12.7 Triple Integrals

Triple Integrals

Fubini's Theorem

Example 1

Evaluate $\iiint_B xy^2z \ dV$ over the box $B = \{(x, y, z) | 0 \leq x \leq 1, -1 \leq y \leq 1, 0 \leq z \leq 2\}$.

Triple Integrals over general regions

Example 2

Evaluate $\iiint_E xz\ dV$ where E is the solid tetrahedron bounded by the four planes $x = y = z = 0$ and $x + y + z = 2$.

Example 3

Evaluate $\iiint_E \sqrt{x^2 + z^2}\, dV$ where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.

Applications

Example 4

Find the volume of the solid bounded by the cylinder $y = x^2$ and the planes $z = 0$, $z = 4$, and $y = 9$.

Example 5

Sketch the solid whose volume is given by the integral

$$\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\, dz\, dy$$

Example 6

Express the integral $\iiint_E f(x,y,z)dV$ in six different ways where E is the solid bounded by the surface $z = 0, x = 0, y = 2. z = y - 2x.$

Density, Centers of Mass, Moments of Inertia

Example 6

Find the center of mass of a solid of constant density that is bounded by the parabolic cylinder $x = y^2$ and the planes $x = z$, $z = 0$, $z = 1$.

HW # 3, 5, 7, 11, 17, 19, 21, 25, 27, 31, 33, 37