

13.4 – Green's Theorem

Green's Theorem

Example 1

Find $\oint_C y^2 dx - x dy$ where C is the rectangle with vertices $(0,0)$, $(2,0)$, $(2,4)$, $(0,4)$.

Example 2

Find $\oint (3\sqrt{x^2 - 5} + 5y)dx + (2x - e^{-y^2})dy$ where C is the circle $x^2 + y^2 = 4$.

Area Computations

Example 3

Find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Green's Theorem for the finite union of simple regions

Example 4

Evaluate $\oint_C y^2 dx + 3xy dy$ where C is the boundary of the semiannular region D in the upper half-plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Regions that are not simply connected

Example 5

Show that if $\vec{F}(x, y) = -\frac{y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j}$ then $\oint_C \vec{F} \cdot d\vec{r} = 2\pi$ for every simple closed path that encloses the origin.

Proof of Theorem from 13.3 about conservative vector fields.