

The shape of “dark matter” halos in disc galaxies according to the modified dynamics

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ABSTRACT

Analyses of halo shapes for disc galaxies are said to give incongruous results. I point out that the modified dynamics (MOND) predicts for disc galaxies a distribution of fictitious dark matter that comprises two components: a pure disc and a rounder halo. The former dominates the true disc in regions of small accelerations, where it controls the z-dynamics in the disc (disc flaring etc.); it has a finite total mass. It also dominates the round component near the centre where the geometry is nearly planar. The second component controls motions far from the plane, has a total enclosed mass that diverges linearly with radius, and determines the rotation curve at large radii. Its ellipticity may be appreciable at small radii but vanishes asymptotically. This prediction of MOND differs from what one expects from galaxy-formation scenarios with dark matter. Analyses to date, which, as they do, assume one component—usually with a constant ellipticity, perforce give conflicting results for the best-value ellipticity, depending on whether they probe the disc or the sphere, small radii or large ones.

Subject headings: gravitation-galaxies: halos, kinematics and dynamics

1. Introduction

The modified dynamics (MOND) predicts that the global structure of the Newtonist’s “dark halo” in disc galaxies is rather more complex than is usually assumed for such halos. It has a disc component and a rounder component with radius-dependent flattening, becoming spherical at large radii. This structure is at odds with what one naturally expects from existing halo-formation scenarios under the dark-matter doctrine.

The purpose of this paper is to point out the general properties of MOND “halos” with the help of analytic results for simple models and situations.

In section 2 I explain how fictitious dark matter arises in MOND and describe its distribution for thin-disc galaxies. Exact expressions for the “dark-matter” distribution in Kuzmin discs, for which the MOND problem can be solved exactly, are given in section 3. In section 4 I discuss generalizations of these results; in particular, the expected z -distribution of the “dark matter” within the disc. Section 5 touches briefly on implications for halo-shape analyses.

2. Fictitious dark matter according to MOND

I work with the formulation of MOND as a modification of Newtonian gravity given in Bekenstein & Milgrom (1984) whereby the gravitational potential, ψ , of a density distribution ρ^* satisfies

$$\vec{\nabla} \cdot [\mu(|\vec{\nabla}\psi|/a_o)\vec{\nabla}\psi] = 4\pi G\rho^* \quad (1)$$

instead of the Poisson equation; here, a_o is the acceleration constant of MOND, $\mu(x) \approx x$ for $x \ll 1$, and $\mu(x) \approx 1$ for $x \gg 1$. A Newtonist, who assumes that ψ solves the Poisson equation, will deduce a mass density $\rho = (4\pi G)^{-1}\Delta\psi$, and will thus find that he requires “dark matter” with density

$$\rho_D = (4\pi G)^{-1}\Delta\psi - \rho^*. \quad (2)$$

Here I investigate the distribution of this fictitious mass density for disc galaxies. For the sake of concreteness I consider a thin, axisymmetric, pure-disc model. The generalization to a galaxy model containing a bulge is straightforward and, in any case, does not qualitatively change the results.

From eqs. (1) and (2) with $\rho^* = 0$, the “dark matter” has a component outside the disc

$$\rho_D = -(4\pi G)^{-1}\vec{\nabla} \ell n \mu(|\vec{\nabla}\psi|/a_o) \cdot \vec{\nabla}\psi \quad (3)$$

It is asymptotically spherical with $\rho_D \rightarrow (4\pi)^{-1}(Ma_o/G)^{1/2}R^{-2}$ at large radii R ; M is the total mass of the disc. This is the component that gives the asymptotically flat rotation curve. There is also a thin-disc component of “dark matter” because the jump condition on the potential is different from the Newtonian jump for the same surface density. In MOND, we have across a thin layer of true surface density Σ^*

$$\delta[\mu(g/a_o)g_z] = 4\pi G\Sigma^*, \quad (4)$$

where $\mathbf{g} = -\vec{\nabla}\psi$ is the (MOND) acceleration, g_z its component perpendicular to the disc, and $\delta[]$ is the jump from one side of the disc to the other. A Newtonist who measure the

the acceleration field will deduce a surface density $\Sigma = (4\pi G)^{-1}\delta[g_z]$. For the $\pm z$ -symmetric case, which I assume all along, we have

$$\Sigma = \Sigma^*/\mu(\mathbf{g}^\pm/a_o), \quad (5)$$

or a “dark-matter” surface density

$$\Sigma_D(r) = [1/\mu(\mathbf{g}^\pm/a_o) - 1]\Sigma^* \equiv \eta(r)\Sigma^*(r), \quad (6)$$

where \mathbf{g}^\pm is the total (MOND) acceleration just outside the disc. It follows from the basic assumptions of MOND that $\eta > 0$, that it is very small where $\mathbf{g} \gg a_o$, and that asymptotically $\eta \propto r$. We also see, generally, that Σ_D is non zero only where there is a true disc. To get $\eta(r)$ we first have to solve the MOND equation for \mathbf{g} .

Note that for a one-dimensional planar mass layer all the “dark matter” lies in the layer itself; outside, the MOND acceleration is constant and so gives $\rho_D = 0$. This means that near the centre of the disc, where the geometry is nearly planar, most of the dark matter lies in the disc and the equipotentials are highly flattened. The Newtonist’s central surface density of the disc is

$$\Sigma_0 = \nu(\tilde{y})\Sigma_0^* > \Sigma_0^*, \quad (7)$$

where $\tilde{y} = 2\pi G\Sigma_0^*/a_o$, Σ_0^* is the true central surface density; the function $\nu(y)$ is defined by $\nu(y) \equiv I^{-1}(y)/y$, where $y = I(x) \equiv x\mu(x)$ (so we have $\nu(y) = 1/\mu[x(y)]$). The argument of $\nu(y)$ is always of the form \mathbf{g}_N/a_o , where \mathbf{g}_N is some Newtonian acceleration, while x is of the form \mathbf{g}/a_o where \mathbf{g} is a MOND acceleration. Regardless of the exact form of $\mu(x)$ we have $\nu(y) \approx 1$ for $y \gg 1$, and $\nu(y) \approx y^{-1/2}$ for $y \ll 1$.

For the widely used form of $\mu(x) = x(1+x^2)^{-1/2}$ we have

$$\nu(y) = [1/2 + (y^{-2} + 1/4)^{1/2}]^{1/2}. \quad (8)$$

Equation(7) is an example where the dark-matter factor $1/\mu(x)$, as appears e.g. in eqs.(3) and (5), calculated at some MOND field value $x = \mathbf{g}/a_o$, can be expressed in terms of the Newtonian-field variable \mathbf{g}_N/a_o . This is possible for all one-dimensional configurations where an exact algebraic relation exists between the Newtonian and MOND accelerations $\mathbf{g}_N = -\vec{\nabla}\varphi$ and $\mathbf{g} = -\vec{\nabla}\psi$ respectively:

$$\mu(\mathbf{g}/a_o)\mathbf{g} = \mathbf{g}_N, \quad (9)$$

or inverting, the MOND acceleration is given by

$$\mathbf{g} = \nu(\mathbf{g}_N/a_o)\mathbf{g}_N. \quad (10)$$

I now demonstrate all the above with analytic expressions for Kuzmin discs.

3. Analysis of the Kuzmin disc

Kuzmin discs have a Newtonian gravitational potential

$$\varphi = -MG/[r^2 + (|z| + h)^2]^{1/2}, \quad (11)$$

where M and h are the mass and scale length of the disc (we use cylindrical coordinates r, z, θ). The potential above the disc ($z > 0$) is that of a point mass M at $-\mathbf{h} \equiv (0, -h, 0)$; the potential below the disc is produced by the same mass at \mathbf{h} . The surface density, $\Sigma_K(r)$, matches the jump in the z -gradient of the potential

$$\Sigma_K(r) = (2\pi G)^{-1} \left. \frac{\partial \varphi}{\partial z} \right|_{z=0^+} = Mh/2\pi(r^2 + h^2)^{3/2} = \Sigma_0(1 + u^2)^{-3/2}, \quad (12)$$

where $\Sigma_0 = M/2\pi h^2$, and $u = r/h$. Everywhere outside the disc the equipotential surfaces are concentric spheres centred at $\pm\mathbf{h}$. Because of this one readily gets the exact MOND solution (Brada & Milgrom 1995). There the algebraic relations (9)(10) hold exactly, where here we now have

$$\mathbf{g}_N(\mathbf{R}) = -MG(\mathbf{R} \pm \mathbf{h})/|\mathbf{R} \pm \mathbf{h}|^3 \quad (13)$$

for the Newtonian acceleration field above (+) and below (–) the disc. We use the parameter $\zeta \equiv MG/h^2 a_o$ to measure how deep in the MOND regime the disc is. For $\zeta \ll 1$ the whole disc is in the MOND regime, and we can write everywhere outside the disc

$$\psi \approx (MGa_o)^{1/2} \ln[r^2 + (|z| + h)^2]^{1/2}. \quad (14)$$

Otherwise this expression is only valid asymptotically (for $R/h \gg \zeta^{1/2}$).

The Newtonist's mass distribution comprises a disc whose surface density is

$$\Sigma(r) = \nu(y)\Sigma_K(r), \quad (15)$$

where

$$y(r) \equiv g_N(r, 0^+)/a_o = MG/a_o(r^2 + h^2) = \zeta/(1 + u^2). \quad (16)$$

So for Kuzmin discs

$$\eta(r) = \{\nu[\zeta/(1 + u^2)] - 1\}. \quad (17)$$

For large values of y (the Newtonian regime), as are found in the inner parts of a disc with $\zeta \gg 1$, we have $\eta \ll 1$. When $\zeta \ll 1$, or at large radii for all ζ , we have

$$\eta(r) \approx \sqrt{\frac{1 + u^2}{\zeta}} - 1 \gg 1, \quad (18)$$

and the “dark disc” dominates. (For the Kuzmin disc where $\Sigma_K(r) \propto r^{-3}$ at large r , $\Sigma_D \propto r^{-2}$ there, and the “dark disc” mass diverges logarithmically.) For the choice $\mu(x) = x(1+x^2)^{-1/2}$ we have from eq.(8) $\eta = [1/2 + (y^{-2} + 1/4)^{1/2}]^{1/2} - 1$.

In addition we need outside the disc a rounder component of ”dark-matter” with density

$$\rho_D(r, z) = -(4\pi G)^{-1} \vec{\nabla} \cdot \mathbf{g} = -\frac{M}{2\pi h^3} \zeta^{-3/2} \hat{y}^{5/2} \nu'(\hat{y}), \quad (19)$$

where $\hat{y} = g_N/a_o = \zeta[u^2 + (1+v)^2]^{-1}$, $v = |z|/h$, and $\nu'(y) = d\nu(y)/dy$. We have $\rho_D \propto R^{-2}$ asymptotically. The equi-density surfaces of this distribution coincide with the equipotentials of ψ and φ : they are spheres centred at $\pm \mathbf{h}$: all quantities are functions of $u^2 + (1+v)^2$. As expected the eccentricities are large near the centre ($u, v \lesssim 1$).

4. Generalizations

For a thin disc with an exponential surface density profile the algebraic relation between Newtonian and MOND accelerations, eqs.(9) and (10), is a very good approximation (as shown numerically in Brada & Milgrom 1995). Expression (17) for η is thus still a good approximation with $y(r) = g_N(r, 0^+)/a_o$; but g_N is now the Newtonian acceleration for the exponential disc. Using the analytic expression for g_r from Freeman (1970), I find $y(r) = \zeta[\exp(-2s) + q^2(s/2)]^{1/2}$, with $s = r/h$, $q(x) = x[I_0(x)K_0(x) - I_1(x)K_1(x)]$, and I_i, K_i modified Bessel functions.

I have so far discussed only the z-integrated “dark-matter” surface density. We need to know how it is distributed in z to predict dynamics in the disc. In the limit where the disc is very thin the term with the radial part of the divergence in eq.(1) can be neglected in the bulk of the disc (where $dg_z/dz \gg dg_r/dr$), and we write

$$\rho^* \approx (4\pi G)^{-1} \frac{\partial}{\partial z} [\mu(g/a_o)g_z]. \quad (20)$$

Or, we can turn this into an expression for the integrated surface density between $\pm z$: $\Sigma^*(r, z) \approx (2\pi G)^{-1} \mu(g/a_o)g_z$; $g = (g_r^2 + g_z^2)^{1/2}$. In our approximation g_r is independent of z , but g_z varies strongly with z . The Newtonist’s integrated surface density is $\Sigma(r, z) \approx (2\pi G)^{-1} g_z = \Sigma^*(r, z)/\mu(g/a_o)$. Just outside the disc g_z becomes independent of z and we get the saturated ratio of Newtonist-to-true surface density as in eq.(5). If we are in a regime where $g_r \gg g_z$ for all z (this is generally valid beyond a few scale lengths) than g , and hence μ , is approximately z independent, and the “dark matter” z distribution is proportional to that of the true density. Otherwise the “dark matter” is more concentrated near the mid-plane than the true density, since g_z , and hence μ , increases with z .

5. Discussion

In recent papers Sackett (1999), Olling & Merrifield (2000), and Ibata & al (2000) note that different techniques for estimating the ellipticity of galactic dark halos give systematically different results. These analyses were applied to different galaxies, so, strictly speaking, are not in conflict with each other. It is, however, alarming that the degree of flattening deduced is so strongly correlated with the type of technique used.

But, if MOND is basically correct one cannot successfully model the dark-matter distribution in disc galaxies by a single-component, elliptical halo of constant ellipticity. Probing the galaxy far from the plane and from the centre will result in rounder halos as in Ibata & al (2000). Probing near the centre might give appreciable eccentricities. For example, the considerable ellipticity that Sackett & al (1994) find for the halo of NGC 4650A reflects mostly constraints from fits to the rotation curve of the smaller, main disc. NGC 4650A is a low-surface-density galaxy, $\zeta \ll 1$, requiring much dark matter even at small radii. This, we saw, is concentrated in the disc, and so attempts to put it in a halo around the central disc requires a highly flattened halo. Indeed, Morishima & Saio (1995) who analyzed the rotation curves of the main disc and the polar ring of this galaxy in a MOND potential derived from eq.(1) find good agreement.

Disc-flaring analyses might also indicate a highly flattened halo. The main reason is that the MOND “dark disc” can greatly enhance the self gravity of the disc and make it dominant in determining the z structure.

It is impossible to make quantitative comparisons with published disc-flaring analyses, as in Becaert & Combes (1997) and Olling (1996b), without delving into the many details of the data and of the assumptions going into such analyses (e.g. the contribution of a stellar disc in addition to that of the studied HI layer, vertical velocity dispersion, etc.). As a demonstration, consider a pure disc of mass M , scale length h , and surface density $\Sigma(r) = (M/2\pi h^2)f(r/h)$. Take r beyond a few scale lengths so that the accumulated mass is roughly saturated. Also assume we are in the MOND regime ($\mu \ll 1$) and at a radius on the flat part of the rotation curve. Neglecting the confining effects of the inner disc and reckoning only with the local self gravity (i.e. assuming pure planar symmetry), the z scale height according to MOND is

$$z_0 \sim \frac{\sigma_z^2(r)\mu(r)}{2\pi G\Sigma(r)}, \quad (21)$$

where σ_z is the z velocity dispersion. Applying instead the expression Olling (1996b) uses to analyze HI flaring in NGC 4244, which neglects disc self gravity and assumes that the confining agent is a flattened halo, we have $z_0 \sim (\sigma_z/V)[2.4q/(1.4 + q)]^{1/2}r$ (taking in his expression a core radius $r_c \ll r$), where V is the (constant) rotational velocity at r , and q is

the flattening. Equating the two expressions we see that to get the scale height for MOND with a halo one would need a q value satisfying $[2.4q/(1.4 + q)]^{1/2} \sim (\sigma_z/V)(h/r)^2/f(r/h)$, where I made use of the MOND relation $V^2 \approx MG/\mu r$. So, for example, for an exponential disc, at 5 scale lengths, and with $\sigma_z/V = 0.1$, a q value of 0.24 is gotten.

Using data for NGC 4244 from Olling (1996a) we have at the last observed point (13 kpc) a gas surface density of $\Sigma \sim 5 \times 10^{-5} \text{gr cm}^{-2}$ (corrected for helium), the MOND factor $\mu \approx V^2/ra_o \sim 0.16$, and the planar velocity dispersion $\sigma \sim 8 \text{km s}^{-1}$. Applying the MOND estimate for the scale height eq.(21), and assuming that the z dispersion equals the planar one, gives $z_0 \sim 1700 \text{ pc}$ compared with Olling’s measured half width at half maximum of 750 pc. We have to remember that σ_z might be somewhat different, that eq.(21) is only approximate, neglecting the global restoring force (see below), etc. Without the MOND factor (“dark disc” contribution) the expected value of z_0 is 6 times larger (i.e. $\sim 10 \text{ kpc}$), so the true disc alone is quite negligible in containing the HI layer in Newtonian dynamics. At 10 kpc the MOND value of z_0 is $\sim 300 \text{ pc}$ ($\mu \sim 0.27$ is larger, and Σ is 10 times larger, than at 13 kpc) compared with a similar value in Olling (1996a). So, the high degree of halo flattening that Olling (1996b) finds for this galaxy may well be largely an artifact of the MOND “dark disc”.

The above was only a demonstration of the importance of the MOND effects in augmenting the self-gravity restoring force of the disc, even in places where the surface density is small. In a detailed analysis one must add the confining force, g_z^* , of the rounder “halo” (the global potential of the disc) to the local, self-gravity component g_z . At very large radii, or beyond some sharp cut-off of the disc, the former will dominate. We can estimate the relative importance of the two effects through the parameter $\beta \equiv g_z/g_z^* \approx 2\pi\Sigma(r)r^3/Mz_0$. This approximation is good inasmuch as the Newtonian radial acceleration can be written as MG/r^2 , so the MOND radial acceleration is $MG/\mu r^2$, and so the restoring force of a spherical potential would be $zMG/\mu r^3$; also $g_z \approx 2\pi G\Sigma(r)/\mu(r)$. Applying this to the above two radii in NGC 4244, with a MOND mass of $3 \times 10^9 M_\odot$, corresponding to an asymptotic velocity of 85 km s^{-1} , I get $\beta \approx 10$ at 10 kpc, and $\beta \approx 1.2$ at 13 kpc. This means that self gravity dominates at 10 kpc and global forces can be neglected, while at 13 kpc the two are of the same order, and the above MOND estimate of z_0 at 13 kpc has to be decreased by a factor of about 2.

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