I am trying to work my way through "Geometric Algebra for Physicists" with several other physicists, astronomers, and mathematitions (I volunteered to lead them) and in doing so fill in the gaps in any proofs (the parts left to the student such as the boundary of a boundary is zero, etc.), but I have hit a brick wall with respect to equation 6.134 (exercise 6.6). I cannot get $\frac{1}{k!} b\cdot(e_1\wedge\cdots\wedge e_n)$ for the r.h.s. of equation 6.134. I keep getting $\frac{1}{(k-1)!} b\cdot(e_1\wedge\cdots\wedge e_n)$ (Also in equation 6.135 should the k be an n. I did not find that in the errata for the book). I would greatly appreciate if you could look at my derivation an explain where I am making a mistake.

I Begin with the definition

$$\mathsf{C} = \sum_{i=0}^{n} (-1)^i\, b\cdot(x_0+\cdots+\breve{x}_i+\cdots+x_n)\,\Delta\,(\breve{x}_i)_{(k-1)} = \sum_{i=0}^{n}\mathsf{C}_i \tag{1}$$

where C_i is defined by

$$\mathsf{C}_i = \left\{ \begin{array}{cc} i=0: & b\cdot(x_1+\cdots+x_n)\,\Delta\,(\breve{x}_0)_{(k-1)} \\ 0<i\le n: & (-1)^i\, b\cdot(x_0+\cdots+\breve{x}_i+\cdots+x_n)\,\Delta\,(\breve{x}_i)_{(k-1)} \end{array} \right\} \tag{2}$$

now define $E_n = e_1\wedge\cdots\wedge e_n$ so that (using the derivation from the section in $D\&L$ on reciprocal frames)

$$(-1)^{i-1} e^i E_n = e_1\wedge\cdots\wedge\breve{e}_i\wedge\cdots\wedge e_n, \quad \forall\, 0<i\le n \tag{3}$$

and

$$\mathsf{C}_{0<i\le n} = \frac{-1}{(k-1)!} b\cdot(x_0+\cdots+\breve{x}_i+\cdots+x_n)\, e^i E_n \tag{4}$$

The main problem in evaluating C_i is that

$$(\breve{x}_0)_{(k-1)} = \frac{1}{(k-1)!}(x_2-x_1)\wedge\cdots\wedge(x_n-x_1) \tag{5}$$

using $e_i = x_i - x_0$ reduces equation 5 to

$$(\breve{x}_0)_{(k-1)} = \frac{1}{(k-1)!}(e_2-e_1)\wedge\cdots\wedge(e_n-e_1) \tag{6}$$

but equation 6 can be expanded into equation 7. The critical point in doing the expansion is that in generating the sum on the r.h.s. of the first line of equation 7 all products containing x_1 (of course all terms in the sum contain x_1 exactly once since we are using the outer product) are put in normal order by bringing the x_1 factor to the front of the product thus requiring the factor of $(-1)^i$ in each term in the sum.

$$\begin{aligned} (e_2 - e_1) \wedge \cdots \wedge (e_n - e_1) &= e_2 \wedge \cdots \wedge e_n - \sum_{i=2}^{n} (-1)^i \, e_1 \wedge e_2 \wedge \cdots \wedge \breve{e}_i \wedge \cdots \wedge e_n \\ &= \sum_{i=1}^{n} (-1)^{i-1} \, e_1 \wedge e_2 \wedge \cdots \wedge \breve{e}_i \wedge \cdots \wedge e_n \\ &= \sum_{i=1}^{n} e^i E_n \end{aligned} \tag{7}$$

or

$$(\breve{x}_0)_{(k-1)} = \frac{1}{(k-1)!} \sum_{i=1}^{n} e^i E_n \tag{8}$$

from equation 7 we have

$$\begin{aligned} \mathsf{C} &= \frac{1}{(k-1)!} \sum_{i=1}^{n} \left(b \cdot (x_1 + \cdots + x_n) - b \cdot (x_0 + \cdots + \breve{x}_i + \cdots + x_n) \right) e^i E_n \\ &= \frac{1}{(k-1)!} \sum_{i=1}^{n} \left(b \cdot (x_i - x_0) \right) e^i E_n \\ &= \frac{1}{(k-1)!} \sum_{i=1}^{n} (b \cdot e_i) \, e^i E_n \\ &= \frac{1}{(k-1)!} \sum_{i=1}^{n} b_i e^i E_n \\ &= \frac{1}{(k-1)!} b E_n \\ &= \frac{1}{(k-1)!} (b \cdot E_n + b \wedge E_n) \\ &= \frac{1}{(k-1)!} b \cdot E_n \end{aligned} \tag{9}$$

any suggestions you make would be greatly appreciated.