

13.2 – Line Integrals

Integration along a smooth parametric plane curve

Example 1

Evaluate $\int_C (2 - xy) ds$ where C is the upper half of the unit circle $x^2 + y^2 = 1$.

Union of Piecewise Smooth Curves

Example 2

Evaluate $\int_C x + y \, ds$ where C consists of the arc $y = x^2$ from $(0,0)$ to $(1,1)$ followed by the vertical line segment from $(1,1)$ to $(1,2)$.

Other Line Integrals

Example 3

Evaluate $\int_C y^2 dx + x dy$ where

- a) C is the line segment from $(-5,3)$ to $(0,2)$.
- b) C is the arc of the parabola $x = 4 - y^2$ from $(-5,3)$ to $(0,2)$.

Orientation of the curve

Line Integrals in Space

Example 4

Evaluate $\int_C y \sin z \, ds$ where C is the circular helix given by the equations $x = \cos t$, $y = \sin t$, $z = t$, $0 \leq t \leq 2\pi$.

Example 5

Evaluate $\int_C z \, dx + x \, dy + y \, dz$ where C is given by $x = t^2$, $y = t^3$, $z = t^2$ and $0 \leq t \leq 1$.

Line Integrals of Vector Fields

Example 6

Find the work done by the force field $\vec{F}(x, y) = x^2 \vec{i} - xy \vec{j}$ in moving a particle along the quarter-circle $\vec{r}(t) = \cos t \vec{i} + \sin t \vec{j}$, $0 \leq t \leq \frac{\pi}{2}$.

HW # 1, 3, 7, 11, 15, 17, 25, 33, 37