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ORBITS IN A NEIGHBORING DWARF GALAXY ACCORDING TO MODIFIED NONRELATIVISTIC DYNAMICS

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ABSTRACT

We study the orbits in the modified nonrelativistic dynamics (MOND) theory within a dwarf galaxy of mass $M_d \sim 10^8\ M_\odot$ at a distance of ~ 100 kpc from a neighboring galaxy of mass $M_g = 5 \times 10^{11}\ M_\odot$, such as ours. It is assumed that a second mass $m \ll M_d$ is gravitationally bound to M_d by a previously calculated potential for the MOND theory. This potential is obtained for a free-falling mass M_d in a constant external gravitational acceleration field $\nabla\phi_g$. The numerical technique of surfaces of section is used to study the stability of the phase-space orbits in the dwarf galaxy. Equatorial orbits with sufficiently small eccentricities $e < 0.65$ are found to be stable with respect to small changes in the initial conditions. (The equatorial plane is perpendicular to the direction of $\nabla\phi_g$, which is along the line joining M_d and M_g.) For decreasing values of the conserved component of the angular momentum, in the direction of $\nabla\phi_g$, equatorial stability is lost.

Subject headings: cosmology: theory — galaxies: dwarf — galaxies: kinematics and dynamics — gravitation

1. INTRODUCTION

The inconsistency between luminous and dynamical mass measurements is well known. An alternative way to explain this inconsistency without dark matter is by modified nonrelativistic dynamics (MOND) for galaxies and galactic systems (Milgrom 1983a, 1983b, 1983c). This theory was later developed into an alternative theory of gravitation by Bekenstein & Milgrom (1984), which we hereafter refer to as the BM theory. MOND agrees with the Tully & Fisher (1977) and Faber & Jackson (1976) laws and explains the dynamics in elliptical and spiral galaxies, without the need for dark matter. It satisfies conservation of energy, momentum, and angular momentum, as well as the weak equivalence principle (Bekenstein & Milgrom 1984).

Recent studies of rotation disks of low surface brightness (LSB) galaxies by Salcedo & Gámez (1999), de Block & McGaugh (1998), and McGaugh & de Block (1998) are in agreement with MOND.

Departure of MOND from Newtonian theory is not connected with a distance scale. In MOND, deviations from Newtonian theory become significant for very small accelerations. The main argument presented is that Newtonian gravitation has not been tested for very weak fields and it could be that the theory is not valid in this regime. Other theories modifying Newtonian theory were reviewed by Bekenstein (1987).

Stability of galactic disks in the BM theory was first studied in a WKB approximation by Milgrom (1989). The WKB scheme deals with perturbation wavelengths $\lambda = 2\pi/k$, much smaller than the involved distances ρ on the disk ($|k\rho| \gg 1$). In Newtonian gravitation, the presence of a dark halo is important in stabilizing the disk against violent bar formation (Ostriker & Peebles 1973). Recent N-body simulations for the BM theory indicate that disks are more stable in MOND than in Newtonian dynamics with dark halos (Brada & Milgrom 1999).

We obtain the conservative Hamiltonian that describes the motion of a particle in a previously calculated potential for the BM theory in § 2. This section also includes a summary of the stability theory for phase-space orbits. Orbits in a dwarf galaxy are discussed in § 3. Our conclusions are presented in § 4.

2. THEORY

In the BM theory, the Poisson equation for determining the gravitational potential is modified to

$$\nabla \cdot \left[\mu\left(\frac{|\nabla\varphi|}{a_0}\right)\nabla\varphi\right] = 4\pi G\rho\ , \tag{1}$$

where $a_0 = 2 \times 10^{-8}$ cm s^{-2} is a constant with the dimension of acceleration, set so as to agree with the Tully-Fisher law (Milgrom 1983b); the function $\mu(x)$ (where $x = |\nabla\varphi|/a_0$) obeys $0 < \mu(x) < 1$, with $\lim_{x\to 0} \mu(x) = x$, and G is the Newtonian gravitational constant. With this particular value of a_0, non-Newtonian effects caused by the solar gravitational field are expected to begin only beyond the Oort cloud (Oort 1963).

Let us assume the existence of a constant external gravitational acceleration $\nabla\phi_g$ due to some source (e.g., a neighboring massive galaxy, such as ours) and a free-falling sphere of mass M_d (e.g., the center of a dwarf galaxy). Sufficiently close to M_d so that $\mu(x) \approx 1$, the solution of equation (1) yields the well-known Newtonian potential

$$\varphi \approx \frac{GM_d}{r}\ .$$

When the solution φ of equation (1) implies gravitational accelerations much bigger than the external gravitational acceleration ($|\nabla\varphi| \gg |\nabla\phi_g|$) but $|\nabla\varphi| < a_0$, spherical symmetry can be assumed, and the potential is obtained from equation (1) in the limit $\mu(x) \approx x$, using Gauss's theorem

$$\varphi \approx (GM_d a_0)^{1/2} \log r\ . \tag{2}$$

In the opposite limit, when $|\nabla\varphi| \ll |\nabla\phi_g|$, Bekenstein & Milgrom (1984) have calculated the approximate potential

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of equation (1) in cylindrical coordinates:

$$\varphi = -\frac{[\mu_g(1 + L_g)^{1/2}]^{-1} M_d G}{[z^2(1-\alpha_g) + \rho^2]^{1/2}} , \tag{3}$$

where z is the direction of $\nabla\phi_g$, $\mu_g = \mu(|\nabla\phi_g|/a_0)$, $L_g = d\ln\mu/d\ln x|_{x=|\nabla\phi_g|/a_0}$, and $\alpha_g = L_g/(1 + L_g)$. In the asymptotic Newtonian limit $|\nabla\phi_g| \gg a_0$, $\mu_g \to 1$, $L_g \to 0$, and $\alpha_g \to 0$. In the non-Newtonian limit, which we call the MOND limit, $|\nabla\phi_g| \ll a_0$,

$$\mu_g(x) \to x , \tag{4}$$

$$L_g(x) \to 1 , \tag{5}$$

$$\alpha_g \to \tfrac{1}{2} . \tag{6}$$

It is to be noted that equation (3) is not a consequence of tidal effects since the external field is constant (Bekenstein & Milgrom 1984).

We study the orbit of a very small mass $m \ll M_d$, such that it does not modify the potential in equation (3), and consider the motion of m, bounded by this potential. This is a central-force type of problem, so that conservation of linear momentum implies that the coordinate dependence of the Hamiltonian is in the relative distance between M_d and m. According to equation (3), and considering the azimuthal symmetry of the potential, the dynamics of the $M_d - m$ system is governed by the conservative Hamiltonian

$$H = \frac{1}{2\mu_r}\left(p_\rho^2 + p_z^2 + \frac{l_z^2}{\rho^2}\right) - \frac{\lambda}{[z^2(1-\alpha_g)+\rho^2]^{1/2}} , \tag{7}$$

$$\lambda = [\mu_g(1 + L_g)^{1/2}]^{-1} G M_d m , \tag{8}$$

where $\mu_r = M_d m/(M_d + m) \approx m$ is the reduced mass and l_z is the angular momentum component in the direction of $\nabla\phi_g$ (the direction joining the neighboring massive galaxy and the dwarf galaxy). Thus, the three-dimensional motion in an axisymmetric potential is reduced to the motion in a plane. In equations (7) and (8), Cartesian coordinates (ρ, z) are used to describe this (nonuniformly) rotating plane, which is often called the meridional plane.

In the MOND limit, when $|\nabla\phi_g| \ll a_0$, equation (3) with the condition $|\nabla\varphi| \ll |\nabla\phi_g|$ yields a minimum distance between M_d and m:

$$r_{\min} = \left[\frac{GM_d}{\sqrt{2}a_0}\right]^{1/2} \frac{a_0}{|\nabla\phi_g|} , \tag{9}$$

where the values for L_g and μ_g are obtained from equations (4)–(6). We note that the forces in the ρ and z directions are not equal.

The Hamiltonian (eq. [7]) has an elliptic equilibrium position at $z = 0$, $\rho = l_z^2/(\mu_r\lambda)$, which is called the guiding center. Expanding the Hamiltonian in a power series about the guiding center, we have

$$H = H_0(I) + H_1(I, \theta) ,$$

$$H_0(I) = \omega_z I_z + \omega_\rho I_\rho , \tag{10}$$

$$\omega_\rho = \frac{\lambda^2\mu_r}{l_z^3} , \quad \omega_z = \frac{\lambda^2\mu_r(1-\alpha_g)^{1/2}}{l_z^3} . \tag{11}$$

This expansion is called the epicycle approximation and is appropriate in the neighborhood of the equilibrium. H_0 is the unperturbed motion and describes the epicycles around the guiding center; the two frequencies ω_ρ and ω_z given in equation (11) are called the epicycle frequency and the vertical frequency, respectively. In phase space, the conservation integrals define a two-dimensional torus. When $j_z\omega_z + j_\rho\omega_\rho = 0$, $j = |j_z| + |j_\rho|$ for integers $j_{z,\rho}$, the linearized frequencies are said to satisfy a resonance of order j. The corresponding torus is called a resonant torus. From equations (6) and (11), the allowed linearized resonances are restricted to $\alpha_g < 1/2$ (see Table 1).

For a resonance of order j, it is possible to put the Hamiltonian into a Birkhoff normal form of degree j by a set of canonical transformations

$$H = K_0(J) + K_1(J, \Phi) ,$$

where K_0 is a polynomial of degree j in the new actions J, and Φ are the new angles (Arnold 1989). K_0 is a good approximation when the perturbation K_1 is sufficiently small. The new frequencies are then

$$\nu_i = \frac{\partial K_0(J)}{\partial J_i} \tag{12}$$

and are functions of the new amplitudes J_i, a phenomenon first discovered by Lindstedt (1882) in connection with nonlinear oscillators.

Nondegenerescence is defined as the nonvanishing of the Hessian determinant

$$\det\left|\frac{\partial\nu_i}{\partial J_k}\right| \neq 0 . \tag{13}$$

There exists a theorem of Kolmogorov (1954) on the behavior of a nonresonant torus under a small perturbation K_1 of a nondegenerate Hamiltonian K_0 (Arnold 1989), which was subsequently proved by Arnold (1961) for Hamiltonian systems and by Moser (1962) for area-preserving maps (see also Arnold 1963 and Moser & Siegel 1971). The theorem, known as KAM in recognition of their work, states the existence of invariant tori, densely filled with phase-space curves winding around them, which are conditionally periodic with the number of independent frequencies equal to the number of degrees of freedom. This theorem is valid if the following condition holds:

$$|(\nu \cdot j)| > C|j|^\tau . \tag{14}$$

In equation (14), C is dependent on the magnitude of the perturbation and τ on the number of degrees of freedom. The existence of an invariant, or KAM, torus surrounding a periodic orbit characterizes the stability. Orbits whose frequencies satisfy equation (14) occupy most of the volume in phase space for sufficiently small C (Moser & Siegel 1971).

TABLE 1

RESONANCES

α_g	ω_z/ω_ρ
$\frac{7}{16}$	$\frac{3}{4}$
$\frac{9}{25}$	$\frac{4}{5}$
$\frac{11}{36}$	$\frac{5}{6}$
$\frac{24}{49}$	$\frac{5}{7}$

The conserved l_z is an isolating integral for the full 3 dof system; the 2 dof system described by equation (7) is a subspace of constant l_z of the original 3 dof system. A surface of section can be constructed in the spirit of Hénon & Heiles (1964). In a 2 dof system—for instance, equation (7)—the dimension of the phase space is four. Since the energy is an isolating integral, motion is constrained to a three-dimensional constant energy surface, for example, ρ, z, and p_z. Successive intersections of the trajectory with the plane $\rho =$ constant and $p_\rho > 0$, described by the set of points z and p_z (or $\dot{z} = dz/dt$), is called a surface of section. Each intersection of the orbit is a point in this plane, and the passage of one point to the next can be considered as a mapping. After an infinite time interval, the points corresponding to a unique orbit fill up a whole area of the surface of section, in the absence of further isolating integrals.

Motion is constrained to the intersection of the surfaces of constant H and I in the presence of a third isolating integral. Consider an orbit whose initial conditions lie in the phase-space region of conserved I. Its points in the surface of section form a smooth curve.

According to the Poincaré-Birkhoff theorem for resonant tori, after the inclusion of the perturbation K_1, we have an even number of periodic orbits in the vicinity of a stable periodic orbit. Periodic orbits are exact resonances of the nonlinear problem. The corresponding surface of section is a fixed point, surrounded by an even number of other fixed points. Half of them are stable, while the other half are unstable. Stable points are surrounded by closed invariant curves. Unstable points are connected by separatrices, forming a structure called a chain of islands. Chaos is present in the vicinity of the separatrices. (For further details see Lichtenberg & Lieberman 1983 and Dunby 1971.)

3. ORBITS IN A DWARF GALAXY

In the following, we describe the motion of a particle m (e.g., a globular cluster) around a spheroid of mass M_d (the nucleus of a dwarf galaxy) with $M_d \gg m$, as shown in Figure 1. We take

$$M_d = 10^8\ M_\odot\ ,$$
$$m = 10^5\ M_\odot\ . \qquad (15)$$

We assume that the M_d-m system is gravitationally bounded to a nearby massive galaxy $M_g = 5 \times 10^{11}\ M_\odot$ (such as the Milky Way), separated by a distance $d = 100$ kpc. From equation (2), the nearby massive galaxy generates a gravitational acceleration on the order of

$$|\nabla\phi_g| \sim (GM_g a_0)^{1/2}/d \sim 5.36^{-1} a_0\ , \qquad (16)$$

within the M_d-m system. The period of the test particle is appreciably less than that of the dwarf galaxy's orbit. Possible resonances between the dwarf galaxy's orbital period and that of the test particle are neglected. According to equation (9), equation (3) is valid for distances greater than $r_{\min} = 1189$ pc, and we choose

$$r \approx 2378 \text{ pc}\ . \qquad (17)$$

Tidal accelerations for distances such as r are neglected, so that $\nabla\phi_g$ is considered to be a constant external gravitational acceleration within the M_d-m system. The geometry is shown in Figure 1.

If the initial conditions are chosen as $z = p_z = 0$, according to equation (7), Kepler's theory is recovered with an effective gravitational constant (eq. [8]). These initial conditions correspond to orbits constrained to the equator of M_d

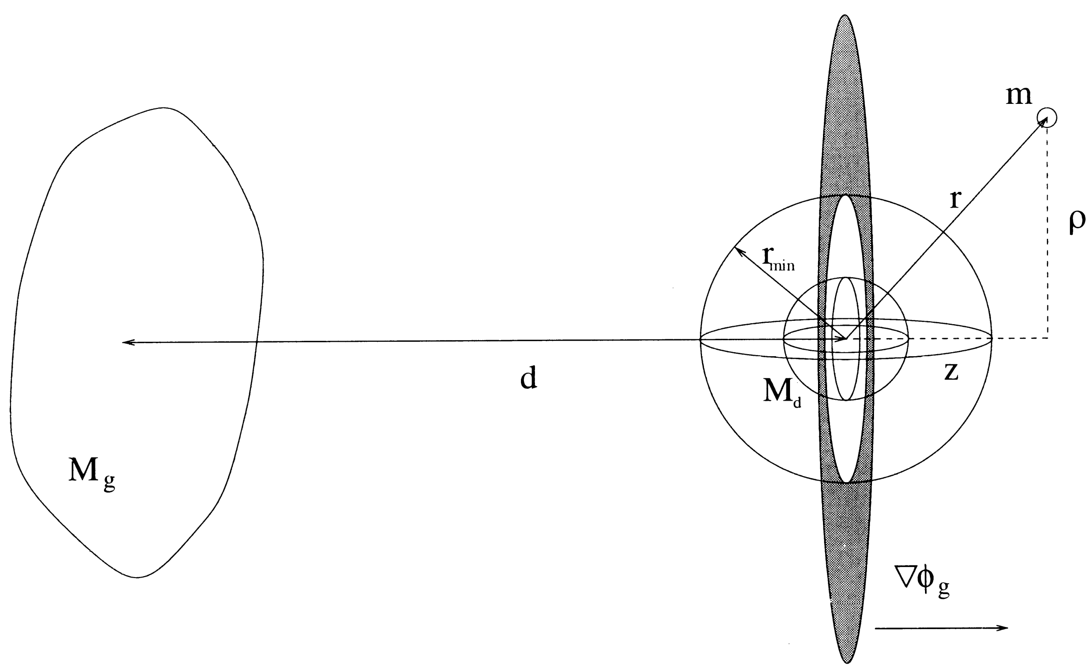

FIG. 1.—Schematic diagram, showing the two masses $M_d = 10^8\ M_\odot$ (e.g., nucleus of a dwarf galaxy) and $m = 10^5\ M_\odot$ (e.g., a globular cluster) and the neighboring galaxy $M_g = 5 \times 10^{11}\ M_\odot$ (e.g., the Milky Way). According to the potential given in eq. (2), the M_d-m system is gravitationally bounded to M_g, at an approximate distance of $d = 100$ kpc. The external gravitational acceleration field due to M_g is $|\nabla\phi_g| \sim 5.36^{-1} a_0$, where $a_0 = 2 \times 10^{-8}$ cm s^{-2}, is the MOND constant. The z direction is the projection of the relative distance r between M_d and m in the direction of the external field $\nabla\phi_g$, and ρ is the orthogonal projection. The equatorial plane is defined by $z = 0$ and is shown in gray; $r_{\min}$ is the minimum distance for which the potential in eq. (3) is valid.

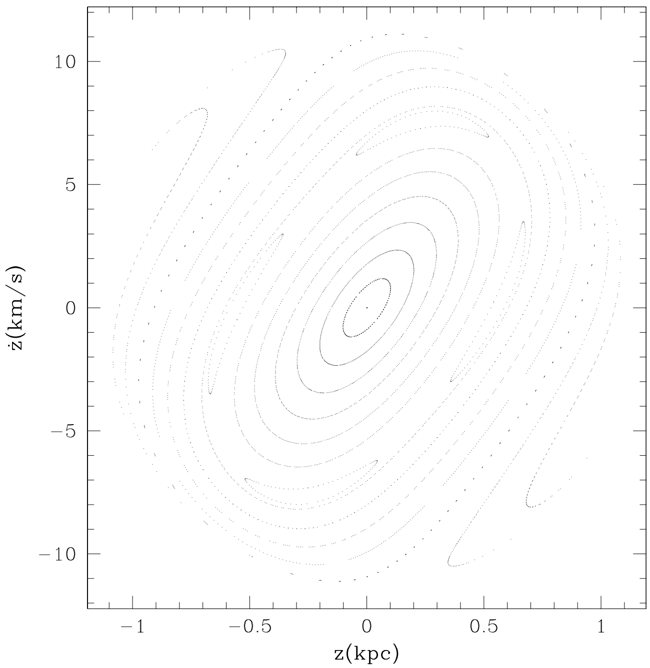

FIG. 2.—Surface of section for $E = -6.817 \times 10^{50}$ ergs and $l_z/\mu_r = 57.090$ kpc km s^{-1}.

and are known as equatorial orbits. (The equatorial plane is defined as the plane containing M_d, perpendicular to the z direction [see Fig. 1].) It then follows that

$$l_z^2 = \mu_r \lambda a(1 - e^2) , \quad (18)$$

$$H = E = -\frac{\lambda}{2a} , \quad (19)$$

$$T = 2\pi a^{3/2}\left(\frac{\mu_r}{\lambda}\right)^{1/2} , \quad (20)$$

where a is the semimajor axis, e the eccentricity, T the period, and E the energy. In Newtonian theory, the relation

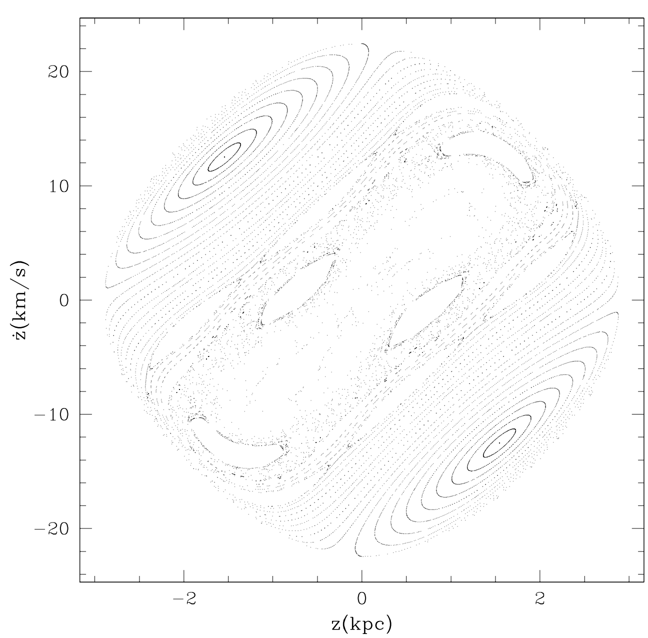

FIG. 3.—Surface of section for $E = -6.817 \times 10^{50}$ ergs and $l_z/\mu_r = 41.201$ kpc km s^{-1}.

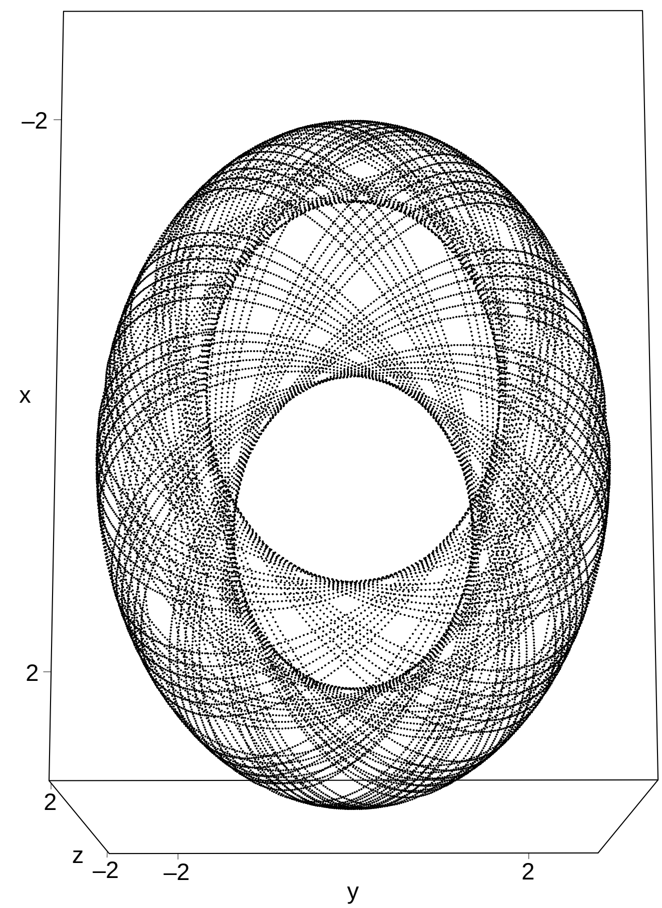

FIG. 4.—Orbit in (x, y, z) space in units of kpc, corresponding to the 2:1 resonance of Fig. 3.

between the period and the semimajor axis is Kepler's third law. The equatorial orbits are uniquely defined by equations (18) and (19).

In the MOND limit, the period predicted by equation (3) for equatorial orbits follows from equations (4)–(6), (8), and (20):

$$T \approx 2\pi r^{3/2}\left[\frac{\sqrt{2}\,|\nabla\phi_g|/a_0}{G(M_d + m)}\right]^{1/2} ,$$

which, according to equations (15), (16), and (17), results in

$$T \approx 5.6 \times 10^8 \text{ yr} . \quad (21)$$

The physical configuration defined by equations (15) and (16) is described in the following manner. We assume the MOND limit in equations (4) and (5) and choose $\alpha_g = 7/16$, corresponding to the linearized lower order resonance of $\frac{3}{4}$ in Table 1. Under these conditions, the surfaces of section for the phase orbits of equation (7) (Figs. 2 and 3) are obtained for an energy $E = -6.817 \times 10^{50}$ ergs, which corresponds to distances of the order of that in equation (17). As a numerical check, the energy was found to be conserved to one part in 10^{10}.

Regions of instability increase for smaller values of l_z. Equatorial orbits are defined as $z = p_z = 0$ in equation (7). For equatorial orbits, l_z is connected to the eccentricity by equation (18). According to equation (14), for equatorial orbits with sufficiently small eccentricities almost the entire phase space is occupied by KAM tori surrounding the stable orbit, as shown in Figure 2. The surface of section in Figure 2 is obtained for $l_z/\mu_r = 57.09$ kpc km s^{-1}, which corresponds to an equatorial orbit with eccentricity $e = 0.4$. It can be seen in Figure 2 that this particular equatorial orbit is stable.

The MOND potential given in equation (3) is valid for distances greater than $r_{\rm min}$, defined in equation (9). This potential predicts equatorial orbits that belong to a plane perpendicular to the external gravitational acceleration produced by M_g (see Fig. 1). We found numerically that the equatorial orbits are stable for eccentricities $e < 0.65$.

With decreasing values of l_z, for example, $l_z/\mu_r = 41.20$ kpc km s^{-1}, the equatorial orbit becomes unstable. There exist other stable periodic orbits, which are not in the plane $z = 0$ (see Fig. 3).

Associated with the upper and lower stable islands in Figure 3, there is an exact 2:1 resonance, which is a periodic orbit in the meridional plane, defined by (ρ, z) coordinates. In three-dimensional space (x, y, z), this periodic orbit corresponds to a quasi-periodic orbit in a spatial region, which is a rounded cylindrical shell surrounding M_d, as shown in Figure 4. A closed orbit in three-dimensional space requires an exact resonance between the azimuthal, vertical, and epicycle frequencies.

4. CONCLUSIONS

There exists a previously calculated potential φ in the MOND theory, given by equation (3), for a free-falling sphere of mass M_d in a constant external gravitational acceleration $\nabla\phi_g$. The potential φ is valid when $|\nabla\phi_g| \gg |\nabla\varphi|$. We assume that the existence of a second mass $m \ll M_d$ does not modify the potential.

The system M_d-m, bounded by the potential φ, is described by the Hamiltonian given in equation (7). Since the potential is axisymmetric, only the component of the angular momentum in the direction of $\nabla\phi_g$, l_z is conserved. As a consequence, when $z = p_z = 0$ and l_z is the total angular momentum, motion is periodic and occurs in a plane perpendicular to $\nabla\phi_g$, which we call the equatorial plane (Fig. 1). Kepler's theory is then recovered with an effective gravitational constant (eq. [8]), and l_z is related to the eccentricity by equation (18).

We analyze the phase space of H, using the numerical technique of surfaces of section. It is found that for eccentricities $\epsilon < 0.65$, the equatorial orbits are stable with respect to small variations in the initial conditions (Fig. 2). For smaller values of l_z, the stability of the equatorial orbit is lost, and there is an increase in the size of the chaotic regions (Fig. 3). There is a disk perpendicular to $\nabla\phi_g$, as well as some regions not in the disk, that are stable against variations of initial conditions.

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