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ARE GALACTIC ROTATION CURVES REALLY FLAT?

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ABSTRACT

In this paper we identify an apparently previously unappreciated regularity in the systematics of galactic rotation curves; namely, we find that at the last detected points in galaxies of widely varying luminosity, the centripetal acceleration is found to have the completely universal form $(v^2/c^2R)_{\rm last} = \gamma_0/2 + \gamma^* N^*/2 + \beta^* N^*/R^2$, where γ_0 and γ^* are new universal constants, β^* is the Schwarzschild radius of the Sun, and N^* is the total amount of visible matter in each galaxy. This regularity points to a possible role for the linear potentials associated with conformal gravity, with the galaxy-independent γ_0 term being found not to be generated from within individual galaxies at all but rather to be of cosmological origin, being due to the global Hubble flow of a necessarily spatially open universe of 3-space scalar curvature $k = -(\gamma_0/2)^2 = -2.3 \times 10^{-60}$ cm^{-2}.

Subject headings: galaxies: kinematics and dynamics — cosmology: theory

1. INTRODUCTION

In discussions of the dynamics of galactic rotation curves it is usually assumed that rotation curves are asymptotically flat at large radial distances, and that whatever is responsible for this non-Keplerian behavior is itself just a purely local phenomenon that arises solely from within the galaxies themselves. In this paper we challenge these two widely accepted notions, and show that once the luminous Newtonian contribution is subtracted out, the resulting velocity discrepancies in individual galaxies are not merely actually growing (and quite rapidly in fact) with distance at the largest available radial distances but, moreover, they are actually growing in a universal manner. Beyond being an interesting model-independent phenomenological regularity in and of itself, and beyond being one that dark matter models of rotation curves should therefore now have to account for, this regularity also points to a role for cosmology in the elucidation of rotation curves, as well as to the possible relevance of the conformal gravity theory of Weyl, which is currently being explored by Mannheim and Kazanas as a candidate alternative to the standard dark matter paradigm. Moreover, with the apparent failure so far of the epochal gravitational microlensing observations to confirm conclusively the existence of the copious spherical dark matter halo that galaxies such as the Milky Way are widely believed to possess, consideration (at least) of alternate gravity would thus not appear to be totally inappropriate. However, since the whole issue of alternate gravity remains contentious at the present time, before investigating (in § 3) any possible dynamical implications of rotation curve systematics, we shall begin (in § 2) by first looking for possible model-independent, purely phenomenological clues in the data themselves, to yield an apparently previously unappreciated phenomenological systematics whose validity is independent of the correctness or otherwise of any candidate galactic dynamical model.

2. PHENOMENOLOGICAL EVIDENCE FOR UNIVERSALLY RISING VELOCITY DISCREPANCIES

Since H II optical studies of galactic rotation curves are confined to the optical disk region, a region which is luminous Newtonian dominated (see, e.g., the fits of Kalnajs 1983 and Kent 1986), to uncover any definitive departures from the standard luminous Newtonian disk Keplerian expectation it becomes necessary to turn to galactic hydrogen gas, which is distributed in galaxies with scale lengths that are typically about 3 times as large as those of the associated optical disks. In their analysis of the available 28 galaxy set of 21 cm line H I rotation curve data, these being the data that actually provide the primary evidence that a pure luminous Newtonian contribution does indeed fail to account adequately for galactic rotation velocities, Casertano & van Gorkom (1991) pointed out that the available data basically fell into three broad categories. Specifically, the rotation curves of low-luminosity galaxies were found to be generally rising at the last detected points, those of intermediate- to high-luminosity galaxies to be flat, and those of the highest luminosity galaxies to be (mildly) falling. Out of this 28 galaxy set, Begeman, Broeils, & Sanders (1991) then identified a particularly reliable 11 galaxy subset, and since these 11 galaxies range over a variation of more than 1000 in luminosity (see Table 1) while still clearly exhibiting the Casertano & van Gorkom (1991) trend (see Fig. 1), this 11 galaxy subset should thus indeed be regarded as being representative of the kind of departures from the luminous Newtonian expectation that have so far been observed, while also being a set which is large and diverse enough to possess any systematics that might characterize these departures.[1]

While the lack of flatness of the low-luminosity galaxies is quite apparent, nonetheless the flatness (or near-flatness) of all the other galaxies is so striking that the rise in the low-luminosity galaxies has essentially been discounted by the general community as being in any way suggestive of a trend, and it is generally assumed that these curves will

[1] While none of the low-luminosity galaxies currently show any flat rotation curve region at all, there is a noticeable turnover in one of these galaxies, viz., DDO 154. However, since this is the most gas-dominated galaxy in the entire sample, random gas pressures or a detected gas warp could be making a substantial contribution to motions in the turnover region. We shall thus ignore any possible ramifications of these last few points here, though clearly if this turnover proves to be a real trend which is then reproduced in other low-luminosity galaxies, it would eventually have to be accounted for, not only here, in fact, but even in the standard dark matter theory and in the modified Newtonian dynamics (MOND) (Milgrom 1983) theory, both of which anticipate a flattening, not a drop, in the DDO 154 rotation curve.

TABLE 1

CHARACTERISTICS OF 11 GALAXY SAMPLE

Galaxy	Distance (Mpc)	Luminosity (10^9 $L_{B_\odot}$)	$(v^2/c^2R)_{\rm last}$ (10^{-30} cm^{-1})	Shift (%)	M/L ($M_\odot$ $L_{B_\odot}^{-1}$)
DDO 154.......	3.80	0.05	1.51	−11	0.71
DDO 170.......	12.01	0.16	1.63	−7	5.36
NGC 1560......	3.00	0.35	2.70	+23	2.01
NGC 3109......	1.70	0.81	1.98	...	0.01
UGC 2259......	9.80	1.02	3.85	+15	3.62
NGC 6503......	5.94	4.80	2.14	...	3.00
NGC 2403......	3.25	7.90	3.31	+15	1.76
NGC 3198......	9.36	9.00	2.67	−15	4.78
NGC 2903......	6.40	15.30	4.86	+14	3.15
NGC 7331......	14.90	54.00	5.51	−16	3.03
NGC 2841......	9.50	20.50	7.25	...	8.26

eventually flatten off, with asymptotically flat rotation curves now being the standard paradigm. However, closer examination of the data reveals a possibly different outcome. Instead of looking at the actual rotation velocities, it is instructive to look at the velocity discrepancy, viz., the excess of the measured rotation velocity over the luminous Newtonian contribution, a contribution whose overall normalization is both highly constrained by the rapid initial rise characteristic of an exponential luminous disk and sharply bounded by the peak of that initial rise, i.e., constrained entirely by the quite well-understood inner rotation region alone. Indeed, as we see from Figure 1, which plots the convenient benchmark maximum optical disk luminous Newtonian contribution (maximum in the sense that the optical disks in the higher luminosity galaxies actually saturate the inner rotation region), the inferred outer region velocity discrepancies are then seen to be far from flat, and in fact actually even to be rising in each and every galaxy in the sample at the last detected data points. Now since the numerical values of the mass-to-light ratios of the optical disks are not known a priori, there is actually some ambiguity in determining at exactly what specific rate these discrepancies do in fact grow, with the benchmark maximum optical disks yielding velocity discrepancies in the higher luminosity galaxies which then grow at the minimum possible rate. However, for such galaxies, even with less than maximum disks, the shape of the optical disk contribution would not be changed. Rather, only the overall normalization would be reduced, to then lead us to velocity discrepancies that would be growing even more rapidly with distance, so that no matter what the optical disk mass-to-light ratios, as long as the optical disk makes some contribution (and the optical disks should anyway not be expected to depart too much from the mass-to-light ratios found in the local solar neighborhood), the discrepancies in the higher luminosity spirals will necessarily be growing (at some rate) with distance out to the currently last detected points. Moreover, since the mass of the H I gas is directly measurable, there is actually no normalization ambiguity at all for the gas-dominated low-luminosity galaxies, with the velocity discrepancies in these galaxies thus unambiguously having to rise precisely as indicated in Figure 1. Consequently, we conclude that all of the galaxies in our sample necessarily possess rising velocity discrepancies out to the last currently detected data points. Since the velocity discrepancies in spiral galaxies are themselves usually explained by spherical dark matter halos, we see that while such halos may eventually lead to asymptotic flatness, their contributions in the detected regions are typically still rising at the farthest points (and unambiguously so in the low-luminosity galaxies), with the flat total velocities that they produce in the bright galaxies actually being achieved by carefully fine-tuning the halo contribution galaxy by galaxy (through the use of two free parameters per halo and thus no less than 22 in total for our complete 11 galaxy sample) to rise at just the same rate as the (usually close to maximum disk) luminous matter contribution is falling. Even for isothermal sphere halos, the asymptotic regime would thus appear to still be some way off, with the case for flat velocity discrepancies not yet being mandated by any available rotation curve data.

Beyond the issue of the shape of the velocity curves, one can also ask whether there is any regularity in the magnitudes of the velocities. For the flat rotation curve galaxies there is indeed such a regularity, viz., the Tully-Fisher law, a phenomenologically established universal relation between the luminosity and the fourth power of the velocity dispersion in the observed flat rotation curve region. Moreover, these same galaxies also appear to possess a second form of universality, which was first noted by Freeman, namely, that the most prominent spiral galaxies all seem to have a common central surface brightness, $\Sigma_0^{\rm F}$. (In passing we note that while there also exist low surface brightness galaxies with $\Sigma_0 < \Sigma_0^{\rm F}$, there do not appear to be any galaxies with $\Sigma_0 > \Sigma_0^{\rm F}$, thus making $\Sigma_0^{\rm F}$ an empirical upper bound on galaxies). While the brighter galaxies thus possess a great deal of universality in addition to having flat rotation curves, this universality is not enjoyed by the nonflat low-luminosity galaxies. Thus it would be of interest to find a universality which also involves the low-luminosity galaxies as well. Given the suggestive fact that the velocity discrepancies are actually rising in all the galaxies, we thus evaluate the total centripetal acceleration at the last data point in each galaxy (except for DDO 154, for which we use the last point before the turnover). As we can see from the fourth column in Table 1, $(v^2/c^2R)_{\rm last}$ (a completely model-independent quantity) is remarkably universal, varying only by a factor of 5 or so over the sample—a number which is altogether less than the factor of 1000 or so by which the luminosity varies in the same sample. Moreover, we see a small but clear trend with increasing mass in this centripetal acceleration. And, in fact, as will become more apparent below, we find that we can parameterize this total acceleration according to the three-component relation

$$(v^2/c^2R)_{\rm last} = \gamma_0/2 + \gamma^* N^*/2 + \beta^* N^*/R^2 \ , \quad (1)$$

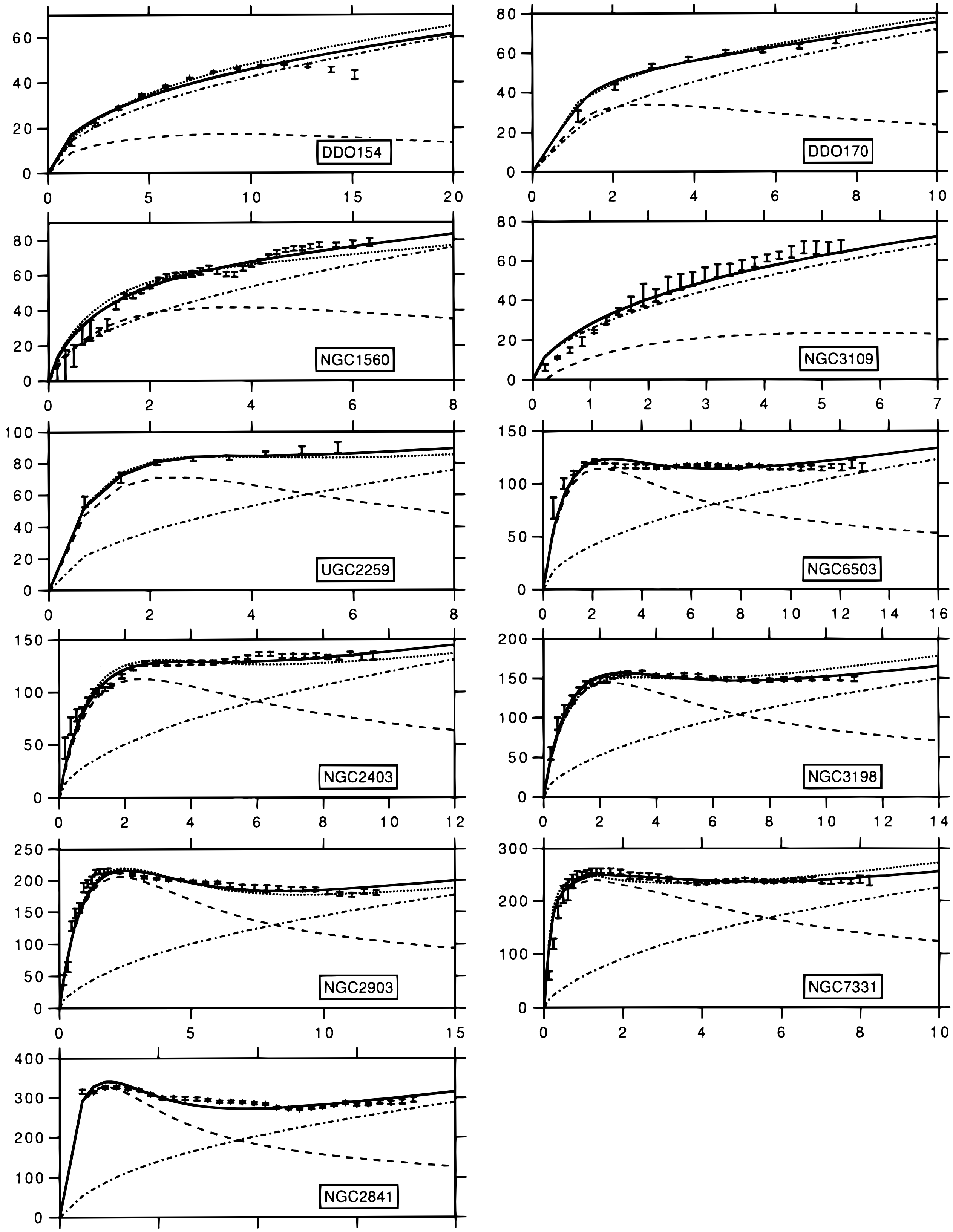

FIG. 1.—Predicted rotational velocity curves associated with conformal gravity for each of the 11 galaxies in the sample. In each graph the bars show the data points with their quoted errors; the full curve shows the overall (adopted distance adjusted) theoretical velocity prediction (in km s^{-1}) as a function of distance from the center of each galaxy (in units of R/R_0, where each time R_0 is each particular galaxy's own optical disk scale length), while the dashed and dash-dotted curves show the velocities that the Newtonian and the linear potentials would produce separately. The dotted curves show the total velocities that would be produced without any adopted distance modification. No dark matter is assumed.

where the two new universal constants γ_0 and γ^* take numerical values 3.06×10^{-30} cm^{-1} and 5.42×10^{-41} cm^{-1}, respectively, where $\beta^* = 1.48 \times 10^5$ cm, and where N^* is the total amount of visible stellar (and gaseous) material in solar mass units in each galaxy.[2] (While the present author was drawn to this regularity via the conformal gravity study presented below, this regularity is an interesting one in and of itself, which now serves as a new constraint on all theories of rotation curves.) As regards this regularity, it is important to realize that there is nothing in any way significant about the actual magnitudes of the radial coordinates, R, of the last detected points in the 11 galaxies, since their locations are fixed purely by the instrumental limits of the various detectors used in measuring the various gas surface brightnesses and not fixed by any dynamics associated with the galaxies themselves. Thus the magnitude of each last measured radial R (a quantity which varies from 8 to 40 kpc or so over the sample) is essentially arbitrary for the galaxies, and yet v^2/R can nonetheless still be universally parameterized. As far as we can see, with the luminous Newtonian contribution being decidedly non-leading at the farthest data points, the only obvious way that this could in fact occur would be if v^2 were in fact growing universally with R, so that the magnitude of v^2/R would not in fact depend on where the last detected points just happened to be located within galaxies. This pattern is clearly not one that would be expected with flat rotation curves, or even in fact that one would think to look for in such a paradigm, and it would instead seem to point to potentials which if anything are actually growing (linearly) with distance rather than falling in the familiar Newtonian manner. Thus it is to this possibility that we now turn.

3. POSSIBLE COSMOLOGICAL ORIGIN FOR UNIVERSALLY RISING VELOCITY DISCREPANCIES

Since our phenomenological analysis points to a possible role for linear potentials in elucidating rotation curves, it is immediately suggested to consider conformal gravity which contains such linear potentials to see whether it can account for the rotation curve phenomenology we have now identified. While conformal gravity (viz., gravity based on the conformal invariant action $I_W = -\alpha \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa}$, where $C^{\lambda\mu\nu\kappa}$ is the conformal Weyl tensor and α is a purely dimensionless coupling constant) dates back to Weyl and Eddington and to the early days of relativity, that it might enable us to dispense with dark matter was recognized only recently by Mannheim and Kazanas on finding (Mannheim & Kazanas 1989; see also Riegert 1984) the exact metric outside of a star in the conformal theory, viz.,

$$ds^2 = B(r)c^2\,dt^2 - dr^2/B(r) - r^2\,d\Omega\ , \tag{2}$$

where

$$B(r) = 1 - 2\beta^*/r + \gamma^* r\ , \tag{3}$$

to thus provide gravitational sources with a new intrinsic length scale $1/\gamma^*$ in addition to the Schwarzschild radius β^* length scale familiar from the standard Einstein theory. Since the above metric generalizes not only Newton but also Schwarzschild, it thus not only meets the classic solar system general relativity tests but also provides for departures from Newton-Einstein on distances large enough that the linear potential term can make itself manifest. For instance, integrating the individual stellar potentials

$$V^*(r) = -\beta^* c^2/r + \gamma^* c^2 r/2 \tag{4}$$

over an infinitesimally thin galactic optical disk with luminous surface matter distribution $\Sigma(R) = \Sigma_0 \exp(-R/R_0)$ and total number of stars $N^* = 2\pi\Sigma_0 R_0^2$ in the standard nonrelativistic weak-gravity way then yields (Mannheim 1993) the centripetal acceleration

$$v^2/R = g_{\text{gal}}^{\text{lum}} = g_\beta^{\text{lum}} + g_\gamma^{\text{lum}}\ , \tag{5}$$

where

$$g_\beta^{\text{lum}} = (N^*\beta^* c^2 r/2R_0^3)[I_0(r/2R_0)K_0(r/2R_0) - I_1(r/2R_0)K_1(r/2R_0)] \tag{6}$$

and

$$g_\gamma^{\text{lum}} = (N^*\gamma^* c^2 r/2R_0) I_1(r/2R_0)K_1(r/2R_0)\ , \tag{7}$$

which is actually found to fit nicely the shapes of the rotation curves of our 11 galaxy sample (Mannheim 1993; Mannheim & Kmetko 1996; Carlson & Lowenstein 1996), but not their overall normalizations, since such a galactic disk would on its own only generate an asymptotic contribution $(v^2/c^2R)_{\text{last}} = \gamma^* N^*/2 + \beta^* N^*/R^2$ and thus lack the N^*-independent $\gamma_0/2$ term found above in our phenomenological analysis of centripetal accelerations.

Apart from the fact that the $\gamma^* N^*/2$ term arises from a non-Newtonian potential, it is otherwise a completely standard, local nonrelativistic term which arises from the local galactic matter distribution and which scales as the total galactic luminosity. However, the additional $\gamma_0/2$ term required for equation (1) is on a very different footing, since it is luminosity independent. Since, moreover, its magnitude given above is of the order of the inverse Hubble radius, it would thus appear to have to have a global, cosmological origin, with cosmology thus needing to provide galaxies with a second linear potential in addition to the one that they themselves internally generate. Now, quite remarkably, it was noted by Mannheim & Kazanas (1989) in their original paper (where they found the generalized exterior Schwarzschild solution discussed above) that cosmology does precisely that. Specifically, they noted the kinematic fact that the general coordinate transformation

$$r = \rho/(1 - \gamma_0\rho/4)^2\ , \quad t = \int d\tau/R(\tau) \tag{8}$$

effects the metric transformation

$$(1 + \gamma_0 r)c^2\,dt^2 - \frac{dr^2}{(1 + \gamma_0 r)} - r^2\,d\Omega$$

$$\to \frac{(1 + \rho\gamma_0/4)^2}{R^2(\tau)(1 - \rho\gamma_0/4)^2}\left[c^2\,d\tau^2 - \frac{R^2(\tau)(d\rho^2 + \rho^2\,d\Omega)}{(1 - \rho^2\gamma_0^2/16)^2}\right]\ , \tag{9}$$

to yield a metric which is conformal to a Robertson-Walker metric with scale factor $R(\tau)$ and (explicitly negative)

[2] In fact, given these values of γ_0, γ^* and β^* as input, this parameterization then serves to determine a value for N^* for each galaxy from its last data point alone, to then enable us below to fit the entire rotation curve of each galaxy in a completely parameter-free way. Interestingly, for the higher luminosity galaxies, the values for N^* inferred this way do in fact turn out to be the maximum disk values.

3-space scalar curvature $k = -\gamma_0^2/4$.[3] Now, and this is the key point, in a geometry which is both homogeneous and isotropic about all points, any observer can serve as the origin for the coordinate ρ; thus in his own local rest frame each observer is able to make the above general coordinate transformation with the use of his own particular ρ. Moreover, since the observer is also free in conformal gravity to make arbitrary conformal transformations as well, that observer will then be able to see the entire Hubble flow appear in his own local static coordinate system as a universal linear potential with a universal acceleration $\gamma_0 c^2/2$ coming directly from the spatial curvature of the universe (a quantity which incidentally is nicely time independent, unlike the time-dependent Hubble parameter itself). Since the internal orbital motions of the stars and gas can be discussed in each galaxy's own rest frame, we thus find that in each such rest frame, each orbiting particle in that specific galaxy will then precisely see the overall Hubble flow acting as a local static universal linear $\gamma_0 r$ potential just as desired. We thus establish a cosmological origin for the universal $\gamma_0 c^2/2$ acceleration needed for equation (1), while also identifying a crucial difference between relativistic and nonrelativistic reasoning. Specifically, in strictly Newtonian physics the only effect of any background would be to put tidal forces on individual galaxies, forces that would not account for the rotational motions of stars and gas but only for a departure therefrom. Relativistically however, since the background produces an effect at the center of each galaxy, the background therefore contributes to the explicit rotational motions of the stars themselves, to thus yield a previously unappreciated but nonetheless apparently quite general consequence of curvature.

In order to now combine the local and global linear potentials, we need to embed each local galaxy in the global Hubble flow and solve the gravitational equations of motion in the presence of $T^{\mu\nu}_{\text{local}} + T^{\mu\nu}_{\text{global}}$. Given the fact that gravity is weak within galaxies, we shall as a first approximation simply add the local and global metrics given above in equations (6), (7), and (9) (it is the very presence of $T^{\mu\nu}_{\text{local}}$ and its associated local geometry, viz., standard static Schwarzschild coordinates, which dictates the appropriate general coordinate transformation needed for eqs. [8] and [9]), to yield the total weak gravity acceleration

$$v^2/R = g_{\text{tot}} = g_\beta^{\text{lum}} + g_\gamma^{\text{lum}} + \gamma_0 c^2/2 , \qquad (10)$$

which can now be fitted to data. With γ_0 and γ^* taking the fixed numerical values given earlier, the fits reduce to just one free parameter per galaxy, viz., the standard optical disk mass-to-light ratio (or, equivalently, the total amount of stars and gas per galaxy, N^*, in solar mass units). Since, unlike dark matter theory, our theory is based on parameters with an absolute scale, it is thus very sensitive to determinations of distances to galaxies. Consequently, we first calculate the total velocity predictions (*dotted curves*) in Figure 1 using the distances (listed in Table 1) quoted by Begeman et al. (1991) (this paper also gives complete data references). Then, again following Begeman et al., we allow for typical uncertainties in the adopted distances to give modest distance shifts of up to $\pm 15\%$ or so. (While larger shifts can actually improve the fits a little in some cases, we have not allowed for shifts of more than this except for NGC 1560, for which a distance estimate of 3.7 Mpc [+23%] has actually been reported in the literature.) With the indicated percentage shifts in adopted distance, with the fitted M/L ratios listed in Table 1, and with $g_{\text{gal}}^{\text{lum}}$ being calculated solely from the known luminous galactic matter [viz., stars and gas, with the optical disk also being given a $\Sigma(R, z) = \Sigma_0 \exp(-R/R_0) \operatorname{sech}^2(z/z_0)/2z_0$ thickness ($z_0 = R_0/5$) modification which is only consequential in the inner rotation region, and with the mass of the hydrogen gas being multiplied by 1.3 to account for the presence of primordial helium], we then obtain the full curve fits of Figure 1, with the dashed and dash-dotted curves showing the velocities that the Newtonian g_β^{lum} and linear $g_\gamma^{\text{lum}} + \gamma_0 c^2/2$ terms would separately produce.[4] No dark matter is assumed, and, as we can see from the fits, none would appear to be needed. Despite the fact that our model is a highly constrained one with very few free parameters, it nonetheless appears to have captured the essence of the data (our fits have smoothed out some of the structure in the data, since we treated the radial dependence of $\Sigma(R, z)$ of the optical disks purely as single exponentials for simplicity), and phenomenologically our fitting would thus appear to be competitive with that of both the standard dark matter model and the MOND alternative. Moreover, if our theory is in fact correct, then it provides us with an actual measurement of the scalar curvature of the universe, something which years of intensive work have yet to accomplish in the standard theory.

We can see from Figure 1 that, at the shifted adopted distances, $(v^2/c^2R)_{\text{last}}$ is indeed remarkably well fitted by $\gamma_0/2 + \gamma^* N^*/2 + \beta^* N^*/R^2$ (this being the asymptotic limit of g_{tot}/c^2) for each and every galaxy in our sample, and that even while the quantity $\gamma^* N^*/2$ does vary enormously with luminosity over our sample, nonetheless the $\gamma_0/2$ term overwhelms it in all but the largest galaxies, so that $(v^2/c^2R)_{\text{last}}$ only shows a mild (but nonetheless significant) dependence on galactic mass. Given the values for γ_0 and γ^* that we obtain from the fits, we see that these two terms would contribute the same amount for galaxies with $N^*_{\text{crit}} = 5.65 \times 10^{10}$ stars, which is indeed toward the high end of our sample.[5] Since our theory is based on rising potentials, it is quite intriguing that it is able to actually (universally) fit the flat high-luminosity rotation curves. To explain this interesting aspect of our theory, we recall that for an exponential disk spiral with surface brightness $\Sigma(R) = \Sigma_0 \exp(-R/R_0)$ the pure luminous Newtonian contribution causes the rotation curve to peak at around $2R_0$ with a normalization which depends on Σ_0. If we now universally match γ_0 to the

[3] In passing we note that in the cosmology discussed in Mannheim 1992, 1995b an open universe with explicitly (very) negative k was in fact realized, with such a universe not suffering from the flatness problem found in the standard cosmology.

[4] That cosmology might impact on rotation curves had already been suggested by Mannheim (1995a) in a paper where only the γ_0 term was considered in addition to g_β^{lum}. It is only with the inclusion of the local g_γ^{lum} as well that the fits can be brought completely into line with the data.

[5] In passing it is intriguing to note that with γ^* being identifiable as the coefficient of the linear potential put out by a typical star such as the Sun, and with N^*_{crit} falling right in the range where the prominent galaxies are located, the asymptotic linear potential produced by a typical galaxy will be of the form $V_\gamma^{\text{lum}}(r) = c^2\gamma^* N^*_{\text{crit}} r/2$, i.e., numerically of order $V_\gamma^{\text{lum}}(r) = c^2\gamma_0 r/2$. Since each local galactic potential becomes of order unity on distance scales of order $r = 1/\gamma_0$, the cooperative effect of all of the galaxies in actually producing the Hubble flow in the first place can thus reasonably be expected to produce a universe whose natural distance scale is in fact $1/\gamma_0$.

Freeman limit value Σ_0^F, then in a Freeman limit galaxy with N_{crit}^* stars (i.e., a galaxy whose entire linear term is then also universally normalized to Σ_0^F), the value of the velocity at, say, $10R_0$ or so (a region where the linear term is already dominant) will be equal to its value at the $2R_0$ Newtonian peak. Further, since at around $6R_0$ the Newtonian contribution has dropped to about half its peak value while the linear contribution there is at about half of its value at $10R_0$, we thus get a total velocity at $6R_0$ equal in magnitude to its values at both $2R_0$ and $10R_0$, and thus a flat rotation curve from $2R_0$ all the way out to about $10R_0$. Freeman limit, N_{crit}^* galaxies thus naturally balance the falling Newtonian contribution against the rising linear one and allow flatness to obtain out to around $10R_0$ or so before the ultimate rise required of the linear potential finally sets in. Further, since we have tuned γ_0 to Σ_0^F in the same galaxies,[6] at around $10R_0$ the velocity obeys $v^4 \sim R_0^2(\gamma_0)^2 \sim R_0^2(\Sigma_0^F)^2 \sim \Sigma_0^F L$, which we recognize as the Tully-Fisher relation. The universal matching of Σ_0^F to γ_0 thus leads to both flatness and Tully-Fisher in N_{crit}^* galaxies. Moreover, recognizing the special status enjoyed by Σ_0^F and N_{crit}^* in our theory, we are now able to explain the trend found by Casertano & van Gorkom (1991). Since the low-luminosity galaxies are both sub-Freeman and sub-N_{crit}^*, the γ_0 term dominates and the rotation curves start to rise immediately. (This parallels the trend identified in dark matter fits, where the low-luminosity galaxies are found to be overwhelmingly dark. In fact, the even more generally found trend of a luminosity-dependent ratio of dark to luminous matter is itself reflected in the two-component form we find for the velocity discrepancy exhibited in $(v^2/c^2R)_{last}$ in eq. [1].) Since the intermediate galaxies are both close to Freeman and close to N_{crit}^*, their rotation curves are both very flat and Tully-Fisher. And since the highest luminosity galaxies have N^* greater than N_{crit}^*, the γ_0 term is temporarily overcome so that the curves actually display a mild initial fall (and the galaxies will still be close to Tully-Fisher unless N^* is altogether larger than N_{crit}^*). In this regard a particularly interesting high-luminosity case is NGC 2841, whose data go out to twice as many scale lengths as the other high-luminosity galaxies. For it the rotation velocity is actually seen to peak in the inner region at 326 ± 3 km s^{-1}, to drop to a low of 271 ± 2 km s^{-1} in the intermediate region, and to then rise back to 294 ± 6 km s^{-1} at the largest distances, a behavior which is quite suggestive of the onset of a delayed rise.

Additionally at this juncture, it is of interest to point out that it is possible to make some contact with Milgrom's MOND alternative. Specifically, for Freeman limit, N_{crit}^* galaxies, we note that in the region (near $6R_0$) where the total β and total γ terms are approximately equal (i.e., where $g_\beta^{lum} \sim (g_\gamma^{lum} + \gamma_0 c^2/2) \sim \gamma_0 c^2$), g_{tot} takes the numerical value $2(\gamma_0 c^2 g_\beta^{lum})^{1/2}$, an expression which we recognize as being of the MOND form on identifying $4\gamma_0 c^2$ ($=1.1 \times 10^{-8}$ cm s^{-2}) with Milgrom's a_0 (a phenomenologically introduced parameter whose fitted numerical value is typically found to be 1.2×10^{-8} cm s^{-2}). With this equivalence we see that while conformal gravity and MOND give very different predictions in the region beyond $10R_0$, they nonetheless give quite similar predictions in the region below $10R_0$, where most of the current measurements have been made. From a theoretical viewpoint we note that while Milgrom developed MOND in order to be able to use a universal acceleration to explain the universal Tully-Fisher relation, the particular choice of the a_0-dominated region form for MOND that he made [viz., $g_{tot} = (a_0 g_\beta^{lum})^{1/2}$] was motivated by the additional assumption of asymptotically flat rotation curves (and thus asymptotic flatness even for the low-luminosity rotation curves as well). As we now see, it is in fact also possible to use a universal acceleration to explain Tully-Fisher in a theory where flatness obtains only as an intermediate phenomenon and not as an asymptotic one.

In conclusion we would like to state that the conformal gravity theory would thus appear capable of explaining the general systematics of galactic rotation curves in a completely natural manner, and that our study suggests that rising rather than flat rotation curves is actually the paradigm, with the luminous Newtonian contribution having inadvertently masked that fact in the higher luminosity galaxies. Moreover, through the cosmological connection we have presented, we believe we have made a case for the existence of a universal linear potential associated with the cosmological Hubble flow, an intriguing possibility that appears to enable us to circumvent the need for galactic dark matter.

[6] Given this correlation, it is plausible that the Freeman limit itself may ultimately arise as an upper bound on the galaxies which are generatable as fluctuations out of the cosmological background, a background which is indeed controlled by the γ_0 scale.

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