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MA110
8-3 Conditional Probability, Intersection, and Independence

What is the probability of rolling a prime number on a single die?

What is the probability of rolling a prime number on a single die, given the information that the number that was rolled was odd?

This is like asking 2 questions:

1) What is the probability that a person develops lung cancer?
2) What is the probability that a person develops lung cancer if we are told they are a heavy smoker?

Conditional Probability

For events *A* and *B* in an arbitrary sample space *S*, we define the **conditional probability of *A* given *B*** as

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \; P(B) \neq 0$$

Example
Suppose we have a spinner where some regions are larger than others. The probability of landing on a given number is given below

e_i	1	2	3	4	5	6
$P(e_i)$	.2	.3	.1	.1	.2	.1

a) What is the probability of landing on a prime number?

b) What is the probability of landing on a prime number, given that it landed on an odd number?

Example
We have collected data on the probability of being in an accident on the last day of the Memorial Day weekend.

		Accident A	No Accident A'	*Totals*
Rain	R	.025	.335	.360
No Rain	R'	.015	.625	.640
	Totals	.040	.960	1.000

1) Find the probability of an accident, rain or no rain.

2) Find the probability of rain, accident or no accident.

3) Find the probability of an accident and rain.

4) Find the probability of an accident, given rain.

Intersection of Events: Product Rule

We want to find a way to calculate $P(A \cap B)$ if it's not given to us or obvious.

Since we already have $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ we could turn this around to be:

$$P(A \cap B) =$$

Since $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)}$ as well, we can write $P(A \cap B) =$

Example
If 49.3% of the US population is male and 35% of males smoke cigarettes, what is the probability that a person selected at random is a male who smokes?

Probability Trees

Example

We have a box that contains 3 blue and 2 white balls. We are to draw 2 balls in succession from the box without replacement. What is the probability of drawing a white on the second ball?

Example

A large computer company *A* subcontracts the manufacture of its circuit boards to two companies, 40% to company *B* and 60% to company *C*. Company *B* in turn subcontracts 70% of the orders it receives from company *A* to company *D* and the remaining 30% to company *E*. When the boards are completed by companies *D*, *E*, and *C*, they are shipped to company *A*. It has been found that 1.5%, 1%, and .5% of the boards from *D*, *E*, and *C* respectively prove defective during the 90 day warranty period. What is the probability that a given board in a computer sold by a company *A* will be defective during the 90-day period?

Independent Events

Let's go back to our example of drawing balls from a box. Remember we have 3 blue and 2 white.

Let: A = white ball on second draw
B = white ball on first draw

Compute $P(A|B)$ and $P(A)$.

Now, let's run the same experiment, but this time let's replace the ball after the first draw. Now, compute $P(A)$ and $P(A|B)$ where A and B are the same as before.

We see that $P(A|B)=P(A)$ in this case and therefore let's rewrite the formula for $P(A\cap B)$ in this case:

$$P(A\cap B)=$$

If A and B are events in our sample space S then we say that ***A* and *B* are independent** if and only if

$$P(A\cap B)=P(A)P(B)$$

Otherwise A and B are said to be **dependent**.

Intuitive definition of independence

Example
In two tosses of a single fair coin show that the events "heads on the first toss" and "heads on the second toss" are independent.

Example
A single card is drawn from a standard deck of 52 cards. Test the following events for independence:

E = the drawn card is a spade
F = the drawn card is a face card

G = the drawn card is a club
H = the drawn card is a heart

Some students get the ideas of independence and mutual exclusivity mixed up. So be careful!!!

Independent Set of Events

In reality we often use out intuition to tell us about independence so that we can then apply the product formulas.

Example

A single die is rolled 6 times. What is the probability of getting the sequence 1, 2, 3, 4, 5, 6?