Montgomery College
Department of Mathematics
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Lesson 19: Factoring a Polynomial Completely

A polynomial is *factored completely* if each factor with more than one term is *prime* and cannot be factored further.

Example 1. $2x^2 - 4x - 30 = (2x + 6)(x - 5)$ is a true statement, but the factoring is not complete. The factor $2x + 6$ can be factored further, and the complete factorization is $2x^2 - 4x - 30 = 2(x + 3)(x - 5)$.

Example 2. $6x^4 + 24x^2 = 6x^2(x^2 + 4)$ is completely factored. The monomial factor $6x^2$ could be written $2 \cdot 3 \cdot x \cdot x$, but this is rarely done. **NOTE**: The binomial factor $x^2 + 4$ can ***not*** be factored.

A FACTORING SCHEME

(1) *Always* factor out common monomial factors FIRST.
(This will simplify additional factoring that may be necessary.)

(2) Look for special patterns:
The difference of squares: $A^2 - B^2 = (A + B)(A - B)$
The sum of squares: $A^2 + B^2$ ***never*** factors

(3) Try factoring by "trial and error" (the reverse of FOIL).

BEFORE YOU STOP

(4) Make sure all binomial and trinomial factors are prime.

(5) Check your factoring by multiplication.

More Examples

Example 3. $2x^2 - 8x + 8 = 2(x^2 - 4x + 4) = 2(x-2)^2$

Example 4. $3x^3 + 9x^2 - 12x = 3x(x^2 + 3x - 4) = 3x(x+4)(x-1)$

Example 5. $x^2 - 5x - 36 = (x-9)(x+4)$

Example 6. $x^2 - 9x + 39$ cannot be factored

Example 7. $2x^2 + 9x + 4 = (2x+1)(x+4)$

Example 8. $4x^2 - 100 = 4(x^2 - 25) = 4(x+5)(x-5)$

Example 9. $4x^2 - 25 = (2x+5)(2x-5)$

Practice Problems

Factor each of the following expressions completely.

1. $x^2 + 8x + 16$
2. $x^2 + 10x + 20$
3. $x^2 - 15x + 36$
4. $4x^2 - 36$
5. $9x^2 + 36$
6. $3x^2 - 21x$
7. $4x^2 - 12x + 9$
8. $4x^2 - 9$
9. $2x^2 + 4x - 6$
10. $2x^2 + x - 3$
11. $x^3 + 2x^2 + x$
12. $50x^2 - 8$
13. $9x^3 + 25x$
14. $16x - 4x^2$
15. $6x^2 + x - 2$
16. $6x^2 - 4x - 2$
17. $3x^4 - 12x^2$
18. $6x^3 - 3x^2 - 3x$
19. $4x^5 - 16x^4 + 16x^3$
20. $20x^2 - 2x - 4$

Answers

1. $(x+4)^2$
2. *not factorable*
3. $(x-3)(x-12)$
4. $4(x+3)(x-3)$
5. $9(x^2+4)$
6. $3x(x-7)$
7. $(2x-3)^2$
8. $(2x+3)(2x-3)$
9. $2(x+3)(x-1)$
10. $(2x+3)(x-1)$
11. $x(x+1)^2$
12. $2(5x+2)(5x-2)$
13. $x(9x^2+25)$
14. $4x(4-x)$
15. $(3x+2)(2x-1)$
16. $2(3x+1)(x-1)$
17. $3x^2(x-2)(x+2)$
18. $3x(x-1)(2x+1)$
19. $4x^3(x-2)^2$
20. $2(2x-1)(5x+2)$

When you have successfully factored each of these 20 polynomials, you may ask your instructor for the paper and pencil quiz for this lesson. Take the quiz in class and return it to your instructor.