

11.5 The Chain Rule

Chain Rule in 1 Variable

The Chain Rule (Case 1)

Example 1

If $z = 4x^2y - 5xy$ with $x = \cos t$ and $y = \sin t$, find dz/dt .

Example 2

The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. If the radius is changing at 0.1 m/s and the height is changing at 0.2 m/s, find the rate at which the volume is changing when $r = 10$ m and $h = 5$ m.

Chain Rule Case 2

Example 3

If $z = x^2y$ with $x = \alpha \sin \beta$ and $y = \alpha \cos \beta$. Find $\partial z / \partial \alpha$ and $\partial z / \partial \beta$.

Chain Rule – Generalized Version

Example 4

If $w = f(x, y, z, t)$ and each of x , y , z , and t are functions of u and v . Find an expression for $\partial w / \partial u$.

Example 5

If $g(s, t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable, show that g satisfies the equation

$$t \frac{\partial g}{\partial s} + s \frac{\partial g}{\partial t} = 0$$

Example 6

If $z = f(x, y)$ has continuous second order-partial derivatives and $x = r^2 + s^2$ and $y = 2rs$, find $\partial z / \partial r$ and $\partial^2 z / \partial r^2$.

Implicit Differentiation

Example 7

Find y' if $3x^2 - 6y^3 = 6x^2y$.

HW # 1, 3, 5, 7, 9, 13, 17, 23, 29, 33, 37