11.6 Directional Derivatives and the Gradient Vector

Directional Derivatives

Suppose we have a surface S given by $z = f(x, y)$. We want to take the derivative at (x_0, y_0) in the direction of an arbitrary vector $\vec{u}$.

Theorem

Example 1

Find the directional derivative $D_{\vec{u}}f(x, y)$ if $f(x, y) = x^2 - 4xy - y^3$ if $\vec{u} = \langle {}^1/_2, {}^{\sqrt{3}}/_2 \rangle$. Find $D_{\vec{u}}f(2,1)$.

Gradient Vector

Example 2

If $f(x, y) = x^2 + 5xy - y^3$ then find $\nabla f(x, y)$.

Directional Derivative

Example 3

If $f(x, y) = x^2 + 5xy - y^3$, find the directional derivative at $(1,2)$ in the direction of $\langle 3, -5 \rangle$.

Functions of three or more variables

Example 4

If $f(x, y, z) = x\ln(yz)$, then find the gradient of f and the directional derivative at $(2,1,1)$ in the direction of $\vec{v} = 3\vec{i} - 2\vec{j} + \vec{k}$.

Maximizing the Directional Derivative

Theorem

Example 5

If $f(x, y) = x\,\mathrm{e}^{\mathrm{y}}$, find the rate of change at the point $P(2,0)$ in the direction from P to $Q(3,1)$. In what direction does f have maximum rate of change? What is this rate of change?

Example 6

The temperature at a point (x, y, z) is given by

$$T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$$

Where T is measured in °C and x, y, z are measured in meters. Find the gradient at the point $(2, -1,2)$ and the maximum rate of increase.

Tangent Planes to Level Surfaces

Normal line to S at P.

Example 7

Find the equations of the tangent plane and normal line at the point $(-2,1,-3)$ to the ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$$

HW # 1, 5, 7, 11, 15, 19, 21, 25, 27, 33, 35, 51