT 133

Basic Trig Functions

Traditionally, a study of trigonometry begins with a consideration of right triangles, using the familiar method of measuring angles in degrees. It is also traditional to label the right angle "C" (upper case). The opposite side is therefore labeled "c" (lower case) , and is called the "hypotenuse" . The two acute angles are labeled with capital letters "A" and "B". The sides opposite them are called "legs" and are labeled "a" and "b", respectively. In this module we will confine our discussion to the acute angles only.

Right triangle trigonometry requires the student to have a clear understanding of the words "opposite" and "adjacent". These words designate relationships. Just as we cannot be everyone's "neighbor", no one side of a right triangle is <u>always</u> opposite or adjacent. We first have to be clear about to which angle we are referring.

Here is a triangle in "standard position" :



The first ratio that we will examine is the length of the side <u>opposite</u>
an angle compared to the length of the hypotenuse. This ratio is known as the "sine" (sin) of the angle.

For angle A :

sine of A = length of opposite side divided by length of the hypotenuse,

or

$\sin A = a/c$

For angle B: $\sin B = b/c$

The second ratio which we will examine is the "cosine" (abbreviated "cos"). Now we are comparing the length of the side adjacent to a chosen angle to the length of the hypotenuse. For angle "A", the adjacent side is "b"—remember that "c" is the hypotenuse.

cos A = b/c and cos B = a/c

Our third ratio is the "tangent" (abbreviated "tan"). Here we will compare the length of the opposite side compared to the adjacent side:

tan A = a/b and tan B = b/a

These are the three basic trigonometric functions. Their values depend upon the size of the angle being used. Therefore,
sin A, cos A and tan A are all functions of angle A. That is why the terms "sin", "cos" and "tan" have no meaning, or value, unless we designate the angle that we are using.

Reciprocal Trig Functions

We have reciprocals of the three basic trig functions:

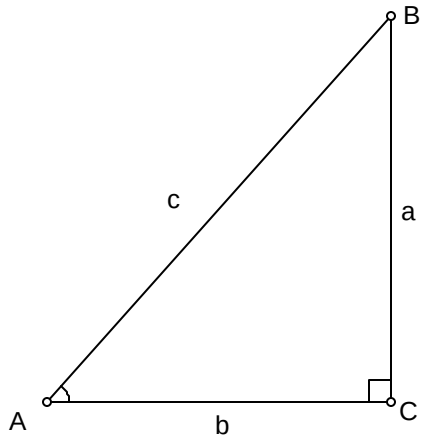
cosecant (csc) A = reciprocal of sin A, so csc A = c/a
secant (sec) A = reciprocal of cos A, so sec A = c/b
cotangent (cot) A = reciprocal of tan A, so cot A = b/a

Remember, these are functions also. They depend upon the size of the included angle. No matter how large a triangle we use,
all 30° angles, for example, have the same values for the three basic and the three reciprocal trigonometric functions. Since we are using ratios, these functions do NOT depend upon the size of the triangle. It really helps to know about similar triangles, which are discussed on pages 355 and 356 of your textbook.

Solving Right Triangles

When we "solve" a right triangle we know the measure of the two acute angles and the length of all three sides. In a right triangle the sum of the two acute angles is 90 °, so knowing the measure of one will allow us to easily calculate the measure of the second acute angle. When neither acute angle's measure is known we must use the "inverse trig functions" which will be included in a later module.

Example 1 :



C = 90° A = 48.21° If *a* = 7.2 m, find *b* and *c*.

Since *a* is opposite angle A we can use the sine function to determine the length of *c*. This approach is not the only way to solve this triangle—we can often find more than one way to solve a right triangle. Many students unnecessarily worry about selecting the "best" method. Find an approach and solve !

sin A = *a*/*c*
sin 48.21° = 7.2/*c*
c sin 48.21° = 7.2
c = 7.2 / sin 48.21°
c = 9.66 m (to hundredths' place)

It is a good idea to use the data that you are most sure of—the values given at the beginning of the exercise. Therefore, let's use the tangent function to find side *b*. The tangent of A uses the opposite side (*a*), whose measure we know, and the adjacent side (*b*).

tan A = *a*/*b*
tan 48.21° = 7.2 / *b*
b tan 48.21° = 7.2
b = 7.2 / tan 48.21°8/28/028/28/02
b = 6.44m (to hundredths' place)

Lastly, B = 90° – 48.21°, or B = 41.79°. We have now solved triangle ABC.