MA 180 Quadratic Functions: Summary and Applications
Section 3.1

Summary

A quadratic function is a function of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. The graph of a quadratic function is always a parabola.

(1) The parabola opens up if a > 0 and down if a < 0.

(2) The **vertex** or turning point of the parabola occurs at $x = -\frac{b}{2a}$, $y = f(-\frac{b}{2a})$.

(3) The **axis of symmetry** is the vertical line $x = -\frac{b}{2a}$.

(4) The **y-intercept** occurs when x = 0, y = c. To find it, substitute x = 0 into the equation and find the corresponding value of y.

(5) The **x-intercepts** occur when y = 0. To find the x-intercepts, substitute y = 0 into the equation and find the corresponding value(s) of x. You will need to solve the equation $ax^2 + bx + c = 0$. You can do this either

(a) **Algebraically** using factoring, if possible, or the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, or

(b) **Graphically**, finding the x-intercepts by using the CALC menu of your calculator.

Examples

1. Let $f(x) = 2x^2 - 2x - 3$.

(a) What is the direction of opening?

(b) Algebraically determine the coordinates of the vertex.

(c) What is the equation of the axis of symmetry?

(d) Algebraically determine all intercepts.

(e) Graph the parabola by hand and check your sketch using your calculator.

2. A ***demand function*** for an item is a relationship between the number of items *x* that will sell (that is, the ***demand*** for the item) and the price *p* charged per item. In general, a demand function is a decreasing function because if a manufacturer wants to sell more items, the price must be lowered.

If the function $p = f(x)$ is a demand function, then the ***revenue R(x)*** received by selling *x* items at a price of *p* dollars per item is $R = (\#\ of\ items\ sold)\cdot(price\ \ per\ \ item) = x\,p$
Suppose that the demand function for a certain item is $p = -3.1x + 524$, $0 \leq x \leq 150$.

(a) Express the revenue R as a function of x.

(b) What is the revenue if 90 items are manufactured and sold?

(c) **Algebraically** determine the coordinates of the vertex of the revenue function.

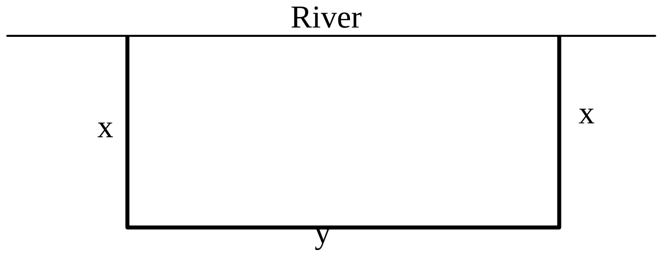
(d) Choose an appropriate window and graph the revenue function. You should see the vertex and intercepts of the function. Use your graph to check that your answer to part (c) is correct.

(e) What quantity x maximizes revenue? What is the maximum revenue?

(f) What price should the company charge to maximize revenue?

3. A rectangular pen that borders a river is going to be enclosed using 250 yards of fencing. No fencing is needed along the river.

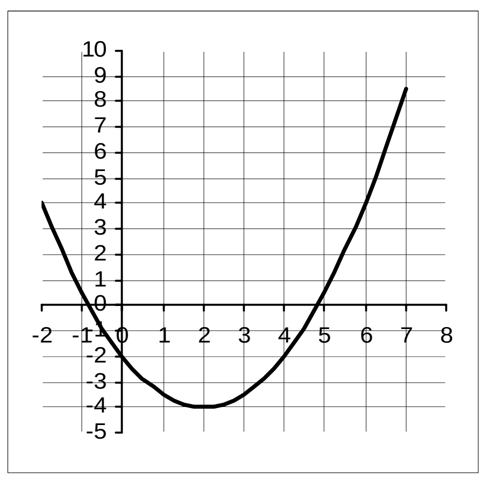
(a) Write an algebraic expression relating x and y to the amount of fencing available.



(b) Express the area *A* as a function of *x*.

(c) Use a graphing utility to graph the area function and sketch. Use the graph to approximate the length and width of the rectangle of maximum area, and verify algebraically.

4. Another form of the equation of a quadratic function is $f(x) = a(x-h)^2 + k$. In this case, the vertex is at (h, k). Again, the parabola opens up if a > 0 and down if a < 0. This form can be used to find the equation of a quadratic function if information about the function, or a picture of the function is given. Use this form of the equation of a quadratic function to find a formula that would produce the given graph. Use your calculator to check your answer



HW for Section 3.1:
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