Vectors in Physics

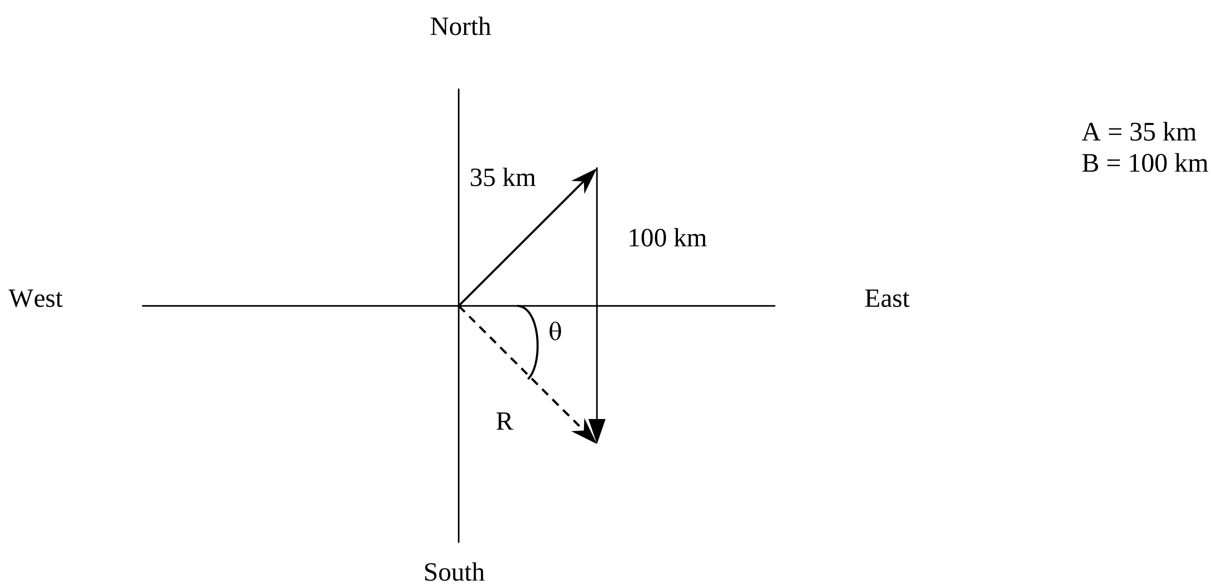
Vectors must have both a magnitude (size) and a direction (angle) associated with them. For example, velocity is a vector so a car travels 50 mi/hr (magnitude) due North (90^o angle).

Combining vectors: when you add vectors you add them head to tail, head to tail and then draw the resultant vector from the first tail to the last head. When subtracting vectors you just flip the direction of the one with the negative sign and then add that vector as you normally would. When adding vectors it is often the easiest to reduce each vector to its x and y components, add the components and use the sum of the components to determine the resultant vector.

Example:

A car travels Northeast for 35 km and then turns South for 100 km. What is the resultant vector?

> First off draw a picture of the problem. Then reduce each leg of the trip into its x and y components (watch for negatives, you need to put them in as needed). Add all the x components and get a total x, add all the y components and get a total y. Using the total x and total y values determine the directions of the resultant vector by using then inverse tangent function. Using the Pythagorean theorem determine the magnitude of the vector.



	x component	y component
A	(35 km)(cos 45^o) = 24.7 km	(35 km)(sin 45^o) = 24.7 km
B	0 = 0	- 100 km = -100 km
R	= 24.7 km	= -75.3 km

The negative sign only refers to the direction of that vector. You don't need to include the sign in the calculations. To get the magnitude of R, use the Pythagorean theorem:

$(24.7 \text{ km})^2 + (75.3 \text{ km})^2 = R^2 \qquad \Rightarrow R = 79.2 \text{ km}$

To determine the direction of the vector solve for θ.

$\tan\theta = \dfrac{75.3}{24.7} = 3.05$

$\tan^{-1} 3.05 = \theta \qquad \Rightarrow \qquad \theta = 72.7^o$ South of East

Note: Again, you don't include the negative sign in the calculation, it is described by the words South of East, i.e. the angle is below the x axis.

Problems:

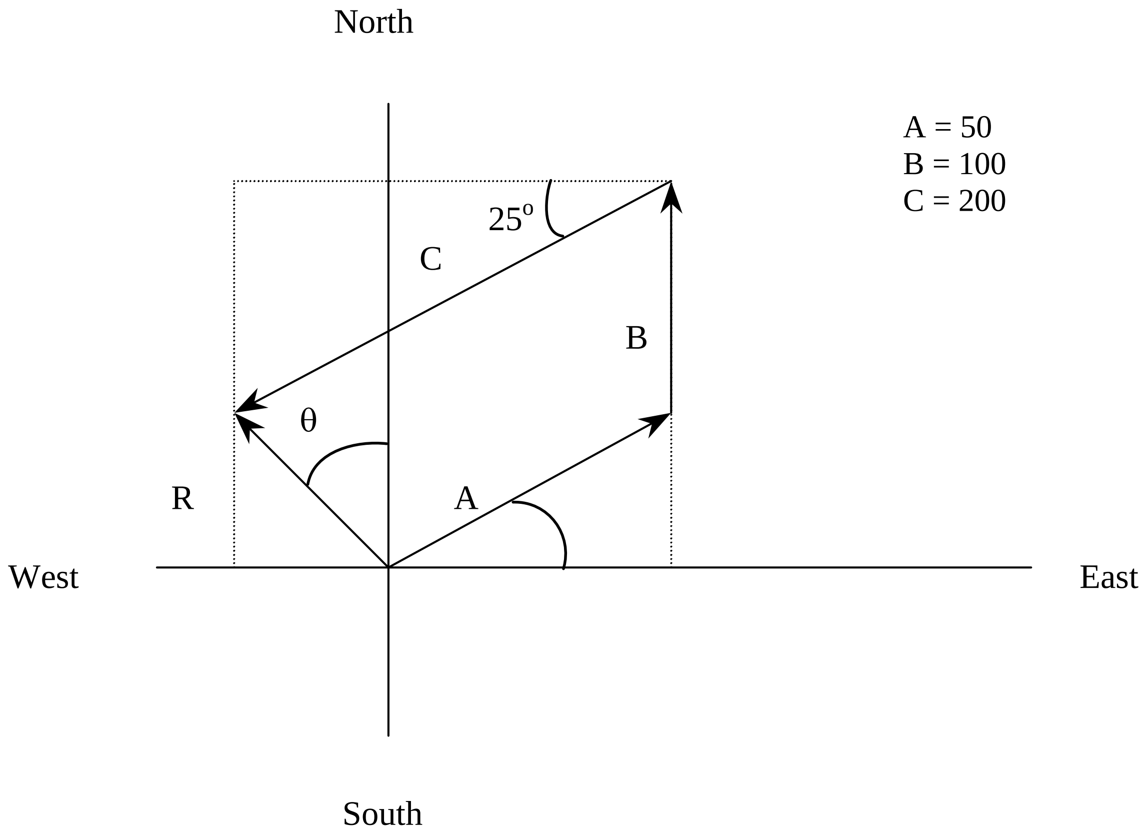
1. Someone in a car drives 50 km 35^o N of East, then turns due North for 100 km and finally turns 25^o South of West for the final 200 km. Determine the magnitude and direction of the resultant vector.

2. Two people are pulling a box, each person is 25o from an imaginary centerline. Person A pulls with a force of 50 N and person B pulls with a force of 75 N. What is the resultant force along the imaginary centerline?

3. A plane flying East at 140 km/hr actually travels a path 15^o North of East at 145 km/hr. What is the velocity of the wind that is moving the plane "off-course"?

Solutions:

1. Someone in a car drives 50 km 35° N of East, then turns due North for 100 km and finally turns 25° South of West for the final 200 km. Determine the magnitude and direction of the resultant vector.



	x component		y component	
A	(50 km)(cos 35°)	= 41	(50 km)(sin 35°)	= 28.7 km
B	0	= 0	100 km	= 100 km
C	(-200 km)(cos 25°)	= -181.3 km	(-200 km)(sin 25 °)	= -84.5 km
R		= -140.3 km		= 44.2 km

The negative sign only refers to the direction of that vector. You don't need to include the sign in the calculations. To get the magnitude of R, use the Pythagorean theorem:

$$(-140.3\text{ km})^2 + (44.2\text{ km})^2 = R^2 \quad \Rightarrow \mathbf{R = 147.1\ km}$$

To determine the direction of the vector solve for θ.

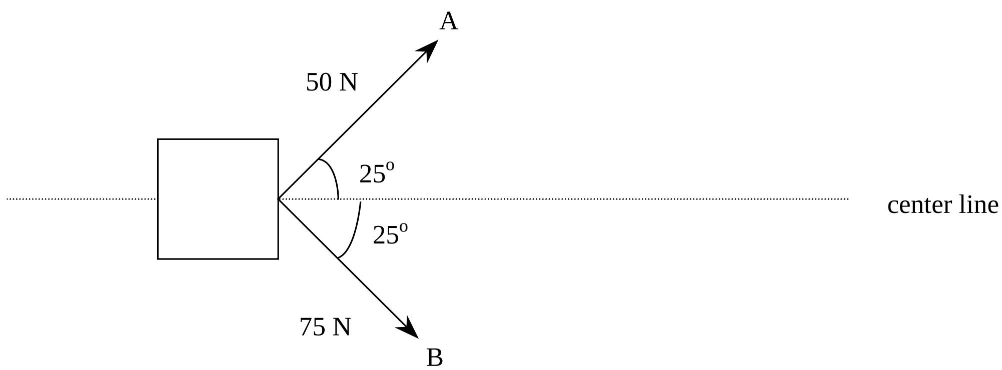
$$\tan\theta = \frac{44.2}{140.3} = 0.36$$

$$\tan^{-1} 0.32 = \theta \quad \Rightarrow \quad \mathbf{\theta = 17.5^o\ West\ of\ North}$$

This could also be express as $19.8^o + 90^o = 109.8^o$ angle

Note: Again, you don't include the negative sign in the calculation, it is described by the words West of North, i.e. the angle is below the x axis.

2. Two people are pulling a box, each person is 25^o from an imaginary centerline. Person A pulls with a force of 50 N and person B pulls with a force of 75 N. What is the resultant force along the imaginary centerline?



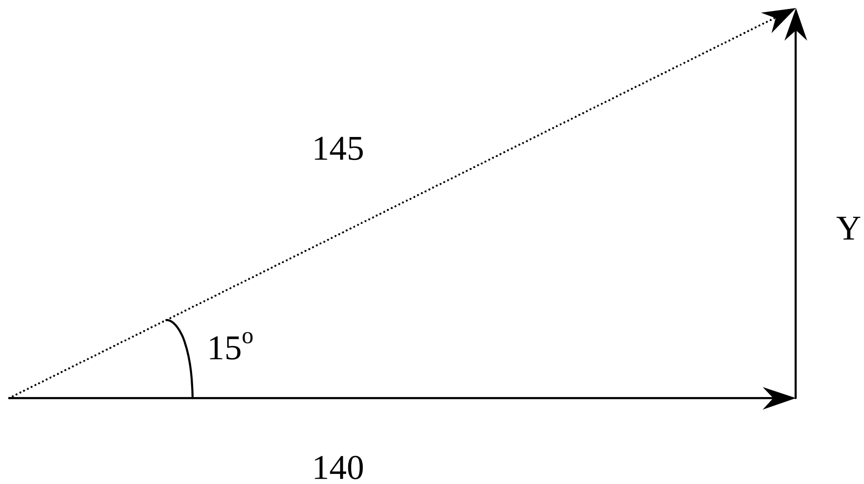
To answer this question you need to get the x component for each force.

A $(50\text{ N})(\cos 25^o) = 45$ N

B $(75\text{ N})(\cos 25^o) = 68$ N

The total force down the centerline is 45 N + 68 N = **113 N**

3. A plane flying East at 140 km/hr actually travels a path 15^o North of East at 145 km/hr. What is the velocity of the wind that is moving the plane "off-course"?



There are 2 ways to solve this problem:

A. Using the Pythagorean theorem

$(140\ km/hr)^2 + y^2 = (145\ km/hr)^2$

$(145\ km/hr)^2 – (140km/hr)^2 = y^2$

y = 37.7 km/hr

B. $\sin 15^o = \dfrac{y}{145 km/hr}$

$(145\ km/hr)(\sin 15^o) = y$

y = 37.5 km/hr

These 2 values for y are close enough.