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LARGE-SCALE FILAMENTS: NEWTONIAN VERSUS MODIFIED DYNAMICS

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ABSTRACT

Eisenstein, Loeb, & Turner (ELT) have recently proposed a method for estimating the dynamical masses of large-scale filaments, whereby the filament is modeled by an infinite, axisymmetric, isothermal, self-gravitating, radially virialized cylinder, for which ELT derive a global relation between the (constant) velocity dispersion and the total line density. We show that the model assumptions of ELT can be relaxed materially: an exact relation between the rms velocity and the line density can be derived for any infinite cylinder (not necessarily axisymmetric) with an arbitrary constituent distribution function (so isothermality need not be assumed). We also consider the same problem in the context of the modified Newtonian dynamics (MOND). After we compare the scaling properties in the two theories, we study two idealized MOND model filaments, one with assumptions similar to those of ELT, which we can only solve numerically, and another, which we solve in closed form. A preliminary application to the same segment of the Perseus-Pisces filament treated by ELT gives MOND M/L estimates of order $10(M/L)_{\odot}$, compared with the Newtonian value $M/L \sim 450(H_0/100\ \text{km s}^{-1}\ \text{Mpc}^{-1})(M/L)_{\odot}$ that ELT find. In spite of the large uncertainties still besetting the analysis, this instance of MOND application is of particular interest because (1) objects of this geometry have not been dealt with before; (2) it pertains to large-scale structure; and (3) the typical accelerations involved are the lowest so far encountered in a semivirialized system—only a few percent of the critical MOND acceleration—leading to a large predicted mass discrepancy.

Subject headings: cosmology: theory — galaxies: clusters: individual (Perseus-Pisces) — gravitation — large-scale structure of universe

1. INTRODUCTION

In a recent paper Eisenstein, Loeb, & Turner (1996, hereafter ELT) describe a method for estimating the dynamical masses of large-scale filaments. Elementary considerations lead one to expect that when such a filament is virialized in the radial direction, its total line density (mass per unit length), μ_0, and its typical velocity dispersion, σ, are related by $G\mu_0 \sim \sigma^2$. For concreteness, ELT consider a model of these filaments consisting of an infinitely long, axially symmetric, self-gravitating cylinder whose constituent velocity dispersions are constant (we call it isothermal even though the velocity distribution need not be "thermal"). For such cylinders, ELT derive the exact relation

$$G\mu_0 = 2\sigma_\perp^2 , \tag{1}$$

where $\sigma_\perp$ is the velocity dispersion along a line of sight perpendicular to the cylinder axis, averaged over the whole cylinder. ELT study carefully the applicability of this relation by testing its performance on N-body–simulated filaments. Finally, they apply this relation to a segment of the Perseus-Pisces filament, and estimate for it a B-band M/L ratio of about $450(M/L)_{\odot}$, implying large quantities of dark matter.

The modified dynamics (MOND) was proposed as an alternative theory to Newtonian dynamics with a view to eliminating the need for dark matter (Milgrom 1983). It posits that in the limit where the typical accelerations in a system governed by gravity are much smaller than some value ($a_0 \sim 10^{-8}$ cm s^{-2})—the deep-MOND limit—the mass M, radius r, and acceleration a are related by $a^2/a_0 \approx MG/r^2$, and not by the standard $a \approx MG/r^2$. So the smaller the mean acceleration in a system, the larger the expected mass discrepancy deduced by a Newtonian analysis.

MOND has been tested extensively on galaxies (Begeman, Broeils, & Sanders 1991; Sanders 1996), and it is natural to check how well it does in the context of large-scale structure. As explained in Milgrom (1989), MOND is not developed enough to afford application to most large-scale structures with overdensity not much exceeding unity or where the Hubble flow is important. Because large-scale filaments seem to be at least radially virialized, they offer a unique opportunity to apply MOND to large-scale structure. Inasmuch as a filament is not completely virialized, and extends to radii where the Hubble flow is important, our analysis can be valid only at smaller radii.

Large-scale structures constitute an extreme from the point of view of MOND because they involve accelerations that are the lowest in the range so far observed, and they carry the usual importance and interest of extremes: Values as low as $0.1a_0$ are measured at the outskirts of dwarf spirals (e.g., Sanders 1996), while, as we shall see below, in the Perseus-Pisces filament a is only a few percent of a_0.

Section 2 brings some generalities concerning cylindrical and spherical structures in Newtonian dynamics and in MOND, and extends the ELT result to more general cylinders. In § 3 we describe two models for self-gravitating infinite cylinders in MOND. In § 4 we consider the Perseus-Pisces filament.

2. VIRIAL RELATIONS FOR CYLINDRICAL AND SPHERICAL SYSTEMS IN NEWTONIAN DYNAMICS AND IN MOND

Virialized spherical systems of size r and velocity dispersion σ have a mean acceleration $\sim\sigma^2/r$ that roughly equals MG/r^2 in Newtonian dynamics, and $(MGa_0/r^2)^{1/2}$ in MOND (very low acceleration limit). Thus, in the former the virial relation $M \sim \sigma^2 r/G$ contains the system's radius, while in the latter the analogous relation $M \sim \sigma^4/Ga_0$ does not. For cylinders, the tables are turned, and, since the Newtonian mean acceleration is $\sim \mu G/r$, while in MOND it is $\sim(\mu G a_0/r)^{1/2}$, we have in Newtonian dynamics $\mu G \sim \sigma^2$, while in MOND we have $\mu G \sim \sigma^4/r a_0$.

We see then that the Newtonian cylindrical case is akin to the spherical, deep-MOND case, the crucial common property being the logarithmic behavior of the potential at infinity. In fact, the isothermal axisymmetric cylinder discussed by ELT, and the deep-MOND, isothermal sphere described in Milgrom (1984), is a special case, with dimensions 2 and 3, of D-dimensional, self-gravitating, isothermal spheres, held together by a potential, φ, that satisfies a Poisson-like equation,

$$\nabla \cdot (|\nabla\varphi|^{\alpha}\nabla\varphi) = A\rho \ . \tag{2}$$

Here $\alpha = D - 2$ is chosen such that the potential of an isolated mass is logarithmic at infinity (for $D = 2$ we get the usual Poisson equation; for $D = 3$ we get deep MOND). The general density law of such spheres is (Milgrom 1996)

$$\rho(r) \propto \frac{(r/r_0)^{-(D-1)\beta}}{[1 + (r/r_0)^{D/(D-1)-\beta}]^{D}} \tag{3}$$

(β is the anisotropy parameter; see the next section). These spheres satisfy a mass-velocity-dispersion relation of the form

$$\langle v^2 \rangle^{D-1} = MA\alpha_D^{-1}\left(\frac{D-1}{D}\right)^{D-1} , \tag{4}$$

where $\langle v^2 \rangle$ is the mass-weighted mean-square velocity of the system, and α_D is the D-dimensional solid angle (2π for $D = 2$, etc.). In fact, Milgrom (1996) proves that this relation is not limited to isothermal spheres but holds for an arbitrary virialized system subject to the above potential equation.

This relation for deep-MOND, isothermal spheres ($D = 3, A = 4\pi G a_0, \alpha_3 = 4\pi$),

$$MGa_0 = \tfrac{9}{4}\langle v^2 \rangle^2 , \tag{5}$$

was first derived in Milgrom (1984). It was later generalized, first by Gerhard & Spergel (1992) to spherically symmetric systems with general velocity distributions, and then by Milgrom (1994) to arbitrary (not necessarily spherical) systems.

Because the results of Milgrom (1996) have not been published yet, we now give the proof of equation (4) for the (relatively simple) special case $D = 2$, which constitutes a generalization of the ELT result to arbitrary cylindrical systems. Following Milgrom (1994), *mutatis mutandis*,, we consider a stationary, self-gravitating system, symmetric under z translations, that is composed of various particle species with masses m_k and distribution functions $f_k(\boldsymbol{r}, \boldsymbol{v})$, $\boldsymbol{r}$ and $\boldsymbol{v}$ being the position and velocity in the x-y plane. As usual, take the second time derivative of the quantity

$$Q \equiv \tfrac{1}{2}\sum_k \int d^2r\, d^2v\, m_k f_k(\boldsymbol{r}, \boldsymbol{v}) r^2 , \tag{6}$$

which must vanish due to the stationarity of the system (if the system is not stationary but still bound, its long-time average vanishes):

$$\sum_k \int d^2r\, d^2v\, m_k f_k(\boldsymbol{r}, \boldsymbol{v}) v^2 + \sum_k \int d^2r\, d^2v\, m_k f_k(\boldsymbol{r}, \boldsymbol{v}) \boldsymbol{r} \cdot \boldsymbol{a} = 0 \ . \tag{7}$$

The first term in equation (7) is the mass-weighted, two-dimensional, mean-square velocity, $\langle v^2 \rangle$, multiplied by μ_0—the total line density of the cylinder. In the second term we put $\boldsymbol{a} = -\nabla\varphi$, where φ is the gravitational potential, to obtain

$$\mu_0\langle v^2 \rangle = \sum_k \int d^2r\, d^2v\, m_k f_k(\boldsymbol{r}, \boldsymbol{v}) \boldsymbol{r} \cdot \nabla\varphi \ . \tag{8}$$

The v integration and the sum over species can now be performed to yield the standard result:

$$\mu_0\langle v^2 \rangle = \int d^2r\, \rho(\boldsymbol{r}) \boldsymbol{r} \cdot \nabla\varphi \ . \tag{9}$$

Now, using the Poisson equation, we can write equation (9) as

$$\mu_0\langle v^2 \rangle = \frac{1}{4\pi G}\int \Delta\varphi\, \boldsymbol{r} \cdot \nabla\varphi\, d^2r , \tag{10}$$

which can be written as

$$\mu_0\langle v^2 \rangle = \frac{1}{4\pi G}\int \nabla \cdot \left[\boldsymbol{r} \cdot \nabla\varphi\nabla\varphi - \frac{1}{2}(\nabla\varphi)^2\boldsymbol{r}\right] d^2r \ . \tag{11}$$

Applying Gauss's theorem to write the integral as a surface integral at infinity, and remembering that in the relevant geometry $\nabla\varphi$ goes asymptotically as $\nabla\varphi \rightarrow -2G\mu_0\, \boldsymbol{r}/r^2$, the integration gives $G\mu_0^2$, and we finally get the general virial relation

$$\langle v^2 \rangle = G\mu_0 \ . \tag{12}$$

When the system is not stationary, but still bound, $\langle v^2 \rangle$ in equation (12) is replaced by its long-time average, $\overline{\langle v^2 \rangle}$.

Note that, in fact, in deriving equation (12) we do not make use of the perfect cylindricity of the system. We only use the Poisson equation and the cylindrical boundary behavior of the potential at infinity. All still hold if the object looks like a uniform, straight line at infinity, i.e., it can have nonuniformities of different kinds: wiggles, modulations of the line density, etc., as long as it is uniform on average. The problem is that in this case the system cannot be expected to be stationary, so we cannot use the instantaneous, observed value of $\langle v^2 \rangle$, and we cannot easily relate it to a line-of-sight velocity dispersion, as we do below.

The two-dimensional mean-square velocity, $\langle v^2 \rangle$, of an astronomical system cannot be measured from our vantage point, and we would like to express it in terms of the one-dimensional, system-integrated mean-square of the velocity component along a line of sight perpendicular to the cylinder axis, $\sigma_\perp$. This can be done, for example, when $\sigma_\perp$ is independent of the azimuthal viewing angle, in which case

$$\sigma_\perp^2 = \tfrac{1}{2}\langle v^2 \rangle , \tag{13}$$

and we may then write

$$2\sigma_\perp^2 = G\mu_0 , \tag{14}$$

which is the relation derived by ELT for their model filament. Two instances in which equation (13) is valid are (*a*) when the velocity ellipsoid is isotropic everywhere (e.g., in a gaseous system) but the system need be neither isothermal nor axisymmetric, and (*b*) when the system is axisymmetric, but then the velocity distribution need be neither isotropic nor isothermal. The ELT model is a special case of instance *b*.

Another point to be made about the ELT model is that it can be extended, at no further cost, to include a very dense core at the center of the filament. If we approximate this core by a line singularity of finite line density $\mu(0)$, then the structure equation of ELT is modified only by replacing the anisotropy parameter β (see next section or ELT for denotation) by $\hat{\beta} \equiv \beta + m_0$, where $m_0 \equiv 2\mu(0)G/\sigma_r^2$. The behavior of the density near the origin is now $\rho \propto r^{-\hat{\beta}}$. Convergence of the mass both near the origin and at infinity is assured for $\hat{\beta} < 2$, which sets an upper limit on the singular line density: $\mu(0) < (2 - \beta)\sigma_r^2/2G$. The relation between the "gas" line density (excluding the singularity) and the "gas" velocity dispersion is now

$$\mu_t(\text{gas}) = \frac{2\sigma_\perp^2}{G} - 2\mu(0) , \tag{15}$$

and the total line density is

$$\mu_{\text{tot}} = \frac{2\sigma_\perp^2}{G} - \mu(0) . \tag{16}$$

Our general relation (eq. [14]) is seen to hold when we note that we must take in it the mass-weighted mean of $\sigma_\perp^2$, including the contribution of the line singularity. The latter must itself have $\sigma_\perp^2(\text{line}) = \mu(0)G/2$. The line singularity can stand, for example, for an ELT model with a radius scale much smaller than that of the "envelope" gas. Such an extended model can help study possible departures from the ELT model. For example, when the "gas" mass is negligible, the singularity line density determines $\sigma_\perp$ through $\mu(0) = \sigma_\perp^2/G$.

3. TWO IDEALIZED MODEL FILAMENTS IN MOND

Obtaining a quantitative inkling of the MOND relation between line density, radius, and velocity requires specific models. We now consider two such filament models. Not enough is known about the velocity distribution and radial-density structure of real filaments to assess the degree of their adequacy. Even if in their very outer parts actual filaments depart greatly from these models, these may be applicable in the inner parts.

The first model makes the same assumptions as ELT's. It consists of an infinitely long, axisymmetric, self-gravitating system such that the azimuthal and radial velocity dispersions—σ_t and σ_r, respectively—are position independent (accelerations along the axis are assumed negligible). We assume that all accelerations are much smaller than a_0, so that the deep MOND limit is taken. The MOND Jeans equation can then be written as

$$\frac{\sigma_r^4}{a_0}\left(\frac{d\ln\rho}{dr} + \frac{\beta}{r}\right)\left|\frac{d\ln\rho}{dr} + \frac{\beta}{r}\right| = -\frac{\mu(r)G}{r} . \tag{17}$$

Here, $\rho(r)$ is the mass density at cylindrical radius r, $\mu(r)$ is the line density within radius r:

$$\mu(r) = 2\pi \int_0^r \rho(r')r'\,dr' , \tag{18}$$

and

$$\beta \equiv 1 - \sigma_t^2/\sigma_r^2 \tag{19}$$

is the anisotropy parameter. The system-integrated rms velocity component along a line of sight perpendicular to the cylinder axis, $\sigma_\perp$, is given by

$$\sigma_\perp^2 = \sigma_r^2\left(1 - \frac{\beta}{2}\right) . \tag{20}$$

Equation (17) is derived along the lines detailed in Milgrom (1984) and is based, e.g., on the formulation of MOND in Bekenstein & Milgrom (1984).

We shall see that the solution of equations (17)–(18) leads to a finite value of the asymptotic μ. Define, then, $\mu_0 \equiv \mu(\infty)$, and use it to define length and density scales by

$$r_0 \equiv \frac{\sigma_r^4}{a_0\,\mu_0\,G} , \quad \rho_0 \equiv \frac{\mu_0}{2\pi r_0^2} , \tag{21}$$

respectively. The deep-MOND Jeans equation can then be written as

$$\zeta'' - s\eta^{-1}\zeta' + \eta^{-1/2}\zeta^{1/2}\zeta' = 0 , \tag{22}$$

where $s \equiv 1 - \beta$, and we use the dimensionless variables

$$\eta \equiv r/r_0 , \quad \lambda \equiv \rho/\rho_0 , \tag{23}$$

and

$$\zeta(\eta) \equiv \mu(\eta r_0)/\mu_0 = \int_0^\eta \lambda(\eta')\eta'\,d\eta' . \tag{24}$$

Equation (22) is to be solved with the boundary conditions $\zeta(0) = 0$ and $\zeta(\infty) = 1$.

Near the origin, the third term in equation (22) can be neglected (it is the only term by which the Newtonian equation differs from the MOND equation), and we can solve the remaining equation to obtain there

$$\zeta'(r \approx 0) \approx b\eta^s , \quad \zeta(r \approx 0) \approx \frac{b}{s+1}\,\eta^{s+1} . \tag{25}$$

Near the origin, thus, $\rho(r) \propto r^{-\beta}$, as in the Newtonian case. Values $\beta < 0$ give nonmonotonic density laws, and we ignore these. [The distribution function for such cylinders can be written as a function of the particle angular momentum, $J = r\sigma_t$, and the energy, $E = v^2/2 + \varphi(r)$, for unit mass particles, $f(v, r) \propto J^{-\beta}\exp(-E/2\sigma_r^2)$, and is increasing with J for $\beta < 0$.]

At large η the second term is negligible, and we may put $\zeta \approx 1$ in the third term. Equation (22) then gives for the asymptotic form of the density $\lambda(\eta \to \infty) \propto \eta^{-1}\exp(-2\eta^{1/2})$, or

$$\rho \propto r^{-1}\exp[-2(r/r_0)^{1/2}] . \tag{26}$$

From equation (25) we see that ζ is increasing near the origin. By equation (22) it cannot decrease at larger radii because if it has a maximum ζ', and hence ζ'' vanishes there and ζ remains constant from that point on, it also cannot increase indefinitely, as can be seen from equation (22), and it must then go to a constant at infinity, as we have assumed all along.

The solution for ζ can be found numerically, by integrating equation (22) out from the origin, starting with equations (25), shooting to find the value of the parameter b for which $\zeta(\infty) = 1$. We can avoid this trial-and-error if we rescale the variables η and ζ:

$$\eta = \gamma\hat{\eta} , \quad \zeta = \omega\hat{\zeta} , \tag{27}$$

such that (*a*) our structure equation (22) remains of the same form in $\hat{\eta}$ and $\hat{\zeta}$, and (*b*) near the origin $d\hat{\zeta}/d\hat{\eta} = \hat{\eta}^s$. This can be shown to hold when $\omega = \gamma^{-1} = b^{1/(2+s)}$. The solution of equation (22) for $\hat{\zeta}(\hat{\eta})$ is unique. The asymptotic value of $\hat{\zeta}$ has to be identified with ω^{-1}, and can be used to find

$$\zeta(\eta) = \omega\hat{\zeta}(\omega\eta) . \quad (28)$$

(The equation can be solved analytically for one, nonphysical value of $\beta = 3/2$.)

We see then that r_0 appears as some radius scale in the density distribution of the isothermal cylinder. If it can be determined for an actual filament together with σ_r, we can determine μ_0 through equation (21) as

$$\mu_0 = \frac{\sigma_r^4}{r_0 a_0 G} , \quad (29)$$

and, in terms of $\sigma_\perp$ from equation (20),

$$\mu_0 = \frac{4\sigma_\perp^4}{(2-\beta)^2 r_0 a_0 G} , \quad (30)$$

In order to use such relations to determine μ_0 for real filaments, we have to relate r_0 to some observable property such as the half-mass radius, the projected half-mass radius, the radius, $R_{1/2}$, at which the surface density reaches a certain fraction of its central value, etc. Take the last, for example; if we write the product $(2-\beta)^2 r_0$ appearing in the denominator of equation (30) as $qR_{1/2}$, we have

$$\mu_0 \sim \frac{4\sigma_\perp^4}{qR_{1/2} a_0 G} . \quad (31)$$

We find numerically that $q \approx 1$ for $\beta = 0$ (isotropic case), $q \approx 1.5$ for $\beta = \frac{1}{2}$, and $q \approx 2$ for $\beta = 1$ (radial orbits).

The second model is contrived to afford analytic solution, but is not less reasonable in default of knowledge of real filaments. It differs from the first in that the velocity dispersions are now assumed to increase from the center out as the fourth root of the radius; thus

$$\sigma_r(r) = S^{1/4} r^{1/4} , \quad (32)$$

and β is still constant (the projected, central velocity dispersion does not vanish). The constant S can be used to construct a quantity

$$\hat{\mu} \equiv \frac{S}{Ga_0} , \quad (33)$$

with the dimensions of a line density, which we use to define the dimensionless line density

$$\zeta(r) \equiv \mu(r)/\hat{\mu} . \quad (34)$$

The Jeans equation can then be written as

$$\left[\frac{1}{\rho}\frac{d(r^{1/2}\rho)}{dr} + \frac{\beta}{r^{1/2}}\right]\left|\frac{1}{\rho}\frac{d(r^{1/2}\rho)}{dr} + \frac{\beta}{r^{1/2}}\right| = -\frac{\zeta(r)}{r} . \quad (35)$$

This, in turn, can be written as

$$\left(\frac{d\ln\rho}{d\ln r} + \delta\right)\left|\frac{d\ln\rho}{d\ln r} + \delta\right| = -\zeta(r) , \quad (36)$$

where $\delta \equiv \beta + \frac{1}{2}$, and

$$\zeta(r) = \frac{2\pi}{\hat{\mu}}\int_0^r \rho(r')r'\,dr' . \quad (37)$$

Near the origin, where $\zeta \approx 0$, we have

$$\rho(r \approx 0) \approx ar^{-\delta} . \quad (38)$$

Since, by definition, $\beta \le 1$, the line density always converges at the origin. However, for $\beta > \frac{1}{2}$ the acceleration diverges there, and the deep-MOND assumption does not hold. These models are thus only good for $\beta \le \frac{1}{2}$.

At large distances, where $\zeta \approx \mu_0/\hat{\mu}$ (μ_0, as before, is the total line density)

$$\rho(r \to \infty) \propto r^{-(\delta+\tau)} , \quad (39)$$

where $\tau \equiv (\mu_0/\hat{\mu})^{1/2}$. As the asymptotic form will be determined by the solution of equations (36)–(37), the value of τ will be read off the solution, and μ_0 will be expressed in terms of $\hat{\mu}$. These equations possess a family of solutions whose members are specified by the choice of the constants, a, dictating the behavior near the origin. Defining

$$\lambda(r) \equiv a^{-1} r^\delta \rho(r) , \quad (40)$$

and redefining the independent variable,

$$x = \left(\frac{r}{r_s}\right)^{2-\delta} , \quad r_s \equiv \left[\frac{8\pi a}{9\hat{\mu}(2-\delta)^3}\right]^{-1/(2-\delta)} , \quad (41)$$

we can write

$$\frac{d\ln\lambda}{d\ln x} = -\frac{3}{2}\left[\int_0^x \lambda(x')dx'\right]^{1/2} . \quad (42)$$

This equation, with the boundary behavior $\lambda(x \approx 0) \approx 1$, has the unique solution

$$\lambda(x) = (1 + x^{1/2})^{-3} . \quad (43)$$

Thus, the full density law is

$$\rho(r) = b\left(\frac{r}{r_s}\right)^{-\delta}\left[1 + \left(\frac{r}{r_s}\right)^{(2-\delta)/2}\right]^{-3} , \quad (44)$$

and

$$\zeta(r) = \frac{9}{4}(2-\delta)^2\left[\frac{(r/r_s)^{1-\delta/2}}{1 + (r/r_s)^{1-\delta/2}}\right]^2 . \quad (45)$$

The radial scale length, r_s, characterizing the density law, is arbitrary, and $b = ar_s^{-\delta}$ is a function of r_s through equation (41). In particular we read from equation (45) that

$$\mu_0 = \frac{9}{16}(3-2\beta)^2\frac{S}{a_0 G} . \quad (46)$$

As the case is with the first model, we can express S in terms of different measures of the velocity dispersion and scale length of observed filaments. For example, we can calculate the system average value of σ_r or $\sigma_\perp$:

$$\frac{2}{2-\beta}\sigma_\perp^2 = \langle\sigma_r^2\rangle = S^{1/2}\frac{\int_0^\infty r'^{3/2}\rho(r')dr'}{\int_0^\infty r'\rho(r')dr'} . \quad (47)$$

The numerator is finite for $\beta < \frac{1}{2}$, and we find

$$\frac{2}{2-\beta}\sigma_\perp^2 = \langle\sigma_r^2\rangle = PS^{1/2}r_s^{1/2} , \quad (48)$$

where

$$P = \frac{4(5-2\beta)}{(3-2\beta)^2}\left(\sin\frac{2\pi}{3-2\beta}\right)^{-1} . \tag{49}$$

The scale radius r_s can be easily related, for example, to the (space) half-mass radius, r_h, containing half the line density:

$$r_s = (\sqrt{2}-1)^{2/(2-\delta)} r_h . \tag{50}$$

Thus, in terms of $\sigma_\perp$ and r_h,

$$\mu_0 = Q\,\frac{\langle\sigma_\perp^2\rangle^2}{a_0\,G r_h} , \tag{51}$$

where

$$Q = \frac{9}{4}(\sqrt{2}-1)^{-4/(3-2\beta)}\left(\frac{3-2\beta}{2-\beta}\right)^2 P^{-2} . \tag{52}$$

For $\beta = -\frac{1}{2}$, $P = \frac{3}{2}$ and $Q \approx 6$. For $\beta = 0$, $P = 40/9(3)^{1/2} \approx 2.6$ and $Q \approx 2.5$. For $\beta = 0.25$, $P \approx 4.9$ and $Q \approx 0.8$. For β approaching $\frac{1}{2}$, Q goes to 0, but then most of the contribution to the mean dispersion comes from larger and larger radii.

4. A PRELIMINARY ANALYSIS OF THE PERSEUS-PISCES SEGMENT

Now we turn to the implications for the Perseus-Pisces superfilament. For reasons explained in their paper, ELT actually apply their method to only a segment of the Perseus-Pisces bridge of length of about 20 h^{-1} Mpc, and diameter of about 4 h^{-1} Mpc ($h = H_0/100$ km s^{-1} Mpc^{-1}). They find a system-integrated, line-of-sight velocity dispersion of $\sigma_\perp \sim 430$ km s^{-1}, and estimate the B-band luminosity within their chosen section at about 4×10^{12} h^{-2} $L_\odot$. Using their σ-μ relation, they get a value of $\mu_0 \approx 8.5\times10^{13}$ $M_\odot$ Mpc^{-1}. All these yield an estimated value of $(M/L)_N \sim 450\,h(\sigma_\perp/430 \text{ km s}^{-1})^2 (M/L)_\odot$. We adopt all the above system-parameter estimates. In addition, we need a radius scale, r, for the filament, and choose to normalize rh to 2 Mpc, which seems the approximate typical radius from Figure 1 of Wegner, Haynes, & Giovanelli (1993).

We can estimate the MOND M/L value in several ways: For example, we can estimate the acceleration a and then correct the Newtonian ELT value by a factor a/a_0. The value of a_0 as deduced by Begeman et al. (1991), normalized properly to the assumed value of H_0, is $a_0 \approx 2\times10^{-8}$ h^2 cm s^{-2}. The acceleration at r is

$$a = \frac{\sigma_r^2}{r}\gamma,\ \gamma \equiv \left|\frac{d\ln\rho}{d\ln r} + \beta\right| , \tag{53}$$

and so

$$\frac{a}{a_0} \approx 1.5\times10^{-2}\gamma\left(\frac{\sigma_r}{430 \text{ km s}^{-1}}\right)^2\left(\frac{rh}{2 \text{ Mpc}}\right)^{-1} h^{-1} . \tag{54}$$

Applying this correction factor to ELT's Newtonian estimate, we get the MOND estimate

$$\left(\frac{M}{L}\right)_M \sim 7\gamma\left(\frac{\sigma_r}{430 \text{ km s}^{-1}}\right)^2\left(\frac{\sigma_\perp}{430 \text{ km s}^{-1}}\right)^2\left(\frac{rh}{2 \text{ Mpc}}\right)^{-1} \times\left(\frac{a_0}{2\times10^{-8}\,h^2 \text{ cm s}^{-2}}\right)^{-1}\left(\frac{M}{L}\right)_\odot . \tag{55}$$

To get another estimator, we can use the above isothermal-cylinder models with all due circumspection. For example, from equation (31) we can write

$$\mu_0 \sim 2.6\times10^{12} q^{-1}\left(\frac{\sigma_\perp}{430 \text{ km s}^{-1}}\right)^4\left(\frac{R_{1/2}}{2 \text{ Mpc}}\right)^{-1} \times\left(\frac{a_0}{2\times10^{-8}\,h^2 \text{ cm s}^{-2}}\right)^{-1} M_\odot \text{ Mpc}^{-1} , \tag{56}$$

where q is between 1 and 2 for β between 0 and 1. With the B-band luminosity estimate of ELT this gives

$$\left(\frac{M}{L}\right)_M \sim 13 q^{-1}\left(\frac{\sigma_\perp}{430 \text{ km s}^{-1}}\right)^4\left(\frac{R_{1/2}h}{2 \text{ Mpc}}\right)^{-1} \times\left(\frac{a_0}{2\times10^{-8}\,h^2 \text{ cm s}^{-2}}\right)^{-1}\left(\frac{M}{L}\right)_\odot . \tag{57}$$

If we use equation (51) from the second model, we obtain an estimator

$$\mu_0 \sim 6\times10^{11} Q\left(\frac{\sigma_\perp}{430 \text{ km s}^{-1}}\right)^4\left(\frac{r_h}{2 \text{ Mpc}}\right)^{-1} \times\left(\frac{a_0}{2\times10^{-8}\,h^2 \text{ cm s}^{-2}}\right)^{-1} M_\odot \text{ Mpc}^{-1} , \tag{58}$$

and

$$\left(\frac{M}{L}\right)_M \sim 3Q\left(\frac{\sigma_\perp}{430 \text{ km s}^{-1}}\right)^4\left(\frac{r_h h}{2 \text{ Mpc}}\right)^{-1} \times\left(\frac{a_0}{2\times10^{-8}\,h^2 \text{ cm s}^{-2}}\right)^{-1}\left(\frac{M}{L}\right)_\odot . \tag{59}$$

ELT discuss in detail many of the uncertainties that plague the analysis, from doubtful model assumptions to difficulties in determining system parameters. We here only stress some of these further, and add some comments peculiar to the MOND analysis. Two important underlying assumptions certainly need confirmation, namely, that the thin-filament geometry is justified (the thin appearance on the sky may be due, at least partly, to projection effects) and that the segment we treat is nearly virialized; some segments of the Perseus-Pisces structure seem to evince ongoing settling down.

Even if the model assumptions are by and large justified, there are several factors that may cause an overestimation of the velocity dispersion $\sigma_\perp$: (1) The contribution from variations in the mean Hubble velocity along the filament. ELT show how this can be largely removed by dividing the filament into segments, and taking the dispersion in each segment separately, but this was not done for the actual Perseus-Pisces analysis. (2) Clumping along the filament that is not taken into account can lead to a substantial overestimate of $\sigma_\perp$, as ELT demonstrate on model filaments from N-body calculations. The Perseus-Pisces segment treated by ELT appears knotty to a degree on Figure 1 of Wegner et al. (1993). (3) Motion along the filament may contribute if this is not exactly perpendicular to the line of sight everywhere. In the case of the Perseus-Pisces segment, causes 1 and 3 seem to be of little consequence (D. Eisenstein (1996), private communication). Such uncertainties in

$\sigma_\perp$ are more crucial in MOND than in Newtonian mass estimates, because the velocity enters the MOND mass estimate in the fourth power, not the second.

Both ELT's and our method are global, and give the total $\mu(\infty)$. The luminosity, however, is taken by ELT only within a certain projected radius (of about 2 Mpc). The actual filament may well extend farther in radius, and the estimated luminosity, uncertain as it is anyway, may be systematically too low because of this, a fact which contributes to an overestimation of M/L.

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