Nicholson

MA110

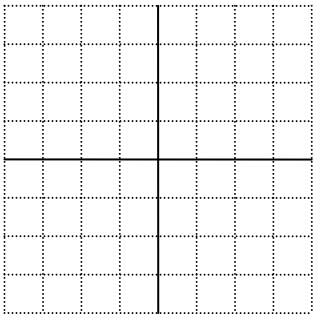
5-1 and 5-2 Systems of Linear Inequalities in Two Variables

We know how to graph linear equations like

$$2x - 3y = 6$$

How do we graph linear inequalities like $2x - 3y < 6$?

First, let's graph the equation:



The line splits the coordinate plane into 2 pieces, known as **half-planes**.

The line itself is the points where the equation $Ax + By = C$ is true, we have equality. Each half-plane is either all the points that have $Ax + By > C$ or $Ax + By < C$. When showing the solutions to a linear inequality on a graph it is customary to shade the half-plane that contains the solutions we are interested in.

Since the line itself is the place where equality occurs, it is included in the solution set of an inequality with $\leq$ or $\geq$ and is no included for inequalities that involve < and >. To show this when we are graphing we use a dashed line to show that the line is not included and a solid line to indicate the line is included.

So we have three steps to graphing the solution set of a linear inequality such as $Ax + By > C$

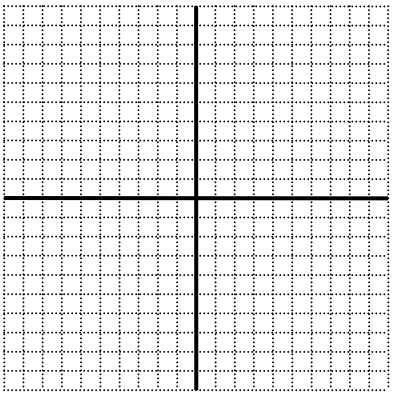
1) First graph the line $Ax + By = C$ as a dashed line if equality is not included and a solid line if it is.

2) Determine which half-plane is the solution region. If unsure, we can choose a test point that is clearly in one region or the other and test it to see if it belongs in the solution set or not.

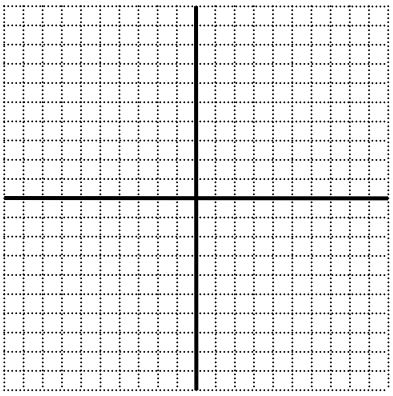
3) Shade the solution region.

Finish graphing the above inequality

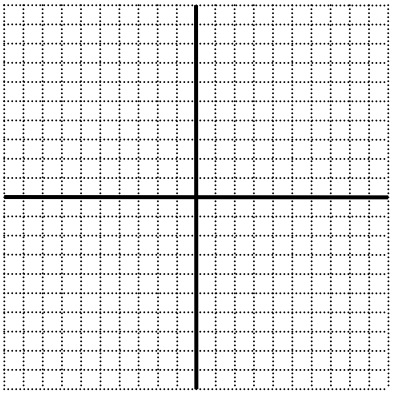
Graph $6x - 3y \leq 18$



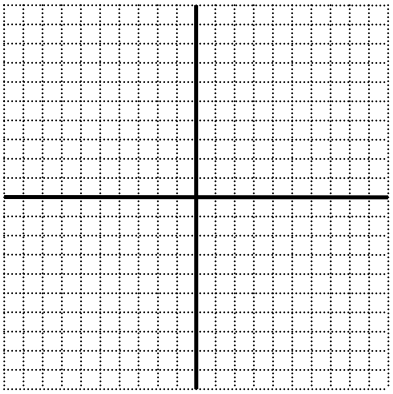
Graph $y > -3$



Graph $x < 3y$



Graph $x \geq 2$



Systems of Linear Inequalities

We now want to solve systems of inequalities. That is, we want to find all values of the variables that solve a collection of inequalities such as:

$$\begin{aligned} 2x &- y \leq 4 \\ 3x &+ 2y > 6 \end{aligned}$$

Is (0,4) a solution to this system?

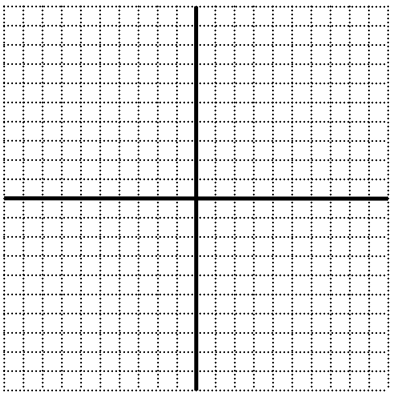
Is (1, 1) a solution to this system?

To solve a system of inequalities we graph each and find where there solution regions overlap.

A **corner point** of a solution region is a point where two or more boundary lines intersect.

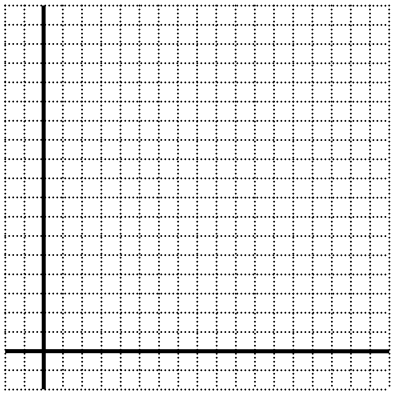
Graph the system from before and find all corner points.

$$\begin{aligned} 2x - y &\le 4 \\ 3x + 2y &> 6 \end{aligned}$$



Solve the following system graphically and find the coordinates of all corner points.

$$\begin{aligned} 2x + y &\le 22 \\ x + y &\le 13 \\ 2x + 5y &\le 50 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$



A solution region is called **bounded** if it can be enclosed within a circle. Otherwise it is called unbounded.

Application

A patient in a hospital is required to have at least 84 units of drug *A* and 120 units of drug *B* each day (Assume an overdosage of either is harmless). Each gram of substance *M* contains 10 units of drug *A* and 8 units of drug *B*, and each gram of substance *N* contains 2 units of drug *A* and 4 units of drug *B*. How many grams of *M* and *N* can be mixed to meet minimum daily requirements?