11.4 Tangent Planes and Linear Approximations

Tangent planes

Take a surface S is given by $z = f(x, y)$ which has continuous partial derivatives. At a point $P(x_0, y_0, z_0)$ on S:

Formula for the Tangent Plane

Example 1

Find the tangent plane to the surface $z = x \ln y$ at the point $(4, 1, 0)$.

Linear Approximations

Use the tangent plane from example 1 to approximate the function $z = x \ln y$ near $(4,1,0)$.

Linearization Formula

Example 2

Consider the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Find f_x and f_y and look at their behavior at the origin.

Differentiable functions

Theorem

Example 3

Show that $f(x, y) = x \sin xy$ is differentiable at $(1,0)$ and find its linearization there. Then use it to approximate $f(1.1, 0.1)$.

Example 4

Consider the heat index chart

Relative humidity (%)										
		50	55	65	70	75	80	85	90	95
Actual Temperature (°F)	90	96	98	100	103	106	109	112	115	119
	92	100	103	105	108	112	115	119	123	128
	94	104	107	111	114	118	122	127	132	137
	96	109	113	116	121	125	130	135	141	146
	98	114	118	123	127	133	138	144	150	157
	100	119	124	129	135	141	147	154	161	168

Find the linear approximation at $(96, 70)$ and use it to approximate the heat index at $(97, 72)$.

Differentials

Example 5

The volume of a cylinder is given by $V = \pi r^2 h.$ Compare ΔV and dV for $r = 3 \pm 0.1\ cm$ and $h = 5\ \pm 0.1\ cm.$

Functions of 3 or more variables

Example 6

A box is found to have measurements 40 cm, 35 cm, and 60 cm respectively. If each measurement is correct to within 0.1 cm, use differentials to find the largest possible error

Tangent Planes to Parametric Surfaces

Example 7

Find the tangent plane to the surface with parametric equations $x = u^2.\ y = v^2, z = u + 2v$ at the point $(1,1,3)$.

11.4 # 1, 3, 9, 11, 15, 19, 23, 29, 33