

13.6 – Surface Integrals

From Line Integrals to Surface Integrals

Example 1

Compute $\iint_S x^2 dS$ where S is the unit sphere $x^2 + y^2 + z^2 = 1$.

Mass, Centers of Mass, Moments of Inertia

Graphs

A special look at surfaces of the form $z = f(x, y)$

Example 2

Evaluate $\iint_S xy \, dS$ where S is the triangular region with vertices $(1,0,0)$, $(0,2,0)$, $(0,0,2)$.

Example 3

Set up the surface integrals to evaluate $\iint_S z \, dS$ where S is the surface whose sides S_1 are given by the cylinder $x^2 + y^2 = 1$, whose bottom S_2 is the disk $x^2 + y^2 \leq 1$ in the plane $z = 0$, and whose top S_3 is the part of the plane $z = 1 + x$ that lies above S_1 .

Oriented surfaces

Orientation of a graph

Orientation of a general surface

Orientation of Sphere of radius a

Closed Surfaces

Surface Integrals of Vector Fields

Example 4

Find the flux of $\vec{F} = \langle z, y, x \rangle$ across the unit sphere $x^2 + y^2 + z^2 = 1$.

Graphs

Electric Flux

Heat Flow

Example 5

The temperature in a metal ball is proportional to the square of the distance from the center of the ball. Find the rate of the heat flow across a sphere S of radius a with center at the center of the ball.