

13.3 – The Fundamental Theorem of Line Integrals

Theorem

Proof:

Example 1

Find the work done by the gravitational field

$$\vec{F}(x) = -\frac{mMG}{|\vec{x}|^3} \vec{x}$$

In moving a particle with mass m from the point $(2,1,5)$ to the point $(5,2,1)$ along a piecewise smooth path.

Path Independence

Theorem

Theorem – Conservative Vector Fields

Proof:

Theorem – Properties of Conservative Vector Fields

Converse?

Open and Simply Connected Regions

Theorem

Example 2

Show the vector field

$$\vec{F}(x, y) = (4x^2 + 2xy)\vec{i} + (x^2 - 3y^2)\vec{j}$$

is conservative. And find f such that $\nabla f = \vec{F}$.

Example 3

If $\vec{F}(x, y, z) = y^2\vec{i} + (2xy + e^{3z})\vec{j} + 3ye^{3z}\vec{k}$, find a function f such that $\vec{F} = \nabla f$,

Conservation of Energy

HW # 3, 5, 7, 11, 13, 15, 19, 33