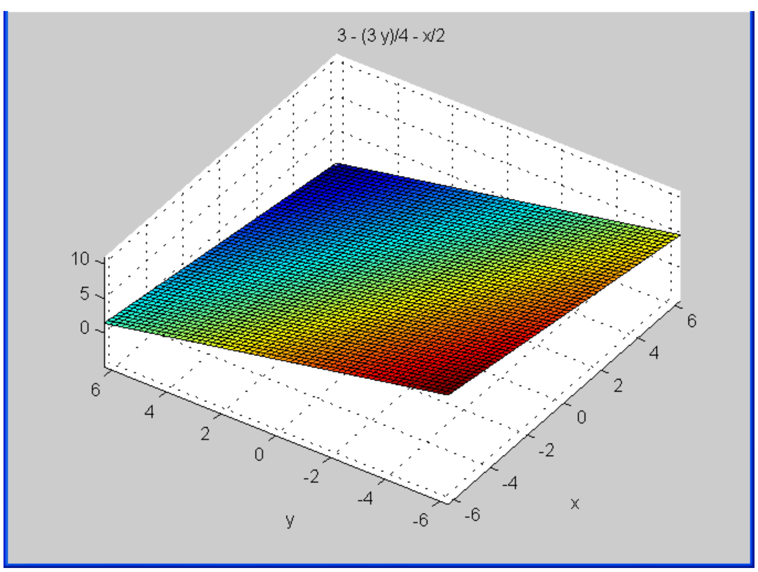
MA 280-Assignment #2: VECTORS AND THE GEOMETRY OF SPACE- Part II

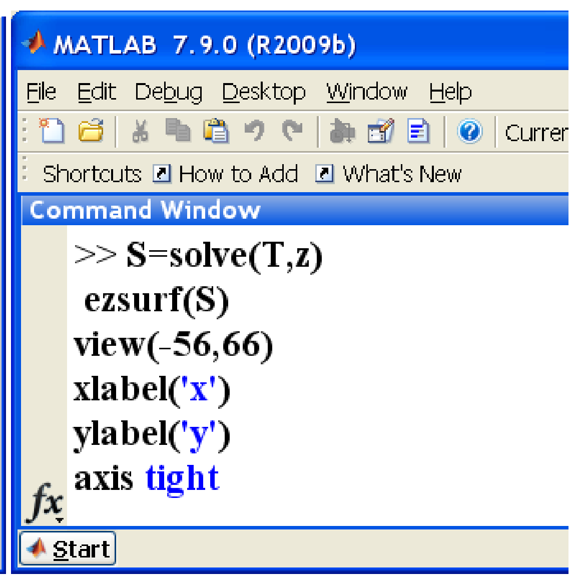
Example 1

We can use the dot product command to write an equation of the plane through the point (2, 4, -1) with normal vector $n = < 2,3,4 >$:

Thus the equation of the desired plane is $T = 0$ or $2x + 3y + 4z - 12 = 0$.

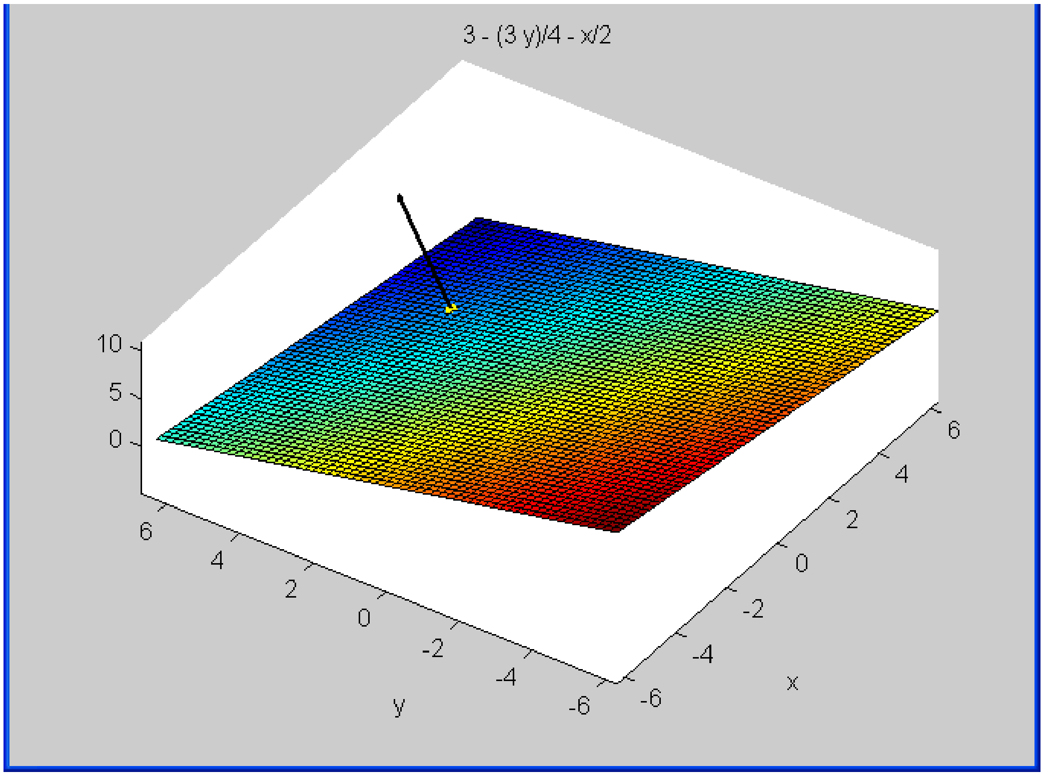
If you wish to visualize the plane, then solve for z in terms of x and y then use the MATLAB command **ezsurf** to plot the plane $z = f(x, y)$:





Note that MATLAB allows you to rotate the figure and select a suitable view. The above figure was captured with the view(-56, 66).

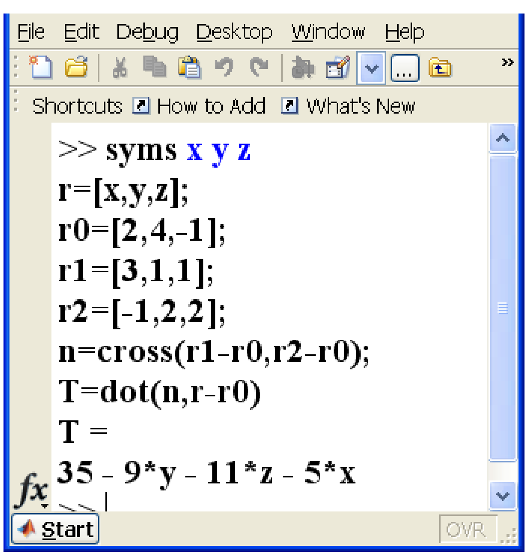
Remark : while it is possible to draw the plane $2x + 3y + 4z - 12 = 0$ together with the normal vector $n = < 2{,}3{,}4 >$ at the point (2, 4, -1), we prefer to postpone the details until later on in the tutorial.

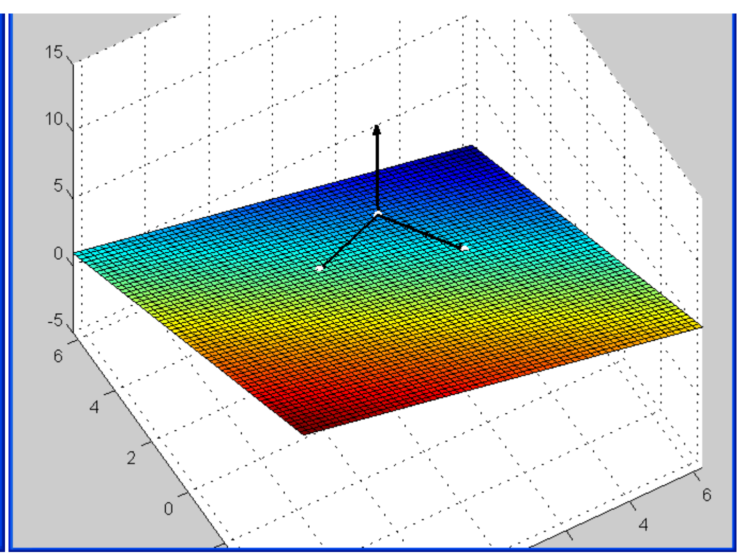


Example 2

Find an equation of the plane through the points (2, 4,-1), (3, 1, 1), and (-1, 2, 2).

Let $r_0 = (2{,}4,-1), r_1 = (3{,}1{,}1), and\ r_2 = (-1, 2{,}2)$. We use the cross product to calculate a normal vector to the plane $\vec{n} = \overrightarrow{r_1 r_0} \times \overrightarrow{r_2 r_0}$.



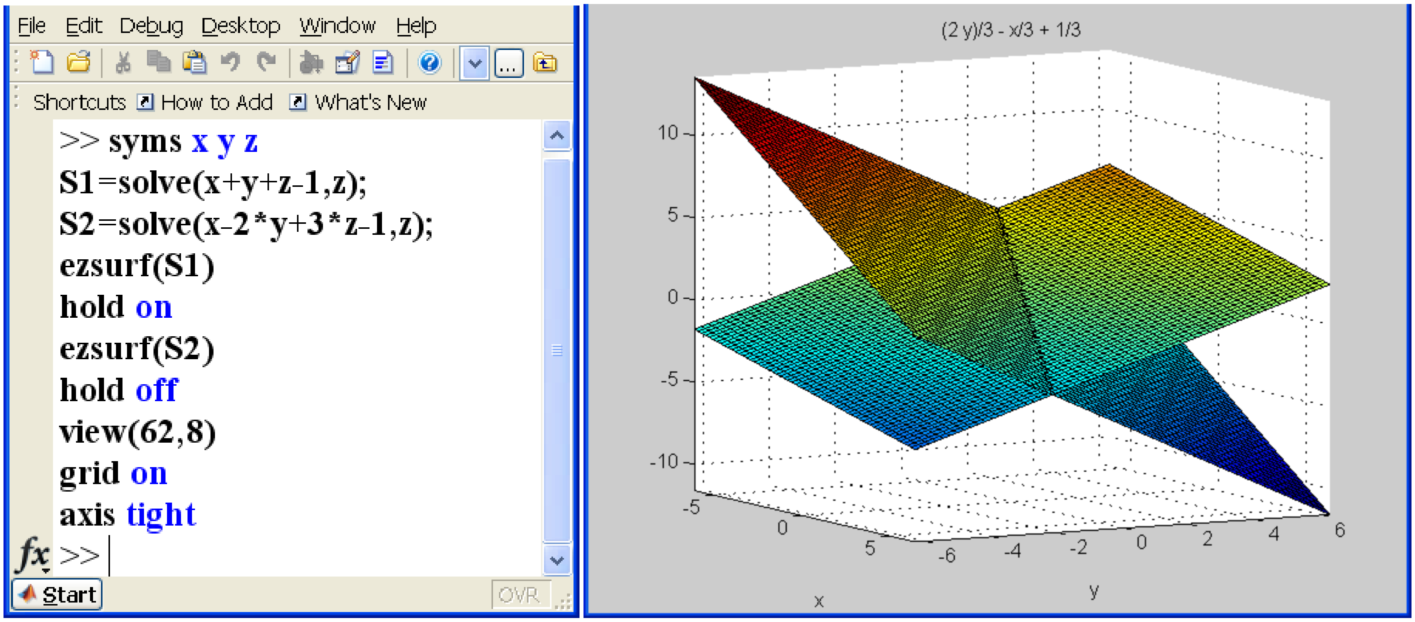


Therefore the equation of the plane is $-5x - 9y - 11z + 35 = 0$.

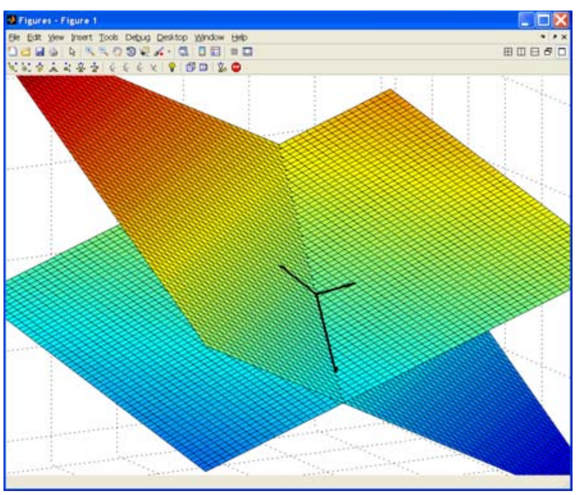
Example 3

Consider the planes $x + y + z = 1$ and $x - 2y + 3z = 1$. Clearly the two planes are not parallel hence they intersect in a line L.

We can solve the above equations for z and use the command **ezsurf** to plot the two planes.



Let $n_1 = < 1,1,1 >$ *and* $n_2 = < 1,-2,3 >$ be the normal vectors to the two planes. One way to determine the line of intersection L is to observe that the cross product $n_1 \times n_2$ is parallel to L. We can verify this fact by drawing $n_1, n_2,$ *and* $n_1 \times n_2$ at the point (1, 0, 0).

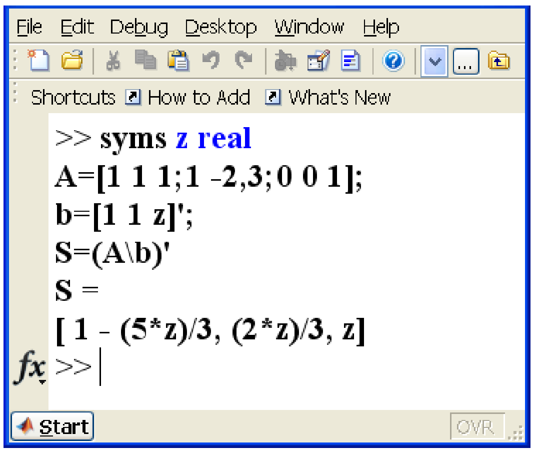


Alternatively, we find two points on L then use them to write an equation of the line of intersection.

Any point in the intersection of the two planes must satisfy the system: $\begin{cases} x+y+z=1 \\ x-2y+3z=1 \end{cases}$ which, as expected, has an infinite number of solutions. Terefore for each pre-assigned value of z we obtain a point in the line L.

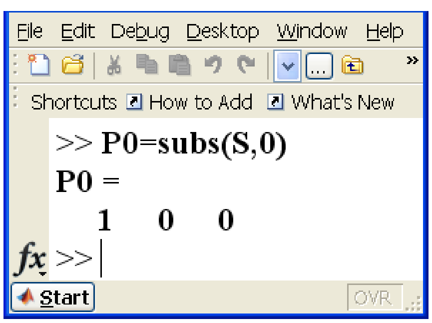
We rewrite the system in the following form: $\begin{cases} x+y+z=1 \\ x-2y+3z=1. \\ z=z \end{cases}$

Put $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 1 \\ z \end{bmatrix}$. We can use MATLAB to solve the system:

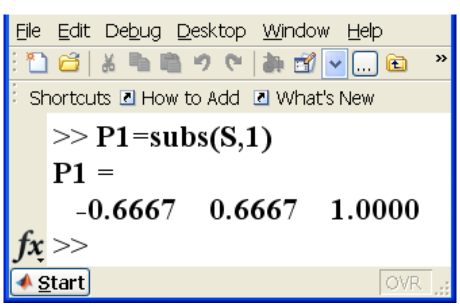


Note that the solution S is given in terms of z; that is for each choice of z we obtain a point in the line of intersection L.

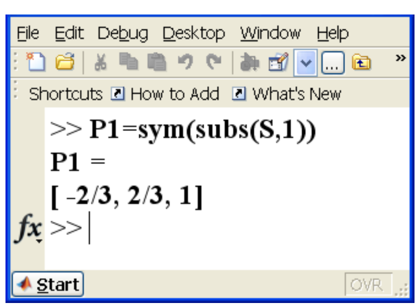
If we let $z = 0$, we obtain a point $P_0 = (1,0,0)$:



and if we let $z = 1$, we obtain a point $P_1 = (-\frac{2}{3}, \frac{2}{3}, 1)$:



or

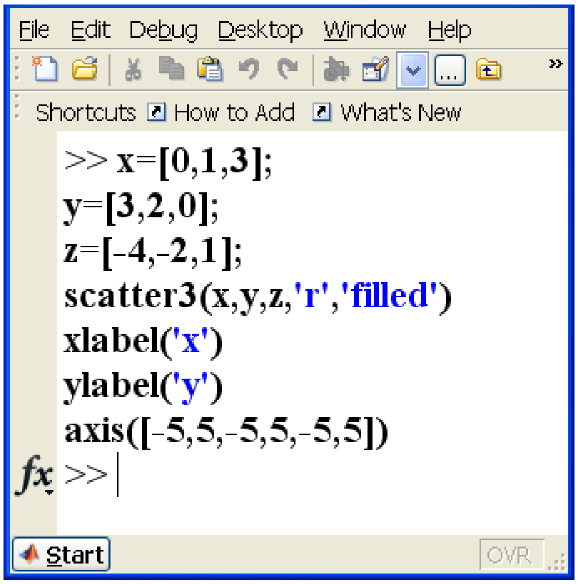


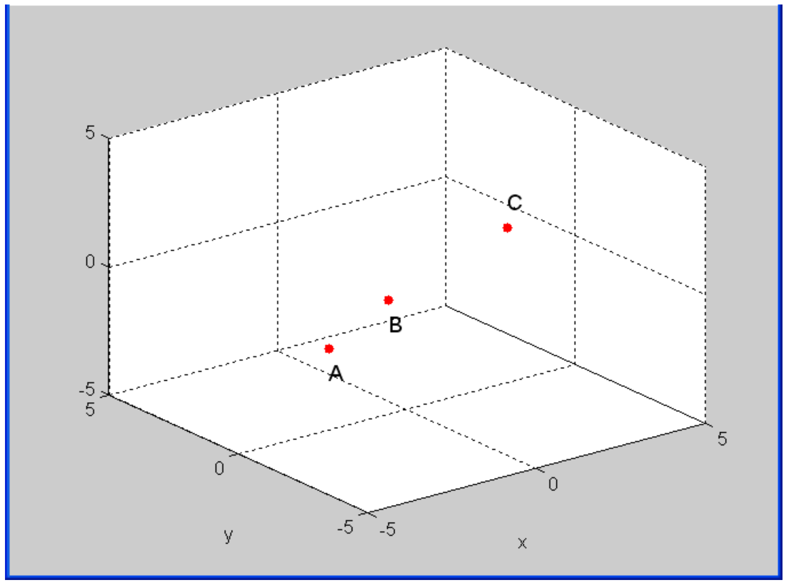
Having two points, we can easily write the equation of the lineof intersection L. We leave the details a an exercise.

Example 4

Occasionally we are able to determine from a graph whether two or more points lie on the same line.

Consider the points $A(0,3,-4), B(,1,2,-2), and\ C(3,0,1)$. We can use the MATLAB command **scatter** to plot the points and determine whether they lie on the same line or not.





Exercise MA 280-2 (Solution at the end of document)

Use MATLAB to

a. Plot the planes $x + y - z = 2$ and $2x - y + 3z = 1$ on the same figure
b. Find two points on the line of intersection of the planes in part(a)
c. Plot the point (-1, 2,1) together with the two points obtained in part (b). Are they on the same line?
d. Find two vectors in the plane that passes through the point (-1, 2, 1) and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.
e. Find an equation of the plane that passes through the point (-1, 2, 1) and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.