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MA110

4-4 Matrices : Basic Operations

Equality of Matrices

Two matrices are equal if :

Addition and Subtraction of Matrices

In order to add or subtract matrices, they must be of the same size. Then we just add or subtract element by element

$$\begin{bmatrix} -2 & 5 & 0 \\ 6 & 5 & 7 \end{bmatrix} + \begin{bmatrix} 8 & -2 & 5 \\ 6 & 0 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} 5 \\ 2 \\ 1 \\ 6 \end{bmatrix} - \begin{bmatrix} 2 \\ -5 \\ 0 \end{bmatrix} =$$

The matrix of all zeros is called the **zero matrix** and is often denoted just "0".

The negative of a matrix *M* is denoted -*M* and is the matrix such that $M + (-M) = 0$.

If $M = \begin{bmatrix} -2 & 6 \\ 4 & 5 \end{bmatrix}$ then $-M =$

Scalar Multiplication

In keeping with previous understanding of multiplication, 2*A* should be equal *A* + *A*. Therefore we can say that if $A = \begin{bmatrix} 2 & 1 \\ -2 & 5 \end{bmatrix}$ then $2A =$

So if we have a matrix *M* and we want to multiply by a number (a scalar) *k* then we multiply each element of *M* by *k*.

$$10\begin{bmatrix} 2 \\ -1 \end{bmatrix} =$$

Matrix Product

The product of a $1\times n$ row matrix and an $n\times 1$ column matrix is given by

$$\begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_1b_1 + a_2b_2 + \cdots + a_nb_n \end{bmatrix}$$

Think of this as having the expression $2x+3y-2z$ which gives us the row vector $\begin{bmatrix} 2 & 3 & -2 \end{bmatrix}$ and if we want to evaluate for $x=1$, $y=0$, and $z=3$ it would be

$$2\cdot 1+3\cdot 0+(-2)\cdot 3$$

which is the same as $\begin{bmatrix} 2 & 3 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

Example
A factory produces a slalom water ski that requires 3 hours labor in the assembly department and 1 hour of labor in the finishing department. Assembly personnel receive \$9 per hour and finishing personnel earn \$6 per hour. What are the labor costs to produce 1 ski?

Matrix Product

If we want to multiply matrix *A* by matrix *B*, then the element in the ith row and jth column is the product of the ith row of *A* with the jth row of *B*.

1) The number of columns of *A* must match the number of columns of *B*.
2) If *A* is an $m\times p$ matrix and *B* is a $p\times n$ matrix then AB is an $m\times n$ matrix.

Find the following

$$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & -2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 0 \\ 5 & 2 \end{bmatrix} =$$

Comments

1) Matrix multiplication is not commutative, meaning that AB and BA may be different. In fact, one may be defined while the other is not (see above)
2) The zero-product property does not hold. That is if $AB = 0$ we can not say that one them is the zero matrix.

Example
If we have the system of equations

$$\begin{array}{rcrcr} 2x & + & y & = & 3 \\ x & - & y & = & 5 \end{array}$$

we can write it as

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Example
A factory produces a slalom water ski that requires 3 hours labor in the assembly department and 1 hour of labor in the finishing department. It also produces a trick ski that requires 5 hours in the assembly department and 1.5 hours in the finishing department. Assembly personnel receive \$9 per hour and finishing personnel earn \$6 per hour. What are the labor costs to produce 1 ski of each type?