

TESTING A COUPLING RELATION BETWEEN VISIBLE AND DARK MATTER IN THE MS 2137–2353 CLUSTER CORE¹

EDMOND GIRAUD

Observatoire de Marseille, Place Le Verrier, 13248 Marseille Cedex 4, France; giraud@observatoire.cnrs-mrs.fr

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ABSTRACT

The empirical coupling relation in $M_d = \gamma M^{1/2} r$ between the halo dark mass distribution M_d and the visible mass M at radius r , previously demonstrated for spiral galaxies, is extrapolated to the case of a simple cluster core with mass distribution constrained by a lens mass model. In the at present unique case of MS 2137–2353, where the lens model density profile may be constrained from $\sim 10 h_{50}^{-1}$ kpc to $\sim 100 h_{50}^{-1}$ kpc because of the presence of a radial arc, the density profile deduced from the coupling relation is found to be consistent with that deduced from lens modeling. The coupling relation predicts very sharp cluster cores. In the case of a cluster such as Abell 2218, where a lens model cannot be tested in the innermost region, the density distribution deduced from the structural relation is more sharply peaked than an isothermal lens model, but the density distributions in the overlap region are consistent.

Subject headings: dark matter — galaxies: halos — galaxies: individual (Abell 2128, MS 2137–2353) — gravitational lensing

1. INTRODUCTION

Galaxy clusters play a crucial role in scenarios of the formation of large-scale systems using the gravitational instability theory. Their global parameters, mass function, and correlation function, as well as their mass distribution and evolution and their internal dynamics, provide parameters for cosmological tests (e.g., reviews by Efstathiou 1991 and Ostriker 1993; simulations of Weinberg & Colle 1992; Navarro, Frenk, & White 1996; Ostriker & Cen 1996; Moore et al. 1998). Rotation curves of disk galaxies and gravitational arcs in cluster cores (as well as their velocity dispersions and X-ray luminosities) are the main convincing evidences for large amounts of dark matter in dense systems (e.g., reviews by Carr 1994 and Blanford & Narayan 1992).

Methods for reconstructing cluster surface density from distorted images of lensed background sources have been developed (Kaiser, Squires, & Broadhurst 1995; Seitz & Schneider 1997 and references therein) and intensively applied to map the dark matter profiles of galaxy clusters (Seitz et al. 1996; Squires et al. 1996, 1997; Tyson, Kochanski, & Dell’Antonio 1998). They allow comparisons of the relative distributions of visible and dark matter. The shape parameters of simple lens clusters such as MS 2137–2353, PKS 0745–191, and Abell 2218 given by the best lens models (i.e., the inferred distribution of dark matter) are those of cD galaxies (Mellier, Fort, & Kneib 1993; Kneib et al. 1995; Allen, Fabian, & Kneib 1996; Hammer et al. 1997). No other form of coupling between the visible and the dark mass cluster distributions has yet been reported.

A coupling relation between visible and dark matter in a cluster core should in principle be the result of the dynamical history of the system and of the relative physical properties of dark and visible material. Demonstrating the existence of such a relation in present-day galaxies or clusters would reveal a key phenomenon in the formation of

large-scale dynamical systems and perhaps shed light on the nature of dark matter. In any case, it would constrain N -body simulations of cluster and galaxy formation and perhaps place bounds on the physical properties of dark matter. Since dark matter halos seem to have shapes that resemble those of the luminous mass distributions, their potentials may determine the shapes of the central galaxies, and one may further ask whether they also determine the radial visible mass or stellar mass distribution. We here explore the possibility of a coupling relation between the stellar mass distribution of a cluster core central galaxy and that of the halo dark matter.

In two recent papers (Giraud 1998a, 1998b), it was shown that the observed kinematics of disk rotating systems are consistent with a coupling relation between the visible (stars + gas) and dark mass distributions, with only the dark-to-visible mass ratio displaying significant variations between galaxies.

The purpose of this paper is to compare the mass distributions deduced from arc models in Abell 2218 and MS 2137–2353 with those deduced from the luminosity profiles of the central galaxies and the above coupling relation. Both clusters benefit from *Hubble Space Telescope* (HST) data, and the case of MS 2137–2353 is crucial because there is a radial arc to constrain the core radius, the central galaxy has a small ellipticity, and the region between the central galaxy and the circular arc is almost free of other objects, all conditions that make MS 2137–2353 the best candidate for testing a coupling relation between dark matter and stellar distribution in a galaxy cluster core.

2. PARAMETERS FOR A ROTATIONALLY SYMMETRIC MASS WITH A DARK HALO

2.1. The Structural Relation

Let $M(|r|)$ be a rotationally symmetric mass distribution of visible matter surrounded by a halo distribution of dark matter $M_d(|r|)$, coupled with the visible mass through a relation of the form

$$M_d(|r|) \approx \gamma M^{1/2}(|r|)|r|, \quad (1)$$

¹ Based on retrieved archives of observations made with the NASA/ESA *Hubble Space Telescope*. The HST is operated by the Space Telescope Science Institute for the AURA, under NASA contract NAS5-26555.

where γ is a measure of the dark-to-visible mass ratio at a well-chosen isodensity radius (see Giraud 1998a, 1998b). In those studies of disk galaxies, we were concerned with flat rotating systems surrounded by spherical halos. The hypothesis here is of a quasi-spherical dynamical system.

2.2. Lens Parameters

Let D_{os} and D_{ls} be the distances of the source from the observer and the lens, respectively, and D_{ol} be the distance of the lens from the observer. Because the dimensions of lenses are small compared to the angular distances of the sources and lenses, the mass distribution can be projected into the lens plane, and the bending angle α can be calculated with sufficient approximation (e.g., Schneider, Ehlers, & Falco 1993) by

$$\alpha = \int \kappa(r') \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^2} d^2r', \quad (2)$$

where

$$\kappa(\mathbf{r}) \equiv \frac{D_{ol} D_{ls}}{D_{os}} \frac{4G}{c^2} [\Sigma_l(\mathbf{r}) + \Sigma_d(\mathbf{r})], \quad (3)$$

with

$$\begin{aligned} \Sigma_l(\mathbf{r}) &= \Sigma_l(|\mathbf{r}|) \equiv \frac{dM(|\mathbf{r}|)}{2\pi|\mathbf{r}| d|\mathbf{r}|}, \\ \Sigma_d(\mathbf{r}) &= \Sigma_d(|\mathbf{r}|) \equiv \frac{d[\gamma M(|\mathbf{r}|)^{1/2} |\mathbf{r}|]}{2\pi|\mathbf{r}| d|\mathbf{r}|}. \end{aligned} \quad (4)$$

The terms Σ_l and Σ_d are the surface densities of the luminous and dark mass components, respectively. The quantity κ is the normalized total surface density. The normalized total mass distribution is given by

$$\begin{aligned} \mu(|\mathbf{r}|) &\equiv \int_0^{|\mathbf{r}|} 2\pi|r'| \kappa(|r'|) d|r'| \\ &= \frac{D_{ol} D_{ls}}{D_{os}} \frac{4G}{c^2} [M(|\mathbf{r}|) + \gamma M(|\mathbf{r}|)^{1/2} |\mathbf{r}|], \end{aligned} \quad (5)$$

so that

$$\alpha(|\mathbf{r}|) = \frac{\mu(|\mathbf{r}|)}{|\mathbf{r}|^2} \mathbf{r}. \quad (6)$$

Since we assume a spherical symmetry, the vector \mathbf{r} and norm $|\mathbf{r}|$ notations will be replaced by the scalar radius r in the rest of the paper. If the dark mass dominates, the total density profile $\Sigma \equiv \Sigma_l + \Sigma_d$ may be significantly different from that of the visible mass. Let us suppose, for example, that the surface density profile deduced from a luminosity profile is a power law in $r^{-\alpha}$. In that case, it is straightforward to show that the total surface density deduced from the structural relation can be written

$$\Sigma(r) = Ar^{-\alpha} + B\gamma r^{-\alpha/2}. \quad (7)$$

For large values of r and γ , the term in $r^{-\alpha/2}$ dominates in Σ , and the luminosity profile has a power index 2 times that of the dark matter surface density. In that case, the M/L ratio will increase with radius. For a luminous exponential

profile, the density profile at large r of a dark halo in $\gamma M^{1/2} r$ is in r^{-1} (see below).

Because the structural relation combined with the shape of the luminosity profile makes definite predictions about the mass profile including dark matter, it can be compared with the density profile deduced from lensing in rich clusters of galaxies.

3. THE DENSITY PROFILE OF THE DARK MATTER IN ABELL 2218 FROM THE LENS MODEL AND FROM THE STRUCTURAL RELATION

The very rich cluster Abell 2218 at redshift $z = 0.175$ presents a system of arcs and arclets that was extensively studied by Pellò et al. (1988, 1992). The central object of Abell 2218 is a cD galaxy with a halo extending to a radial distance of $26''$. There are two arcs at radial distances $r_1 \approx 20''.7$ and $r_2 \approx 22''.1$ from the nucleus of the cD galaxy, and two other arcs around a second bright galaxy (Pellò et al. 1992). Kneib et al. (1995) derived a lensing model with two clumps, which reproduces well the four arcs seen in the cluster. The most important characteristic of this model is that the centers, orientations, and ellipticities of the mass distribution are deduced from the geometrical parameters of the envelopes of the two bright galaxies. The model requires a mass-to-light ratio $M/L_R = 80$ within the ellipse traced by the arc at $r_2 = 22''.1$ ($\sim 85 h_{50}^{-1}$ kpc), which increases up to $M/L_R = 180$ within the most extended ellipse constrained by the arcs ($r \approx 256 h_{50}^{-1}$ kpc). More recent mass models based on *HST* data (Squires et al. 1996; Kneib et al. 1996, hereafter K96) do not differ significantly from those deduced from ground-based data, but they are improved by the inclusion of weakly lensed objects. Values of $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega = 1$ are used to compare with the model of K96. The main component of the mass model is centered on the bright cD galaxy (No. 391 in the numbering scheme of Le Borgne, Pellò, & Sanahuja 1992), and has an ellipticity $a/b = 1.37$, whereas the cD ellipticity is in the range 1.45–1.55. The analytical form of the mass distribution used in Kneib et al. papers is a pseudoisothermal sphere with core radius $r_c = 45\text{--}76 h_{50}^{-1}$ kpc and a velocity dispersion of $1202\text{--}1335 \text{ km s}^{-1}$. The important parameter of a lens mass model constrained by a distorted background object at radius r is the surface density at r . The model is validated by the observations at radial distance of from a few kpc less than the innermost arc out to the outermost weakly lensed objects. The innermost arcs constrain the gradient of the surface density at that position. At smaller radial distances, the density profile is given by the choice of the analytical form. We consider here the region surrounding the bright cD galaxy out to the two arcs, No. 359 and No. 384, where secondary components of the lens mass model have little effect. The structural relation gives the surface density profile approximately out to these arcs. Therefore, a comparison is possible in the overlap region.

The luminosity profile of the central galaxy is measured on a stacked image made from three individual *HST* exposures acquired by K96 and retrieved from public archives. These are WFPC2 images taken with the F702W filter for a total integration time of 6500 s. Each individual exposure is offset by an integer number of pixels, and the frames have been registered and coadded using median filtering to correct for cosmic rays and bad pixels. The best fit to the profile is obtained with a double exponential disk in

$\exp(r/r_{d1})$ and $\exp(r/r_{d2})$ (Fig. 1). The empirical values of r_{d1} and r_{d2} are found to be the same on the two axes: $r_{d1} = 31.9 h_{50}^{-1}$ kpc and $r_{d2} = 7.3 h_{50}^{-1}$ kpc on the major axis, and $r_{d2} = 7.0 h_{50}^{-1}$ kpc on the minor axis.

If there is a structural relation of the form $M_d(r) = \gamma M^{1/2}(r)r$ between the stellar mass distribution of the central galaxy and the dark mass distribution in the cluster core, the dark matter density will have a profile in

$$\Sigma_d(r) = \sqrt{\frac{\sigma_1}{2\pi}} \gamma \left[\frac{r_{d1}}{r} U_{d1}(r) + \frac{1}{2} \frac{r}{r_{d1}} \frac{\exp(-r/r_{d1})}{U_{d1}(r)} \right] + \sqrt{\frac{\sigma_2}{2\pi}} \gamma \left[\frac{r_{d2}}{r} U_{d2}(r) + \frac{1}{2} \frac{r}{r_{d2}} \frac{\exp(-r/r_{d2})}{U_{d2}(r)} \right], \quad (8)$$

where

$$U_{d1,2}(r) = \sqrt{1 - \left(1 + \frac{r}{r_{d1,2}}\right) \exp\left(-\frac{r}{r_{d1,2}}\right)}. \quad (9)$$

This density profile, which is flatter than the luminosity profile, is compared with the profile of the lens component associated with object No. 359 (the cluster core) in Figure 2, which may be commented on as follows. (1) In the inner region, where the lens mass model is not constrained by the observations, the model density deduced from the structural relation is sharply peaked and very different from the lens mass model. (2) In the small region of overlap near the two arcs, the gradients are similar. (3) At larger r , where the model density deduced from the structural relation is extrapolated, the two models are still consistent. Points (2) and (3) are not surprising, because in the case of an exponential profile the gravitational potential of a halo in $M_d = \gamma M^{1/2}r$ is flat (at sufficiently large r , the term in r_d/r dominates in the expression of Σ_d). Removing the uncertainty on the mass profile in the central region would require a study of cases with radial arcs very close to the mass center. In this

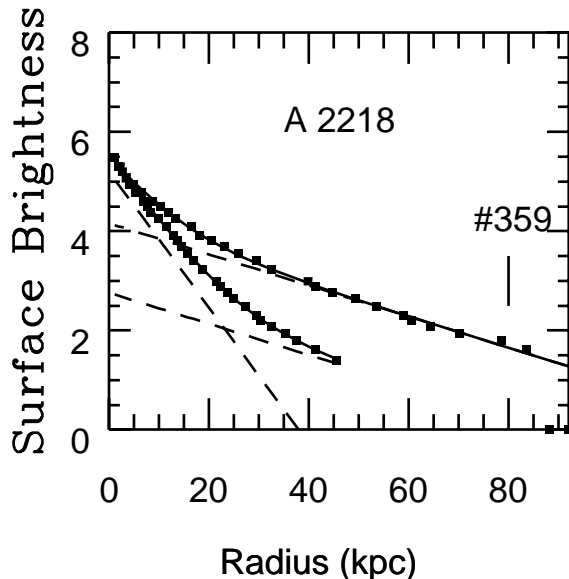


FIG. 1.—Luminosity profiles along the major and minor axes of the cD galaxy in Abell 2218 from *HST* data, and the double exponential fits to the observations. The position of arc No. 359 is shown.

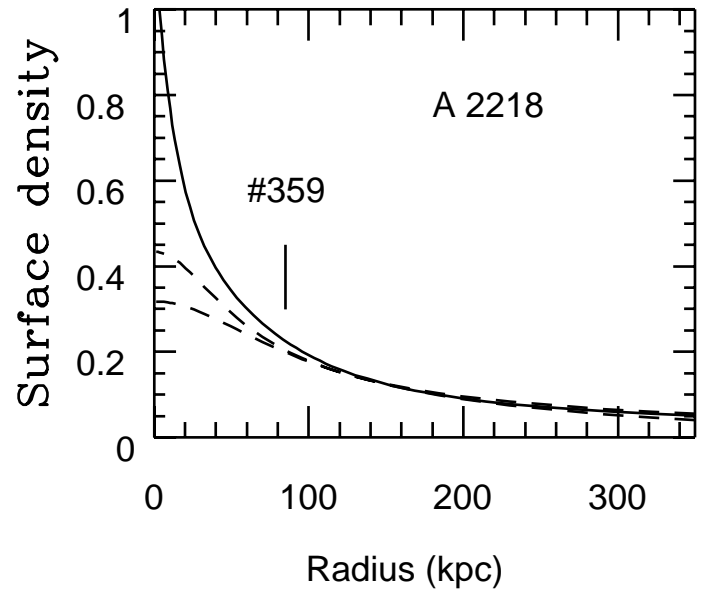


FIG. 2.—Dark matter surface density profile from the coupling relation between the stellar mass distribution and the dark matter distribution (solid line), compared with the lens models of K95 and K96 (dashed lines), both along the major axis. The position of arc No. 359 is shown. The curve from the coupling model is extrapolated toward larger radii. At smaller r , the curve of the lens model depends on the analytical form chosen for the lens mass distribution.

respect, the radial arc of MS 2137–2353 provides a unique opportunity for comparing the density profiles obtained from the advocated structural relation to that of a lens mass model.

4. THE DENSITY PROFILE OF THE DARK MATTER IN MS 2137–2353 FROM THE LENS MODEL AND FROM THE STRUCTURAL RELATION

The core of the X-ray cluster MS 2137–2353 is dominated by a giant galaxy at $z = 0.315$. Two arc systems, including a tangential arc and a radial arc, both close to the central galaxy, have been discovered and modeled (Fort et al. 1992; Mellier et al. 1993; Hammer et al. 1997, hereafter H97). They are images of two distinct sources, S_1 and S_2 , at similar redshift. The four amplified images of S_1 and the three images of S_2 have revealed that the mass distribution in the cluster core has geometrical parameters in rather good agreement with those of the visible light (H97). The most important characteristic of this model is that the center, orientation, and ellipticity of the mass distribution are consistent with the geometrical parameters of the central galaxy. The luminosity profile is measured on a stacked image made from eight of the WFPC2 images taken with the F702W filter, acquired by H97 and retrieved from public archives. The total integration time is 18,400 s. The data reduction is the same as for Abell 2218. In the present case, the luminosity profile turns out to be a power law in $(r/r_c)^{1-2\beta}$, with $\beta = 1.34$. Figure 3 shows the integrated luminosity, or the mass in stars for a constant M/L (stars) ratio, which is in $(r/r_c)^{0.31}$, with $r_c = 2''.6 \pm 0''.2$ ($14.7 \pm 1.1 h_{50}^{-1}$ kpc), out to $r = 38 h_{50}^{-1}$ kpc.

The hypothesis of a structural relation between the visible mass distribution and the dark matter profile yields a total

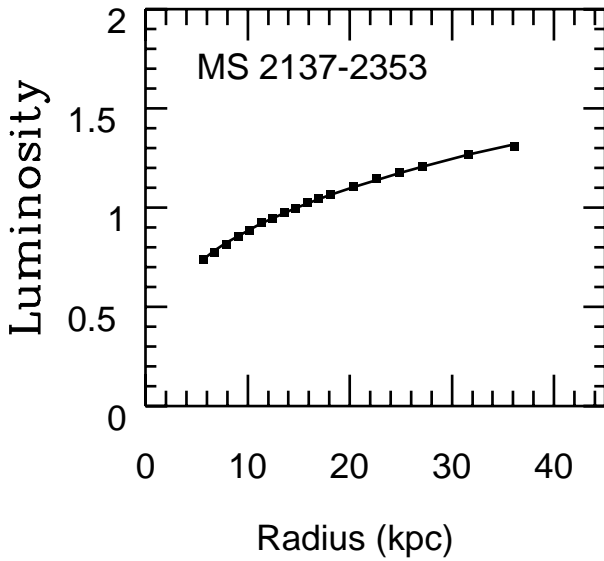


FIG. 3.—Integrated luminosity profile measured on the coadded frame of MS 2137–2353 as a function of radius. The solid line shows the best fit to the data.

density profile

$$\Sigma\left(\frac{r}{r_c}\right) \propto \left(\frac{r}{r_c}\right)^{1-2\beta} + \frac{5-2\beta}{2(3-2\beta)} \gamma r_c M^{1/2}(r_{\text{crit}}) \times \left(\frac{r_{\text{crit}}}{r_c}\right)^{(3-2\beta)/2} \left(\frac{r}{r_c}\right)^{(1-2\beta)/2}, \quad (10)$$

or, in numerical values,

$$\Sigma\left(\frac{r}{r_c}\right) \propto \left(\frac{r}{r_c}\right)^{-1.68} + [11, 57] \left(\frac{r}{r_c}\right)^{-0.84}, \quad (11)$$

where the range [11, 57] corresponds to a range of [1, 5] in $M/L_R(\text{stars})$. For $r > r_c$, the second term is >11 times the first term; therefore, at $r > r_c$ the surface density profile of the dark matter predicted by the structural relation and the visible mass is a power law in $(r/r_c)^{1-2\beta'} = (r/r_c)^{-0.84}$, that is, $\beta' = 0.92$.

The lens model density profile of the dark matter adopted by H97 is also of the form $\rho = \rho_0[1 + (r/r_c)^2]^{-\beta'}$. The presence of the radial arc, which extends very close to the mass center ($3''.4$), implies a very small value for the core radius r_c : $2''.2 \pm 0''.8$ (see Fig. 3 of H97), very close indeed to the $2''.6 \pm 0''.2$ deduced from the structural relation.

The optimal value of β' for the reconstruction of the sources, $\beta' = 0.875 \pm 0.045$ from H97, is much flatter than that of the visible light, $\beta = 1.34$, but is in good agreement with the power index $\beta' = 0.92$ deduced from the structural relation (Fig. 4) in the region where the two models can be compared. Lens models from ground-based images gave $\beta' \approx 1$ (Mellier et al. 1993). The major difference in galaxy halos is that the parameter γ in the mass model is of the order of 15 times that of disk galaxies.

5. CONCLUSION

A coupling relation in $M_d = \gamma M^{1/2} r$ between dark mass and visible mass, previously demonstrated for disk rotating systems, has been tested in the cases of two well-studied cluster cores, those of Abell 2218 and MS 2137–2353, with dark matter profiles deduced from lens modeling.

In Abell 2218, the extrapolated density distribution, deduced from the coupling relation in the region of the cD galaxy, is consistent with that deduced from a lens mass model at radii where the lens model is constrained by observed arcs and arclets. In the inner region, where the mass distribution is not constrained by lens modeling, the density distribution of dark matter deduced from the coupling relation is much more sharply peaked than that deduced from the isothermal lens model. Very small core radii are indeed consistent with statistical approaches (Wu & Hammer 1993).

The simple unimodal cluster core of MS 2137–2353 provides us with a unique opportunity for comparing the dark mass profile deduced from the coupling relation with that of a lens mass model, because there is a radial arc very close to the mass center, as well as a second tangential arc. In that case, the coupling relation appears to be consistent with the lens model distribution. Both the core radius, $r_c = 2''.6$, and the power index of the dark mass profile, $\beta' = 0.92$, deduced from the luminosity profile of the central galaxy and the coupling relation are in agreement out to the radius of the tangential arc with those obtained for the reconstruction of the sources ($r_c = 2''.2 \pm 0''.8$, $\beta' = 0.875 \pm 0.045$) through the gravitational lens model of Hammer et al. (1997). The dark mass dominates the luminous mass except in the very inner region, $r < 2''$, of the central galaxy. The model M/L ratio increases with radius. The existence of a radial arc and its observation with the *HST* were extremely important for comparing the core radii.

Since only one cluster is known to have a radial arc, the present work goes far to validate the coupling relation in clusters, and there is likely to be a large family of relations that would do as well for only one cluster. Nevertheless, this result should be taken into consideration because the test was done on the best cluster candidate available at present, MS 2137–2353, which has (1) excellent *HST* data, (2) a

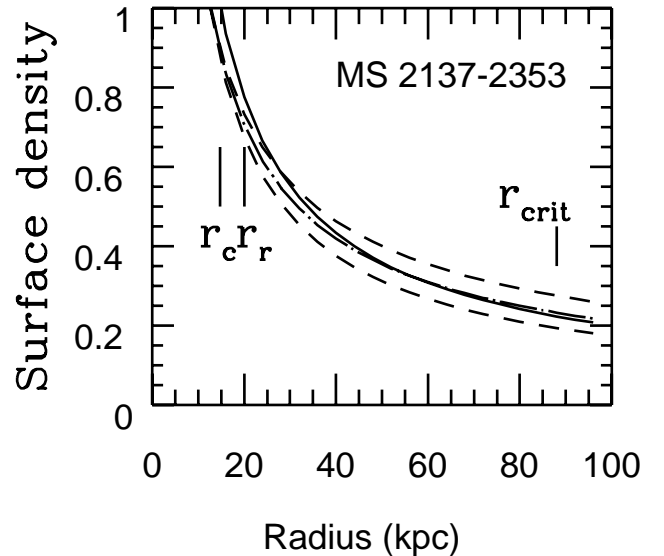


FIG. 4.—Dark matter surface density profile, $\sigma_d(r) \propto (r/14.7)^{-0.84}$, deduced from the structural relation and the luminosity profile of the central galaxy in MS 2137–2353 (solid line), compared with two lens mass models (dashed lines) obtained by using the best lens estimate for the core radius $r_c = 12.5$ kpc and the limit exponents $\sigma_{\text{min}}(r) \propto (r/12.5)^{-0.66}$ and $\sigma_{\text{max}}(r) \propto (r/12.5)^{-0.84}$ (from H97). The position of the tangential arc is r_{crit} ; that of the radial arc extends from 19 kpc outward; it is marked by r_r .

radial arc to constrain the core radius, (3) a central galaxy with small ellipticity, and (4) a region between the central galaxy and the circular arc almost free of other objects.

The structural relation should result from the dynamical history of cluster cores and the relative properties of visible and dark matter. A challenge for the dark matter hypothesis is to find the physical mechanism responsible for the structural relation between the dark halo distribution and the visible (stars & gas) mass distribution in spiral galaxies. The present paper suggests that the same mechanism may be at work in cluster cores.

The dark matter density profiles of the MS 2137–2353 cluster core deduced from both the structural relation and the lens model are steeper than the CDM $\rho \propto r^{-1}$ universal profile of Navarro, Frenk, & White (1996). Recent simulations with higher resolutions (Moore et al. 1998) yield

steeper density profiles, $\rho \propto r^{-1.4}$, which are in better agreement with the present paper. The Navarro et al. (1996) profiles are, however, more concentrated than allowed by the observations of dwarf galaxies (see their Fig. 12).

Since the modified dynamics of Milgrom (1983) (MOND), recently tested by McGaugh & de Blok (1998, and references therein), is often successful in describing the rotation curves of galaxies from their luminous density profiles, it may also give the right bending angles. However, MOND's fundamental constant, a_0 , is proportional to γ^2 (see Appendix), which is not a constant, and is an indicator of the dark-to-luminous mass ratio (Giraud 1998b). The value of γ for MS 2137–2353 is about 15 times that of disk galaxies. Consequently, the required value of a_0 may be off by a large factor.

APPENDIX A

THE STRUCTURAL RELATION AND MODIFIED DYNAMICS

Let a spherical luminous mass distribution $M(r)$ be surrounded by a dark halo with mass $M_d(r)$. Let V_c be the circular velocity defined by $V_c^2(r) \equiv GM_d(r)/r$. The energy required to unbind a mass m from a halo with circular velocity V_c is $E \approx mV_c^2$. If the mass distribution in the dark halo is of the form $M_d(r) = \gamma M^{1/2}(r)r$, the circular velocity is

$$V_c^2 = G\gamma M^{1/2}. \quad (\text{A1})$$

The basis of Milgrom's (1983) modified dynamics is the assumption that in the low-acceleration regime, $a \ll a_0$, the acceleration of a particle at a distance r from a mass M satisfies approximately $a^2/a_0 \approx GM/r^2$, where a_0 is a constant with the dimension of an acceleration. Let m_g be the gravitational mass of a body moving in a static force field F with acceleration a . In MOND, Newton's second law is replaced by $F = m_g g_N = m_g \mu(a/a_0)a$, where $\mu(x \gg 1) \approx 1$ and $\mu(x \ll 1) \approx x$. The simple form $\mu(x) = x(1 + x^2)^{-1/2}$ is often used in calculating rotation curves (Kent 1987; Begeman, Broeils, & Sanders 1991; Sanders 1996; McGaugh & de Blok 1998). At a large galactic radius, $g_N \approx MG/r^2$ and $a = V^2/r$, from which one gets the asymptotic velocity

$$V_{\text{lim}}^4 \approx M G a_0. \quad (\text{A2})$$

The energy needed to unbind a particle of mass m must be the same whether the force field comes from a dark halo or from modified dynamics. Therefore,

$$a_0 \approx G\gamma^2. \quad (\text{A3})$$

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