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A STRATIFIED FRAMEWORK FOR SCALAR-TENSOR THEORIES OF MODIFIED DYNAMICS

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ABSTRACT

Although the modified Newtonian dynamics (MOND) proposed by Milgrom successfully accounts for the systematics of galaxy rotation curves and cluster dynamics without invoking dark matter, the idea remains a largely ad hoc modification of Newtonian dynamics with no basis in deeper theory. Nonstandard scalar-tensor theories have been suggested as a theoretical basis for MOND; however, any such theory with the usual conformal relation between the Einstein and physical metrics fails to predict the degree of light deflection observed in distant clusters of galaxies. The prediction is that there should be no discrepancy between the detectable mass in stars and gas and the lensing mass, in sharp contradiction to the observations (Bekenstein & Sanders). In the present paper, I demonstrate that one can write down a framework for scalar-tensor theories that predict the MOND phenomenology for the low-velocity ($v \ll c$) dynamics of galaxies and clusters of galaxies and are consistent with observations of extragalactic gravitational lenses, provided that one drops the requirement of the Lorentz invariance of gravitational dynamics. This leads to "preferred-frame" theories characterized by a nonconformal relation between the two metrics. I describe a toy theory in which the local environment (the solar system, binary pulsars) is protected from detectable preferred-frame effects by the very same nonstandard (aquadratic) scalar Lagrangian that gives rise to the MOND phenomenology. Although this particular theory is also contrived, it represents a limiting case for two-field theories of MOND and is consistent with a wide range of gravitational phenomena. Moreover, it is a cosmological effective theory which may explain the near numerical coincidence between the MOND acceleration parameter and the present value of the Hubble parameter multiplied by the speed of light.

Subject headings: cosmology: theory — galaxies: distances and redshifts — gravitational lensing — relativity

1. INTRODUCTION

More than 10 years ago Milgrom proposed a modification of Newtonian dynamics (MOND) as an explanation of mass discrepancies in astronomical systems with low internal acceleration (Milgrom 1983a, 1983b, 1983c). As an alternative to dark matter, the idea has proved to be amazingly resilient in spite of sustained attacks from several quarters (e.g., The & White 1988; Lake 1989; Gerhard & Spergel 1992). This is due in large part to the successes of the simple MOND prescription on a phenomenological level, many of which were previewed by Milgrom in his original papers: asymptotically flat rotation curves of spiral galaxies, the observed form of the luminosity-velocity correlations for spiral and elliptical galaxies (the Tully-Fisher and Faber-Jackson relations), the existence of a critical maximum surface density in spirals and ellipticals (the Freeman and Fish laws), the appearance of large mass discrepancies in low surface brightness systems (e.g., dwarf spheroidals and low surface brightness spirals), the magnitude of the discrepancy in clusters of galaxies, and the magnitude of Virgocentric inflow.

In addition, Milgrom used MOND to make rather specific predictions which have been borne out by subsequent observations. There are two notable examples: The first is the prediction that the rotation curves in luminous high surface brightness galaxies should decline to an asymptotically flat value, while the rotation curve in low-luminosity, low surface brightness galaxies should slowly rise to the asymptotically flat value (Milgrom 1983b). This effect has been observed subsequently by Casertano & van Gorkom (1991). Second, there was the suggestion, on the basis of high MOND mass-to-light ratios of clusters of galaxies, that hot X-ray–emitting gas may make a very substantial contribution to the total observable mass of clusters (Milgrom 1983c). This has now been well established by *ROSAT* observations (Böhringer, Schwarz, & Briel 1993); indeed, the predicted MOND mass agrees remarkably well with the observed hot gas mass for a number of clusters (Sanders 1994).

But perhaps the most outstanding success of MOND has been in connection with the observed extended rotation curves of spiral galaxies. It is not that MOND, in some general sense, predicts flat rotation curves; the simple MOND formula predicts the precise form of the rotation curve of a spiral galaxy from the observed distribution of stars and gas. In a sample of galaxies with well-observed gas kinematics, there is remarkable agreement between the observed and predicted rotation curves using a single value of the MOND acceleration parameter a_0 (Begeman, Broeils, & Sanders 1991; Sanders 1996). At the very least, one can say that the MOND prescription provides a far more economical fitting algorithm for galaxy rotation curves than do multiparameter dark halo models.

In spite of these successes, the idea is not yet taken seriously by most physicists and astronomers. The reason for this, to some extent, is the absence of a solid theoretical underpinning of the idea; MOND remains an ad hoc, empirically motivated modification of Newton's law without connection to a more familiar theoretical framework. There have been several attempts at writing down a relativistic (i.e., generally covariant) theory which reduces to MOND in the limit of low accelerations (Bekenstein & Milgrom 1984, hereafter BM; Bekenstein 1988; Sanders 1986, 1988; Romatka 1992), but these theories all contain anomalies, or they are inconsistent with the classical local gravity tests. Moreover, they are designed to reproduce MOND in the low force limit but are not based upon some more general principle in the spirit of general relativity (GR) or modern gauge theories of particle physics.

There is one aspect of MOND which renders the idea less ad hoc and which would seem to point the way to a more substantial theoretical basis. That is the near numerical coincidence between the empirically derived acceleration parameter, a_0, and cH_0, the Hubble parameter multiplied by the speed of light. This appears to give a cosmological significance to the acceleration parameter, and the implied relation between local dynamics and the expansion of the universe seems distinctly Machian (Milgrom 1983a, 1994). With respect to the theoretical basis of MOND, this numerical coincidence suggests that the proper theory should be an *effective* theory; that is to say, the MOND phenomenology would only be predicted when the theory is considered in a cosmological background. Upon reflection it is evident that such an effective theory cannot be provided by GR because, in the context of GR, there is no cosmological influence of this order on local gravitational physics. However, various scalar-tensor theories do offer the possibility of such a connection.

Two types of scalar-tensor theories have been suggested to provide a theoretical basis for MOND: the so-called "aquadratic Lagrangian" (AQUAL) theories (BM; Sanders 1986) and a general class of two-scalar theories, of which "phase coupling gravity" (PCG) is the most discussed example (Bekenstein 1988; Sanders 1988, 1989; Romatka 1992). The AQUAL theories suffer, unavoidably, from causality violations (superluminal propagation of scalar waves) if the MOND phenomenology is reproduced (BM; Bekenstein 1988), and PCG, apparently in any form, provides no stable background solution for the two additional fields (Bekenstein 1992). But a more practical problem with these, and indeed with all scalar-tensor theories in which the scalar field enters as a conformal factor multiplying the Einstein metric, is the failure to predict the gravitational deflection of light at the level apparently observed in rich clusters of galaxies (Bekenstein & Sanders 1994). That is to say, if one wishes to replace dark matter by a modified theory of gravity of the standard scalar-tensor type, then the scalar field produces no enhanced deflection of light; the observed deflection would be due only to the detectable matter, implying that the mass of a system determined via gravitational lensing in the context of GR should be substantially less than the conventional virial mass. This apparently is not the case (Fort & Mellier 1994).

With a view toward resolving the light-bending problem for scalar-tensor theories, Bekenstein & Sanders (1994) considered a more general relation between the Einstein and physical metrics: the *disformal* transformation, a relation which includes both the scalar field and its gradient (Bekenstein 1992). The result was discouraging: if the propagation of classical gravitational waves is to be causal, then the sign of the disformal term must be such that the light bending is actually reduced over that predicted by GR.

However, it now appears that the form of the transformation considered by Bekenstein & Sanders (1994) is not the most general relation between the gravitational and physical metrics, and that a clue to the more general transformation is suggested by a class of theories known historically as "stratified" theories (Ni 1972). Here the physical metric is related to the Einstein metric via a conformal factor involving the scalar field and additional terms usually constructed from a nondynamical vector field. In some preferred frame, assumed to be the cosmological frame, the vector has only a time component, and spacelike strata of the physical and Einstein metrics are conformally related (historically, the "Einstein" metric is assumed to be the Minkowski metric, so the theory is conformally flat on spacelike strata, but we will not make that restriction in the definition of stratified theories considered here). In stratified theories the light bending in the weak field limit can be equivalent to that predicted by GR; in fact, the original motivation for such theories was to overcome the absence of light bending predicted by conformally flat scalar theories of gravity such as that of Nordström (Misner, Thorne, & Wheeler 1973).

Aesthetically, such theories may be criticized because, unlike GR, they contain a priori elements such as a nondynamical vector field and, in their original form, prior geometry described by the Minkowski metric. This is certainly contrary to the spirit if not the letter of general covariance. Because they give a special status to the cosmic frame, such theories, philosophically, go some way back toward the prerelativity concepts of absolute space and time. But aesthetics aside, the earlier stratified theories are all ruled out because they predict various local preferred-frame effects (such as Earth tides) at a level that should have been detected (Will 1993).

In the present paper, I resurrect the idea of stratified theories to provide a framework for scalar-tensor theories of MOND in which the deflection of light bears the same relation to the weak field force as in GR. To achieve this, I introduce an additional vector field, here assumed to be nondynamical, into the matter Lagrangian in the form of a stratified theory. But an additional element is that the vector field is also introduced into the scalar field Lagrangian to form a new invariant which becomes the square of the scalar cosmic time derivative ($\dot{\phi}^2$) in the preferred cosmological frame (also an aspect of the generalized stratified theory of Lee, Lightman, & Ni 1974). This allows one to write a cosmological effective theory of MOND, i.e., one in which the acceleration parameter a_0 is not put in by hand but is identified naturally with the cosmic time derivative of the scalar field.

I describe a toy theory in which the total attraction, to high precision, is Newtonian in the high acceleration limit (e.g., near the Sun), but the phenomenology is basically that of MOND in the low acceleration limit. In this particular example, the scalar field Lagrangian is of the unconventional aquadratic or AQUAL form, although two-scalar theories like PCG are also possible. However, unlike AQUAL, PCG, or any scalar-tensor theory with a conformal relation between the two metrics, this theory produces gravitational deflection of light at the level predicted by GR with dark matter. Locally (i.e., in the solar system) the stratified aquadratic theories are weakly coupled (but not arbitrarily weak) scalar-tensor theories, but unlike traditional scalar-tensor theories (e.g., Brans-Dicke) the predicted deflection of light about the Sun is identical to that in GR. And, unlike the traditional stratified theories, the predicted local preferred-frame effects may be suppressed by a large (but not arbitrarily large) factor; in fact, the current experimental limits on preferred-frame effects are already at or near the minimum level predicted by this theory. There is, moreover, one additional predicted effect that might well be observable: a secular cosmic variation in the gravitational constant.

The basic goal here is to write down a relativistic theory of MOND, however contrived, in which the cosmological background determines the value of a_0 and which is consistent with local and extragalactic phenomenology—in particular with the observed deflection of light by clusters of galaxies. It would seem to be important to demonstrate that this is possible, particularly in view of the negative result of Bekenstein & Sanders (1994) on conformally or disformally coupled scalar fields. The principal conclusion is that *stratified scalar-tensor* theories, with a nonstandard scalar field Lagrangian, can be consistent with the observations of galaxy rotation curves and cosmic gravitational lenses as well as, at the present levels of experimental precision, local gravitational dynamics.

2. THE EFFECTS OF COSMOLOGY ON LOCAL GRAVITATIONAL DYNAMICS

In the context of GR, cosmology has an insignificant effect on local gravitational dynamics. Israelit & Rosen (1990) considered the equation of motion of a particle in a cosmological background and demonstrated that any additional acceleration on a particle orbiting a galaxy at distance r is on the order of rH_0^2, i.e., of the same magnitude as the effect of a possible cosmological constant. Therefore, MOND effects that are postulated to be present on galactic scales at accelerations of cH_0 cannot possibly arise due to the influence of cosmology in the context of pure GR.

The general arguments for this absence of significant cosmological effects were elucidated earlier by Will & Nordtvedt (1972) and are paraphrased below: Consider a local gravitational system (the solar system, the Galaxy, a cluster of galaxies) embedded in the universe. How does cosmology affect the gravitational physics of this local system? We divide the solution of a set of field equations into two parts: a cosmological solution and a local solution. From this viewpoint, cosmology establishes boundary conditions for the various fields generated by the local system; i.e., the local system "feels" the cosmology via its asymptotic field values. Now the cosmological metric is the Robertson-Walker metric, which, on scales small compared to cH_0^{-1} and short compared to H_0^{-1}, is the Minkowski metric. In GR the metric field ($g_{\mu\nu}$) is the only field; therefore, the local field must become Minkowskian, that of empty space, far from the mass concentration. From the Birkhoff theorem we know that a spherically symmetric gravitational field in empty space must be static, with a metric given by the Schwarzschild solution. This means that weak field gravity is Newtonian (G is unaffected by the presence of matter), that there is no cosmic time dependence of G (local gravitational physics is time-reversible), and that, due to the invariance properties of the Minkowski metric, there are no preferred-frame effects for systems in uniform relative motion.

In standard scalar-tensor theory there are two fields ($g_{\mu\nu}$ and ϕ), and, as the theory is usually written, the scalar interacts with matter jointly with $g_{\mu\nu}$ via a conformal transformation of the metric; i.e., the form of the interaction Lagrangian is taken to be

$$L_I = L_I[\psi(\phi)^2 g_{\mu\nu} \ldots] , \quad (1)$$

where ψ is a function of the scalar field. In empty space, far from the mass concentration, the physical metric is conformally flat. The conformal function can be factored and appears as a time-variable gravitational constant or universal mass function. Thus the boundary condition on $g_{\mu\nu}$ remains Minkowskian and there are no preferred-frame effects. Moreover, in standard scalar-tensor (i.e., Brans & Dicke 1961; Nordtvedt 1970; Wagoner 1970), with the usual quadratic Lagrangian ($\phi_{,\alpha}\phi^{,\alpha}$), the scalar "force" is essentially Newtonian and scales as the mass. The variation of ϕ with position is generally small compared to the cosmological value (insignificant variation of G near a mass concentration), but G is a function of cosmic time. Therefore, gravitational physics is not time-reversible.

A theory such as Bekenstein's (1988) "phase coupling gravity" is a "two-scalar plus tensor" theory and, as such, offers the possibility of a more dramatic cosmological effect on local dynamics. Here, of the two scalar fields q and ϕ, only one (ϕ) couples to matter (jointly with $g_{\mu\nu}$ as in eq. [1]), and the two fields interact via the kinetic term of the matter-coupling field; i.e., the scalar field Lagrangian is given by

$$L_s = \tfrac{1}{2}(q^2\phi_{,\alpha}\phi^{,\alpha} + q_{,\alpha}q^{,\alpha} + V(q^2)) . \quad (2)$$

Thus, in the field equation q^2 appears with respect to the gradient of ϕ in a form analogous to the MOND function μ in the BM field equation (eq. [6a] below); i.e.,

$$(q^2\phi^{\alpha})_{;\alpha} = \frac{4\pi\eta G}{c^4}\, T , \quad (3)$$

where η is a dimensionless parameter describing the strength of the scalar field coupling and T is the contracted energy-momentum tensor as usual. Cosmology sets the asymptotic value of q, which may be very different from its value near a local mass concentration. This implies that local gravitational physics may be strongly non-Newtonian (the scalar force is not necessarily $1/r^2$, nor is the coupling to mass necessarily linear), which would seem to be ideal for an effective theory of MOND. Indeed it has been demonstrated (Sanders 1989) that with an appropriately chosen, but somewhat unnatural, self-interaction potential for the scalar field the predicted phenomenology is basically that of MOND. The scalar force exceeds the usual Newtonian force at accelerations below an a_0 which is identified with the cosmic time derivative of ϕ. At accelerations much below a_0 the scalar force also becomes inverse square but exceeds the Newtonian force by a factor δ (typically assumed to be 10). It can be shown that

$$a_0 = 1.5\delta\Omega_0\, cH_0(t_0 H_0) , \quad (4)$$

where Ω_0 is the usual density parameter of the universe and t_0 is the age of the universe. It is evident that with PCG in a cosmological setting, the MOND coincidence is explained.

However, PCG has two serious failures: the first concerns solar system dynamics. On the scale of the solar system, PCG may be considered as a Brans-Dicke theory with a weakly variable Brans-Dicke parameter ω. The problem is that in the form of the theory described by Sanders (1989) ω in the solar system is much smaller than the experimental lower limit of about 1000. While it may be possible to avoid this problem by choosing a different form for the self-interaction potential, the second failure is more fundamental. As noted in the Introduction, for every scalar-tensor theory in which the interaction with matter is described by equation (1), the scalar field has no effect on the motion of photons and therefore could not explain the enhanced deflection apparently observed in clusters of galaxies (Fort & Mellier 1994) and, possibly, in individual galaxies (Brainerd, Blandford, & Smail 1996). Therefore, PCG in its original form cannot be a viable theoretical replacement for dark matter.

A second scalar-tensor theoretical framework for MOND is provided by theories with aquadratic Lagrangians for the kinetic term of the scalar field; i.e.,

$$L_s = \frac{1}{2} F(X) \frac{a_0^2}{c^4} , \tag{5a}$$

where

$$X = \frac{\phi_{,\alpha} \phi^{,\alpha} c^4}{a_0^2} \tag{5b}$$

and $F(X)$ is an arbitrary positive function of its unitless argument. Combined with equation (1), this leads to the BM field equation:

$$(\mu \phi^{,\alpha})_{;\alpha} = \frac{4\pi G T}{c^4} , \tag{6a}$$

with

$$\mu = dF/dX = F'(X) . \tag{6b}$$

To yield MOND phenomenology, $F'(X)$ must asymptotically approach $X^{1/2}$ in the limit of small X.

The original AQUAL theory of BM, as well as its trivial revision by Sanders (1986), is in no sense a cosmological effective theory. The acceleration parameter is written in by hand, and the theory has no obvious cosmological extension (as originally written, the Lagrangian becomes imaginary if the scalar 4-gradient has only a time component). However, by making use of an additional field, a nondynamic vector field, it is possible to write down an AQUAL theory in which the cosmic time derivative of the scalar field, $\dot{\phi}$, can be inserted separately into the scalar field Lagrangian. Since, in the appropriate units, $\dot{\phi}$ has a current value on the order of cH_0, this provides an obvious mechanism for a cosmologically imposed critical acceleration on the order of a_0. Moreover, given the vector field, such an AQUAL theory can be written quite naturally as a stratified theory. Then with the appropriate coupling of the scalar to the Einstein metric and to the vector field, the problem of light bending is solved.

3. STRATIFIED SCALAR-TENSOR THEORIES AND THE DEFLECTION OF LIGHT

3.1. *A Stratified Theory including General Relativity*

In the historical stratified theories, several of which were designed to produce light bending equal to that predicted by GR, the physical metric was constructed from a nondynamical vector field and the Minkowski metric, i.e., a prior geometry unrelated to the distribution of mass or energy (Ni 1972). Here, because we wish to retain GR in the strong field limit, we replace the Minkowski metric by the Einstein metric. Then, in the spirit of stratified theories, the physical metric, $\tilde{g}_{\mu\nu}$, is related to the Einstein metric, $g_{\mu\nu}$, via the transformation

$$\tilde{g}_{\mu\nu} = u(\phi) g_{\mu\nu} - w(\phi) A_\mu A_\nu , \tag{7}$$

where $u(\phi)$ and $w(\phi)$ are at this point unspecified functions of the scalar field ϕ, and the dynamics of $g_{\mu\nu}$ is derived from the Hilbert action

$$S_g = \frac{c^4}{16\pi G} \int R[g_{\mu\nu}] \sqrt{-g}\, d^4x ; \tag{8}$$

i.e., the theory includes GR and, for $u = 1$, $w = 0$, reduces to GR. We specify that A^μ is a nondynamical vector field with the only nonzero component pointing in the positive time direction in the cosmic frame and tuned to the Einstein metric such that

$$g^{\mu\nu} A_\mu A_\nu = -1 \tag{9}$$

(in some theories $A_\mu = t_{,\mu}$, where t is a nondynamical cosmic time scalar). Thus, any frame at rest with respect to the universe (i.e., the cosmic background radiation) becomes a preferred frame where the theory takes its simplest form.

A word is required about the definition of a nondynamical vector field in a theory in which spacetime is not a priori Minkowskian. In a spacetime with a high degree of symmetry (i.e., Robertson-Walker), the vector A^μ can be uniquely defined as the unit vector orthogonal to spacelike hypersurfaces. However, if we permit mass concentrations, as in the real universe, the definition of a nondynamical vector field becomes ambiguous, as the entire spacetime cannot necessarily be globally sliced by such surfaces (J. D. Bekenstein 1995, private communication). There are several possibilities for removing this ambiguity, but it may be that a fully consistent theory requires that the vector field be dynamical. For now, because in problems of galactic or solar system dynamics $g_{\mu\nu} \approx \eta_{\mu\nu}$ (with appropriately chosen physical units), we assume that A^μ remains strictly timelike in any almost Minkowskian frame at rest with respect to the cosmic frame.

With the coupling described by equation (7), the particle action in the Einstein frame is given by

$$S_p = -mc \int \left\{ -[u(\phi) g_{\mu\nu} - w(\phi) A_\mu A_\nu] \frac{dx^\mu}{dp} \frac{dx^\nu}{dp} \right\}^{1/2} dp , \tag{10}$$

where p is some parameter along the path of the particle. Extremizing the action with respect to variations in x^μ in the usual way and setting $dp = d\tau$ (the invariant interval), we find the covariant equation of motion:

$$\frac{dU_\nu}{d\tau} = \frac{1}{2} g_{\mu\kappa,\nu} U^\kappa U^\mu u(\phi) + F_\nu , \tag{11}$$

where $U_\nu = \tilde{g}_{\mu\nu} dx^\mu / d\tau$ is the covariant velocity. The first term on the right-hand side is the usual Einstein-Newton gravitational force, and F_ν is the scalar force (in the Einstein frame the motion is nongeodesic) given by

$$F_\nu = \frac{\phi_{,\nu}}{2} \left[-\frac{u'}{u} + \left(\frac{u' w}{u} - w' \right) A_\kappa A_\mu U^\kappa U^\mu \right] , \tag{12}$$

where the prime indicates the derivative of u or w with respect to ϕ.

Because we are interested here in the equivalent Newtonian force (the ordinary force or 3-force) on slow or fast particles, we may also set $dp = dt$ (time in some specific frame) in equation (10) and rewrite the equation of motion as

$$\frac{dp_i}{dt} = -\frac{1}{2} m(g_{00,i} + g_{jk,i} v^j v^k) u(\phi) + F_i \,, \tag{13}$$

where

$$p_i = m v_i(-\tilde{g}_{00} - \tilde{g}_{jk} v^j v^k)^{-1/2} \tag{14}$$

is the 3-momentum and $\boldsymbol{v}$ is the 3-velocity (the Greek indices refer to spatial coordinates). The first term on the right-hand side again represents the Einstein-Newton force, and F_i is the ordinary scalar force given now by

$$F_i = m \frac{\phi_{,i}}{2} \left[(u' g_{\mu\nu} - w' A_\mu A_\nu) \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \right] \left[-\tilde{g}_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \right]^{-1/2} . \tag{15}$$

In the weak field limit we may set $g_{\mu\nu} \approx \eta_{\mu\nu}$. This, in effect, is defining the measure of time and length such that gravitational radiation propagates with unit velocity. Then, in the preferred frame where $\boldsymbol{A}$ is strictly timelike, we find for the ordinary scalar force

$$F_i = -m \frac{\phi_{,i}}{2} (u' + w' - u' v^2)[(u + w) - u v^2]^{-1/2} . \tag{16}$$

Dividing by the relativistic mass,

$$m' = m u^{1/2}[(1 + w/u) - v^2]^{-1/2} \,, \tag{17}$$

we then determine the ordinary acceleration induced by the scalar field as

$$f_i = -\frac{\phi_{,i}}{2} (u' + w' - u' v^2) u^{-1} \,, \tag{18}$$

where the particle speed v approaches the limit $c' = (1 + w/u)^{1/2}$, which is variable and may exceed unity as measured in units in which the Einstein metric is asymptotically Minkowskian. Setting $v = 0$ gives the scalar acceleration on slow-moving particles, and setting $v = c'$ gives the acceleration of relativistic particles or photons. If we let k be the ratio of the scalar acceleration on photons to that on slow particles, we find the simple result that

$$k = \left(w' - \frac{u' w}{u} \right) (u' + w')^{-1} . \tag{19}$$

That is to say, in the weak field static limit, the deflection of a photon from a straight-line path would be given by

$$\theta = \frac{2}{c^2} \int f_{\rm N}^{\perp} dz + \frac{k}{c^2} \int f^{\perp} dz \,, \tag{20}$$

integrating along the path. Here $f_{\rm N}^{\perp}$ is the perpendicular component of the usual Newtonian acceleration (i.e., resulting from the weak field limit of GR, the first term on the right-hand side of eq. [13]), and $f^{\perp}$ is the perpendicular component of the scalar force on slow particles (eq. [16] with $v = 0$). For the usual scalar-tensor theory with a conformally coupled scalar field, $w(\phi) = 0$, which tells us immediately that $k = 0$; i.e., the scalar field does not affect the motion of photons at all.

3.2. *General Constraints Determine the Free Functions and Light Bending*

Several general physical considerations uniquely determine the form of the functions $u(\phi)$ and $w(\phi)$ and hence the relation of the light bending to the weak field force (k in eq. [20]). First of all, the condition that the physical and Einstein metrics have the same signature requires that $u(\phi) > 0$. Further, it is reasonable to expect that physics should be invariant to a global transformation of physical units, and the appearance of a special direction in stratified theories implies that unit transformations may differ in directions parallel and perpendicular to $\boldsymbol{A}$ (J. D. Bekenstein 1995, private communication). Representing such a transformation as a shift in the zero of ϕ, as in conformal theory, and considering coordinates in which $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are diagonal, such invariance implies that

$$u(\phi) = e^{-\phi} \tag{21}$$

and

$$w(\phi) + u(\phi) = e^{\beta\phi} . \tag{22}$$

One more condition allows us to specify β (following an argument by Dicke 1957). It is evident that the electromagnetic invariant should contain the *physical* and not the gravitational metric, i.e.,

$$L_{\rm em} = \tilde{g}_{\alpha\mu} \tilde{g}_{\beta\nu} F^{\alpha\beta} F^{\mu\nu} \,, \tag{23}$$

because this yields trajectories for light corresponding to null geodesics of the physical metric. Then, in Maxwell's equations, the effective dielectric parameter and permeability of empty space are given by

$$\epsilon = u(\phi) \tag{24}$$

and

$$\mu = \frac{1}{u(\phi) + w(\phi)} \,, \tag{25}$$

in units such that the Einstein metric is asymptotically Minkowskian. But the unitless physical constants, such as the fine-structure constant,

$$\alpha = \frac{e^2}{h} \left(\frac{\mu}{\epsilon} \right)^{1/2} , \tag{26}$$

should be independent of the choice of physical units. Then it follows that $\mu = \epsilon$ and $\beta = 1$ in equation (22). Thus we find that

$$w(\phi) = e^{\phi} - e^{-\phi} \,, \tag{27}$$

and that $u(\phi)$ and $w(\phi)$ have the form generally assumed in the traditional stratified theories (Ni 1972). Note that while light propagates along null geodesics of the physical metric, gravitational radiation propagates along null geodesics of the Einstein metric. In units such that the physical metric is asymptotically Minkowskian, the velocity of propagation of gravitational radiation becomes $c_g = e^{-\phi}$. Therefore, causal propagation of gravitational waves requires that $\phi > 0$; this should follow from any sensible cosmology.

Given the form of $u(\phi)$ and $w(\phi)$, we find from equation (19) that $k = 2$ as in the historical stratified theories; i.e., the weak field expression for the deflection of photons, equation (20), has the same relation to the total acceleration on slow particles as in GR. This has an immediate observational consequence: any replacement of dark matter by a stratified scalar-tensor theory yields light bending exactly equivalent to that of GR plus dark matter; i.e., the lensing mass of a cluster determined by the usual formula should be equal to that of the conventional virial mass.

The overall conclusion of this section is that the use of the nonconformal relation between the physical and Einstein metrics (eq. [7]) involving an additional cosmic field permits the scalar field to influence the deflection of photons in a manner not anticipated by Bekenstein & Sanders (1994). Indeed, given very general constraints on the framework free functions, the relation of the deflection angle to the weak field force is identical to that in GR.

4. AQUADRATIC STRATIFIED THEORY

4.1. *A Generalized Field Action*

Having introduced the nondynamical cosmological unit vector $\boldsymbol{A}$ into the general relation between the physical metric and the Einstein metric (eq. [7]), we may also use it to form a second scalar field invariant; i.e., in addition to the usual invariant

$$I = g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} \tag{28a}$$

there is also

$$J = A^{\mu}A^{\nu}\phi_{,\mu}\phi_{,\nu} \tag{28b}$$

(here the theory will be written in the Einstein frame). Because in the cosmological frame A^{μ} is postulated to be timelike, this allows us to insert the cosmic time derivative of ϕ directly into the scalar field Lagrangian instead of introducing a new dimensional parameter, a_0 (as in the aquadratic theories of BM and Sanders 1986). Moreover, if

$$K = I + J\ , \tag{29}$$

then K becomes the square of the spatial gradient of ϕ in the preferred frame. Thus we can manipulate the spatial and time derivatives independently in the preferred frame at the level of the field action.

The most general theory involving J and K is described by the action

$$S_{\phi} = \frac{c^4}{8\pi G}\int JQ\left(\frac{K}{J}\right)\sqrt{-g}\,d^4x\ , \tag{30}$$

where $Q(X)$ is any real function of its argument X. In particular, if $Q(X) = X - 1$, we are left (in the preferred frame) with the usual quadratic scalar field Lagrangian, which, of course, yields an inverse-square attraction for the scalar force; the more general form yields AQUAL theories, but with no new dimensional parameters.

The dynamics of the theory comes from the total action

$$S = S_g + S_{\phi} + S_m\ , \tag{31}$$

where S_g is the gravitational action (eq. [8]) and the matter action S_m is given by S_p (eq. [10]) summed over particles. Finding the extremum of the action with respect to ϕ gives the field equations

$$\frac{1}{\sqrt{-\tilde{g}}}(\sqrt{-g}P^{\alpha\beta}\phi_{,\beta})_{,\alpha} = \frac{4\pi G}{c^4}\tilde{T}^{\mu\nu}[g_{\mu\nu}e^{-\phi} + (e^{\phi} + e^{-\phi})A_{\mu}A_{\nu}]\ , \tag{32a}$$

where

$$P^{\alpha\beta} = g^{\alpha\beta}Q'(X) + A^{\alpha}A^{\beta}[Q(X) + Q'(X) - XQ'(X)] \tag{32b}$$

with $Q' = dQ/dX$. In the preferred frame this becomes

$$P^{ij} = g^{ij}Q'(X) \quad (i, j = 1, 2, 3) \tag{32c}$$

and

$$P^{tt} = Q(X) - XQ'(X)\ . \tag{32d}$$

The source is expressed in terms of the energy-momentum tensor

$$\tilde{T}^{\mu\nu} = -\frac{2}{\sqrt{-\tilde{g}}}\frac{\delta}{\delta\tilde{g}^{\mu\nu}}S_m \tag{32e}$$

in the physical frame, so that the density ρ and pressure p are those quantities actually measured by an observer. This resembles the AQUAL field equation of BM, except that the scalar function of the invariant $\phi_{,\alpha}\phi^{,\alpha}$ [$F(X)$ in the notation of BM, eq. (5a)] has been replaced by a tensor $P^{\alpha\beta}$. The complete theory includes the usual Einstein field equation for $g_{\mu\nu}$, but with unconventional terms for the contribution of the scalar field to the energy-momentum tensor.

Recalling that $X = (\nabla\phi\cdot\nabla\phi)/\dot{\phi}^2$ in the preferred frame, let us choose

$$Q(X) = F(X) - \kappa\ , \tag{33}$$

where κ is a number on the order of unity included to provide a cosmological solution, and F is to be identified with the function of $(\nabla\phi/a_0)^2$ in the BM aquadratic theory (referred to below as the BM function). There are no obvious a priori restrictions on the form of F, but in order to reproduce both MOND phenomenology on the scale of galaxies and precise inverse-square attraction in the solar system, it must be the case that $F(X) \propto X^{3/2}$ in the limit where $X \ll 1$ (see BM) and $F(X) \propto X$, where $X \gg 1$. This is because in the quasi-static case (no variation of ϕ on timescales short compared to the Hubble time) the MOND function

$$\mu(x) = F'(X) \tag{34a}$$

(see eqs. [6a], [32a], [32c]), as in BM theory (eq. [6b]), then has the appropriate asymptotic behavior (Milgrom 1983a). Here

$$x = \sqrt{X} = |\nabla\phi/\dot{\phi}|c\ , \tag{34b}$$

with the cosmic time derivative $\dot{\phi}$ playing the role of a_0 (here the speed of light is explicitly included to make the physical units clearer below).

4.2. *The Cosmological Origin of* a_0

The identification of $\dot{\phi}$ with a_0 becomes evident when we consider, with equations (32) and (33), the cosmic evolution of ϕ. With no spatial gradients $P^{tt} = -\kappa$, and equation (32a) becomes

$$e^{\phi}\frac{d}{dt}(R^3\dot{\phi}) = -\frac{4\pi G}{\kappa}(3pe^{-\phi} + \rho e^{\phi})R^3\ , \tag{35}$$

where R is the cosmic scale factor. It is clear from equation (35) that pressure enters as a source in the same way as density; in particular, in contrast to standard scalar-tensor theory, the radiation contributes to the source of ϕ at the level of twice the corresponding density for nonrelativistic material. This is nicely consistent with the result derived in the previous section where we see that the scalar force produces twice as much acceleration on photons as on slow-moving particles (see also Dicke 1957). But in addition we see that in a pressureless universe the evolution for ϕ is identical in form to that in standard conformally coupled scalar-tensor theory. In particular, we find that

$$\dot{\phi} = -\frac{4\pi G\rho t}{\kappa}\,, \tag{36}$$

where t is cosmic time. Here an integration constant has been arbitrarily set to zero. At the present epoch this becomes

$$\dot{\phi}_0 = -1.5\,\frac{\Omega_0}{\kappa}\,H_0^2\,t_0\,, \tag{37}$$

where t_0 is the present age of the universe.

Now consider the solution of equations (32) and (33) about a point mass at rest in the preferred cosmological frame. This is simplified by assuming that the solution for ϕ about the mass concentration is quasi-static; i.e., there is no time dependence on timescales short compared to the Hubble time. Furthermore, we assume that $\dot{\phi}_0$ has very weak r-dependence and so appears as a constant in the spatial equation. One then finds, from equations (32) and (33) that

$$x\mu(x) = \frac{GM}{r^2c^2\,|\,\dot{\phi}_0\,|}\,. \tag{38}$$

Here a factor e^{ϕ} has been absorbed into the definition of G. We set

$$\mu(x) = \tfrac{1}{2}k_c\,x \tag{39}$$

in the limit $x \ll 1$ (the required form in the MOND regime), where k_c is a number between 0.1 and 1 with a physical meaning to be described below. Then, in the low acceleration limit, equation (38) becomes

$$(\nabla\phi)^2 = \frac{2GM}{k_c\,r^2c^3}\,|\,\dot{\phi}\,|\,. \tag{40}$$

From equation (18) we find that the scalar force on slow particles is

$$f_s = \tfrac{1}{2}\nabla\phi c^2\,, \tag{41}$$

or

$$f_s = \left(\frac{GMc\,|\,\dot{\phi}\,|}{2k_c\,r^2}\right)^{1/2}. \tag{42}$$

This is identical to the MOND expression in the low acceleration limit with

$$a_0 = \frac{c\,|\,\dot{\phi}\,|}{2k_c}\,, \tag{43}$$

or, with equation (37),

$$a_0 = \frac{3}{4k_c}\,\frac{\Omega_0}{\kappa}\,(t_0\,H_0)cH_0\,. \tag{44}$$

Thus the possibility of separating the time and space gradients of ϕ in the preferred frame can provide a cosmologically effective theory for MOND.

5. A LIMITING THEORY

5.1. *Weak Field Constraints on the Lagrangian*

Stratification can solve the light-bending problem of scalar-tensor theories while providing a framework for cosmological effective theories of MOND. But can one construct such a theory that is consistent both with the MOND phenomenology and with local gravitational dynamics? In the weak field limit, scalar-tensor theories may be considered as two-field theories of gravity where, in addition to the usual Newtonian force, f_N, there is a scalar force, f_s (a "fifth" force) which is given by equation (41). The simplest such aquadratic theory would be one in which the BM function is

$$F(X) = \tfrac{1}{3}X^{3/2} \tag{45}$$

in equation (33). This yields a scalar force about a point mass with the form of equation (42) (i.e., falling as $1/r$) which exceeds the Newtonian force below accelerations of a_0, i.e., beyond a critical radius given by

$$r_0 = \left(\frac{GM_\odot}{a_0}\right)^{1/2}. \tag{46}$$

The total force in the solar system would then be

$$f_\odot = \frac{GM_\odot}{r^2} + \left(\frac{GM_\odot\,a_0}{r^2}\right)^{1/2}. \tag{47}$$

The problem is that the deviation of $f_\odot$ from inverse-square attraction would severely violate the experimental constraints imposed by planetary precession and limits on the variation of Kepler's constant, $K_\odot = GM_\odot$. For example, in the outer solar system where the deviation would be the largest, the predicted fractional variation in Kepler's constant at distance r from the Sun is $\Delta K_\odot/K_\odot = r/r_0$. At the orbit of Neptune this is 4.2×10^{-3} (with $a_0 = 1.2 \times 10^{-8}$ cm s^{-2}), which is more than a factor of 1000 larger than the existing observational upper limit on $\Delta K_\odot/K_\odot$ ($\leq 2 \times 10^{-6}$) between the orbit of Neptune and the inner planets (Anderson et al. 1995).

Therefore, a theory is required in which the total attraction in the solar system is inverse square to very high precision while yielding MOND phenomenology on the scale of galaxies. A toy theory that can meet these requirements is defined by

$$F(X) = X/\eta \tag{48a}$$

in the limit where $X \geq 1$ and

$$F(X) = \tfrac{1}{3}k_c(1 - X^{3/2})^{-1}\,, \tag{48b}$$

where $X < 1$ (the MOND limit). Here η and k_c are parameters of the theory (in the complete theory, eq. [33], it must be that $\kappa > k_c$ for the existence of stable scalar waves).

With equations (34), (41), and (43) the MOND function becomes

$$\mu(x) = 1/\eta\,, \quad x > 1\,, \tag{49a}$$

$$\mu(x) = \tfrac{1}{2}k_c\,x(1 - x^3)^{-2}\,, \quad x < 1\,, \tag{49b}$$

where the scalar force is given by $f_s = k_c x a_0$. Therefore, k_c is the transition scalar force, in units of a_0, between the high and low acceleration limits of the theory.

Equations (38), (41), and (49) may be solved numerically for the scalar force as a function of distance from a point mass, and this is shown in Figure 1 in the case where $\eta = 1.25 \times 10^{-5}$ and $k_c = 0.34$ (these values will be justified below). Here we see the Newtonian and scalar forces, in units of a_0, as a function of radius, in units of r_0 (eq. [46]). The vertical dashed line corresponds to the position of the orbit of Neptune assuming that the point mass is a solar mass and taking $a_0 = 1.2 \times 10^{-8}$ cm s^{-2} as implied by galaxy rotation curves (Begeman et al. 1991). It is evident that the total attraction $(f_N + f_s)$ is inverse square to the orbit of Neptune; at larger distance the scalar force remains nearly constant at the level of $k_c a_0$, while the total acceleration decreases by about 4 orders of magnitude to a value near a_0; at still larger radii f_s dominates the total attraction, falling as $1/r$. It should be emphasized that there is no theoretical motivation for this assumed form of the BM function (eqs. [48a] and [48b]). It is the form required if the theory is to be consistent with the inverse- square law in the solar system while yielding modified dynamics at accelerations below cH_0.

Equations (38), (41), and (49) may also be solved algebraically for f_s about an extended spherically symmetric mass distribution, where now M is replaced by M_r, the enclosed mass at radius r. In Figure 2 we see the predicted rotation curves for several spherical galaxies with exponential density distributions; the various values of the mass and length scale are indicated. It is evident that the curves are asymptotically flat and structureless with the asymptotic velocity scaling as $M^{1/4}$ as in MOND. However, if k_c is smaller than about 0.3, rotation curves first decline before rising to the asymptotic flat value, in contradiction to the observed form.

5.2. *Post-Newtonian Constraints on the Parameters of the Theory*

Consistency with local gravitational dynamics strongly constrains the values of η and k_c as well as the form of F near the transition acceleration (eqs. [48a] and [48b]). In the high acceleration limit and *in the preferred frame*, the Lagrangian becomes that of a weakly coupled scalar field, as in Brans-Dicke theory with a large value of the Brans-Dicke parameter ω. However, the theory differs from the standard scalar-tensor theories in that the relation between the Einstein and physical metrics is nonconformal. Moreover, as is well known from the measurement of the CMB dipole anisotropy, we are not in the preferred frame: the solar system is moving with a velocity of 370 km s^{-1} with respect to the cosmological frame; therefore, in addition to those relativistic effects associated with a weakly coupled scalar field, geophysical and orbital preferred-frame effects (Nordtvedt & Will 1972; Ni 1972) must also be present at

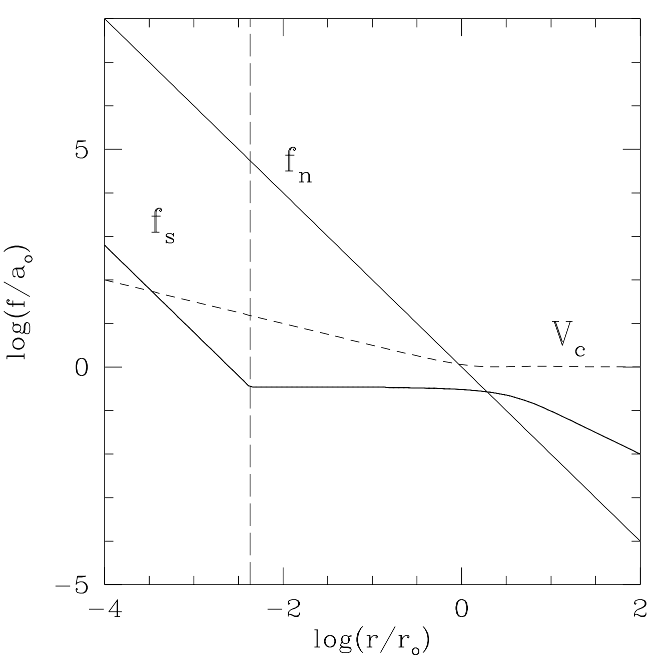

FIG. 1.—A log-log plot of the Newtonian force f_N and the scalar force f_s as a function of distance r from a point mass M (*solid curves*). The scalar force is plotted for the theory described by eqs. (48a) and(48b), where $\eta = 1.25 \times 10^{-5}$ and $k_c = f_s(\text{Neptune})/a_0 = 0.34$. The force is given in units of a_0 and the radius in terms of $r_0 = (GM/a_0)^{1/2}$. The asymptotic behavior of the scalar force is consistent with perfect inverse-square attraction in the solar system to the orbit of Neptune, indicated by the vertical dashed line (where $a_0 = 1.2 \times 10^{-8}$ cm s^{-2}), but approaches $1/r$ at low accelerations (the MOND limit). Smaller values of η further suppress preferred-frame effects, but at the expense of inverse-square attraction within the orbit of Neptune. Smaller values of k_c increase the extent of inverse-square attraction (for a given value of η), but at the expense of asymptotically flat rotation curves. Also shown by the dashed curve is the circular velocity V_c in units of the MOND asymptotic velocity $(GMa_0)^{1/4}$. This is Keplerian inside the transition radius but approaches unity asymptotically.

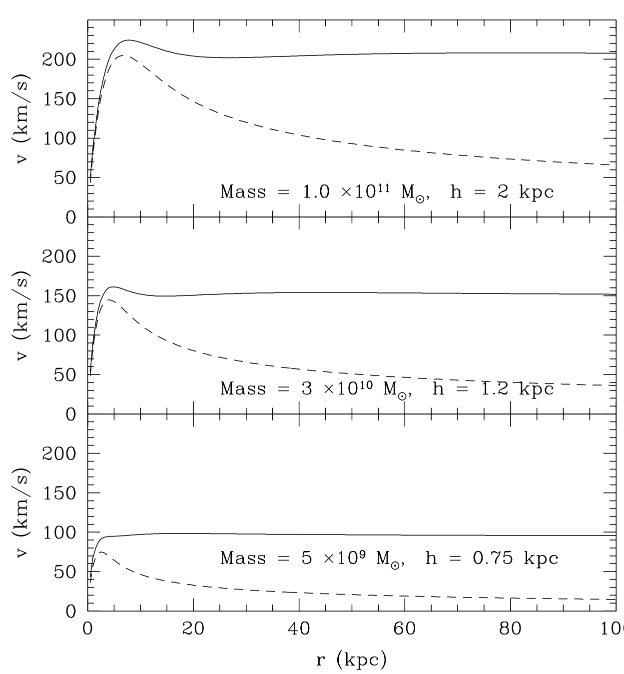

FIG. 2.—Predicted rotation curves of spherical galaxies having an exponential density distribution. The solid curve shows the total rotational velocity as a function of radius. This includes, in addition to the usual Newtonian force, the scalar force which is derived from the toy theory described by eqs. (48a) and (48b), i.e., the theory is identical to that giving the force about a point mass shown in Fig. 1. The dashed curve is the rotation curve resulting from the Newtonian force alone. The masses of the galaxies (in solar units) and the exponential scale lengths are indicated on the figure. The rotation curves are seen to be asymptotically featureless and flat; the asymptotic velocity scales as the one-fourth power of the mass as in MOND.

some level. These include diurnal solid Earth tides, an annual variation of the Earth's rotation frequency, and additional contributions to the anomalous precession of planetary orbits.

For comparison with GR, the magnitude of relativistic and preferred-frame effects peculiar to any alternative theory are conveniently expressed in terms of the parameterized post-Newtonian (PPN) formalism. For the candidate theory the standard parameters can be evaluated following the procedure outlined by Will (1993), and it is found that

$$\gamma = 1 , \tag{50a}$$

$$\beta = \left(\frac{1-\eta/2}{1+\eta/2}\right)^2 , \tag{50b}$$

$$\alpha_1 = \frac{-4\eta}{1+\eta/2} , \tag{50c}$$

$$\alpha_2 = \alpha_3 = 0 . \tag{50d}$$

Those PPN parameters associated with the violation of energy-momentum conservation are zero; i.e., the theory is "semiconservative" in the terminology of Will (1993).

The value of γ is the same as in GR, implying an identical predicted deflection of light about the Sun (not surprising, since the theory was designed with this in mind) as well as identical predictions for radar echo delay. The parameter β ($=1$ in GR) enters into the expression for anomalous relativistic precession of planetary orbits, but the strongest expermental limit is provided by the lunar laser ranging test of the equivalence principle (Dickey et al. 1994). This constrains $\beta < 10^{-4}$; therefore, from equation (50b), it must be the case that $\eta < 10^{-4}$.

The various preferred-frame effects are expressed in terms of the velocity of the solar system (or Earth) with respect to the cosmological frame, w, to second order in w/c, times various combinations of the three post-Newtonian parameters α_1, α_2, and α_3 (in GR all are zero). Will & Nordtvedt (1972) demonstrated that for all standard Lagrangian-based stratified theories (conformally flat on spacelike strata in the preferred universal rest frame) it is the case that $\alpha_2 = \alpha_3 = 0$, and if the light bending is equivalent to that predicted by GR ($k = 2$ in eq. [20]), $\alpha_1 = -8$. However, in the present case, where the physical metric is constructed from the Einstein metric and not from the Minkowski metric (eq. [7]), the preferred-frame effects are suppressed by roughly a factor of 4η (in the limit where η becomes very large, $\alpha_1 \to -8$ as in the standard stratified theories). Combined solar system data constrain $|\alpha_1| < 4 \times 10^{-4}$ (Will 1993). A constraint on the strong field equivalent of this parameter, $\hat{\alpha}_1$, implied by binary pulsar data, is $|\hat{\alpha}_1| < 1.7 \times 10^{-4}$ (Bell et al. 1996). But very recently it has been pointed out that current lunar ranging already constrains α_1 at a level below 10^{-4} (Müller, Nordtvedt, & Vokruoulický 1996); therefore, if we take an experimental upper limit of $|\alpha_1| < 5 \times 10^{-5}$, it must be the case that $\eta < 1.25 \times 10^{-5}$ if the proposed theory is to be viable (i.e., in the limiting case $\alpha_1 = -5 \times 10^{-5}$, which is consistent with the present limit of $-8 \pm 9 \times 10^{-5}$ determined by Müller et al. 1996).

While the nondetection of preferred-frame effects at this level of precision provide an upper limit on η, the two requirements of inverse-square attraction in the solar system and of asymptotically flat, featureless galaxy rotation curves provide, in effect, a lower limit, as well as determining the value of k_s. The total attraction in the high acceleration limit is

$$f_\odot = \left(1 + \frac{\eta}{2}\right)\frac{GM_\odot}{r^2} , \tag{51}$$

and this should extend at least to the orbit of Neptune to ensure consistency with the experimental result of Anderson et al. (1995). Taking $a_0 = 1.2 \times 10^{-8}$ cm s^{-2} as above, this means that inverse-square attraction should extend to $(f_\odot/a_0)_{\rm Nep} = 5.5 \times 10^4$. In other words, the high acceleration limit of the theory (eq. [48a]) should apply down to a transition scalar acceleration of

$$k_c = f_s/a_0 \le (\eta/2)(f_\odot/a_0)_{\rm Nep} . \tag{52}$$

But, as noted above, the prediction of flat, featureless rotation curves as in Figure 2 requires that k_c be greater than 0.3. This, combined with equation (52) and the upper limit on η set by the experimental limit on α_1, requires that

$$0.30 < k_c < 0.35$$

and

$$1.0 \times 10^{-5} < \eta < 1.3 \times 10^{-5} .$$

Thus the window of viability for this theory is very small indeed. In particular, the lower limit on η implies that local preferred-frame effects should soon be detectable at the level of $\alpha_1 \ge 4 \times 10^{-5}$ if this theory is correct.

In a general sense, a stratified theory constructed from the Einstein metric, as opposed to the Minkowski metric, can predict very weak local preferred-frame effects because it is a two-field theory with a nonstandard scalar field action: in addition to the scalar force there is the usual Einstein-Newton force. It is the scalar force that ties the solar system to the cosmological frame; the local tensor field is not influenced by motion with respect to this frame. Because of the peculiar aquadratic scalar-field Lagrangian it is this usual Einstein-Newton force (the first term in eq. [13]) which becomes dominant in the limit of large accelerations (in the solar system or on the surface of the Earth). In effect, the preferred-frame effects are suppressed by the factor $f_s/f_{\rm N}$, the ratio of the scalar force to the Newtonian force. Thus the very same scalar-field Lagrangian that yields MOND phenomenology on the scale of galaxies suppresses local preferred-frame effects.

On the scale of galaxies, the theory is not Newtonian, so it is inappropriate to speak of post-Newtonian parameters. But, because the scalar force dominates the Einstein-Newton force on this scale, the preferred-frame effects should be present with their full magnitude. It is not clear that this could influence the structure of galaxies.

With respect to the original binary pulsar, the candidate theory, as a scalar-tensor theory, would lead to the emission of dipole radiation in addition to the usual quadrupole gravitational radiation. However, because the scalar field is so weakly coupled in the high acceleration limit, it is expected that the dipole radiation would also be suppressed by a factor of η. Thus there is not likely to be a predicted contradiction with the observed rate of orbital decay in the binary pulsar, although this has not yet been worked out in detail.

There is one additional local scalar-tensor effect that cannot be suppressed. As in any scalar-tensor theory, there is a cosmic variation of the gravitational constant. In this case the magnitude of this effect is

$$\frac{\dot{G}}{G} = -\dot{\phi}_0 . \quad (53)$$

For the toy theory considered here, with the use of equation (43), this becomes

$$\frac{\dot{G}}{G} = \frac{2k_c a_0}{c} \approx 7.0 \times 10^{-12}\left(\frac{a_0}{10^{-8}\ \mathrm{cm\ s^{-2}}}\right) \mathrm{yr}^{-1} . \quad (54)$$

There is an similar expression in the context of PCG (Sanders 1989), and this suggests that $\dot{G}/G \approx a_0/c$ applies to any cosmological effective theory for MOND based upon scalar-tensor theory. Determination of $\dot{G}/G$ by ranging measurements are already at levels of precision below 10^{-11} yr^{-1} (Will 1993); thus time variation of G should also soon be detected if MOND is correct and scalar-tensor theory is its basis.

In summary, aquadratic stratified theories of modified dynamics are very strongly constrained by three observational requirements: the necessity of producing almost perfect inverse-square attraction in the solar system out to Neptune; the avoidance of detectable preferred-frame effects at the location of the Earth (at least at the present levels of experimental precision); and the necessity of predicting MOND phenomenology on the scale of galaxies and clusters of galaxies. The toy theory described by equations (48a) and (48b) barely satisfies these requirements. Values of η much larger than 10^{-5} are ruled out by the present constraints on local preferred-frame effects. Smaller values of η suppress preferred-frame effects, but at the expense either of inverse-square attraction in the solar system or of asymptotically flat, structureless galaxy rotation curves. Changing the theory in such a way that the scalar force falls more rapidly with radius in the transition region between inverse-square and MOND attraction (a BM function intermediate between eqs. [45] and [48a, b]) makes matters worse; such theories are already ruled out because they violate the constraints on either local inverse-square attraction or preferred-frame effects. A scalar force that actually increases with radius could work, but such a theory is impossible in the context of one-scalar aquadratic theory. So in that sense, equations (48a) and (48b) describe a limiting case for aquadratic stratified theories in which the scalar force decreases monotonically with radius; if this theory is not viable, then no such theories are viable.

6. CONCLUSIONS

As first emphasized by Milgrom, the near numerical coincidence of the MOND acceleration parameter a_0 with cH_0 provides a very important clue to the theoretical basis of MOND: the cosmological background affects local dynamics in a way that is closely approximated by the MOND prescription. Such considerations clearly rule out GR because this theory predicts no such direct cosmological influence of this magnitude on local dynamics. AQUAL, an unconventional scalar-tensor theory for MOND (BM; Sanders 1986), is in no sense such an effective theory because a_0 is explicitly written in by hand and the theory has no cosmological limit. PCG (Bekenstein 1988), as one of a class of scalar-tensor theories characterized by two scalar fields coupled in the kinetic term of one of them, is such an effective theory with the cosmic time derivative of the matter-coupling field playing the role of a_0 (Sanders 1989). However, AQUAL, PCG, and all scalar-tensor theories in which the field couples to matter as a conformal factor multiplying the Einstein metric fail to reproduce the observed deflection of light by clusters of galaxies (Bekenstein & Sanders 1994).

This problem can be solved by reintroducing and generalizing the concept of conformal coupling only on spacelike strata of a preferred universal frame. The traditional means of singling out such a preferred frame is through the introduction of a nondynamical universal vector field with the only nonzero component being the time component in the preferred cosmic frame. The essential new ingredient presented in this paper is that the introduction of such a cosmic vector field can, at the same time, both solve the light-bending problem of scalar-tensor theories and also permit an aquadratic scalar field Lagrangian to be written without the explicit introduction of a new dimensional parameter a_0. That is to say, with this single new element one can write a cosmological effective scalar-tensor theory of MOND which also predicts the degree of the gravitational deflection of light actually observed in cosmic gravitational lenses.

It should be emphasized, however, that this is not a traditional stratified theory, in that the physical metric is constructed from the Einstein metric and not the Minkowski metric (eq. [7]). The only a priori element is the vector field. Therefore, Einstein's field equations are retained (with additional source terms), and, in the particular candidate theory considered here, the traditional Einstein-Newton force becomes dominant in the high acceleration regime; such theories are indistinguishable from GR to high precision on the scale of the inner solar system and binary pulsar. For accelerations comparable to those prevailing in the inner solar system, the toy theory considered here reduces to a weakly coupled scalar-tensor theory which, because of the nonconformal relation between the two metrics, differs from Brans-Dicke theory in that the predicted light deflection and radar echo delay are precisely the same as in GR, although preferred-frame effects are present at some level. However, the dominance of the Einstein-Newton force on this scale also implies that the inevitable preferred-frame effects are suppressed by a factor of 4η, where η is a parameter of the theory that can be small but not arbitrarily small. Moreover, theories of this form make rather precise predictions on the cosmic variation of the gravitational constant ($\approx a_0/c$). Of course, one could reasonably expect that any relativistic generalization of MOND would lead to local deviations from the predictions of GR at some level. From this point of view the continued design of local gravity tests with higher precision is a valuable activity.

The structure of this class of theories (eqs. [32a]–[32e]) appears, at least superficially, to be similar to the aquadratic theories of BM and of Sanders (1986). These earlier AQUAL theories have been considered unphysical because of the predicted superluminal propagation of scalar waves and the implied violation of causality (Bekenstein 1988). The same objection does not necessarily apply to theories of this general type because the replacement of the MOND function μ by a tensor $P^{\alpha\beta}$ (eq. [32a]) does, in fact, give the theory a different structure. In general, the properties of stability and causality depend upon the precise form of F and the value of κ (eq. [33]). Whether or not causality and stability can be reconciled with the cosmological origin of

a_0 is the subject of a later paper on AQUAL and two-scalar preferred-frame theories.

The introduction of an a priori field is an unattractive element in any theory. It might well be that in a consistent theory the vector field must be dynamical—a universal vector field coupled to gravity as in the theory of Will & Nordtvedt (1972), or perhaps the normalized scalar field gradient itself as in Bekenstein's (1992) disformal transformation. Dynamical or not, the vector singles out a preferred frame; in more fully dynamical theories the preferred frame effects might be further suppressed by the appearance of cosmological matching parameters as in the generalized stratified theory of Lee et al. (1974). But it should be noted that the vector field as written here does more than select a preferred frame; it also breaks the time-reversal invariance of gravitational physics in a fundamental way. It literally is the arrow of time written in by hand.

From an observational point of view there clearly is a universal preferred frame—that in which the CMB dipole vanishes. There is also a universal cosmic time which appears to possess a sense of direction not present in the spacelike dimensions of this frame. One might speculate that cosmology is described by a preferred-frame, time-irreversible theory of gravity—a stratified theory with long-range interaction primarily mediated by a scalar field—while local gravitational dynamics, i.e., at accelerations higher than the natural cosmic value of cH_0, are described by GR. A consequence of such a supposition is the necessity of scalar field dynamics similar to that of the toy theory described here, i.e., aquadratic scalar-tensor theory or an equivalent two-scalar theory like PCG. That is to say, the reconciliation of preferred-frame cosmology with general relativistic local dynamics (very weak local preferred-frame effects) would *require* the modified dynamics at low acceleration. But such speculation is only suggested by the observational appearance of a preferred cosmic frame and, at present, has no justification in deeper theory.

In summary, the result of Bekenstein & Sanders (1994) on gravitational lensing in the context of scalar-tensor theories (i.e., that the lensing mass of a cluster should be substantially less than the virial mass) dealt a serious blow to such theories as a foundation for MOND. The essential result here is that scalar-tensor theories can be cured of this ailment apparently only at the expense of rewriting them as stratified theories; but then the Lorentz invariance of gravitational dynamics is broken and local preferred-frame effects are inevitable. The nonstandard scalar-field Lagrangian giving rise to MOND phenomenology allows preferred-frame effects to be suppressed, but not by an arbitrary factor due to the necessity of producing asymptotically flat spiral galaxy rotation curves while satisfying the strict experimental limits on deviations from inverse-square attraction in the outer solar system. The nondetection of preferred-frame effects at a level of a factor of 3 below current limits would not necessarily rule out MOND, but it would inflict serious damage on stratified scalar-tensor theories of MOND. It is clear that if one adopts the point of view that the mass discrepancy in large astronomical systems is due to an incomplete understanding of gravity rather than to dark matter, then consistency with phenomena ranging from gravitational lensing by clusters of galaxies to galaxy rotation curves to planetary motion already imposes stringent conditions on acceptable field theories.

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