13.8 – Divergence Theorem

Divergence Theorem

Let E be a simple solid region and let S be the boundary surface of E, given with positive (outward) orientation. Let $\vec{F}$ be a vector field whose components have continuous partial derivatives on an open region in $\mathbb{R}^3$ that contains E. Then

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} \vec{F} \, dV$$

In other words:

Under certain conditions, the flux of $\vec{F}$ across the boundary surface of E is the same as the triple integral of the divergence of $\vec{F}$ over E.

Example 1

Find the flux of the vector field $\vec{F} = \langle x, y, z \rangle$ over the unit sphere $x^2 + y^2 + z^2 = 1$.

Example 2

Evaluate $\iint \vec{F} \cdot d\vec{S}$ where $\vec{F} = \langle x^2 + yz^3, 2xy + e^{-xz}, \sin(xy) \rangle$ over the unit cube with one corner at the origin and the opposite corner at $(1,1,1)$.

Example 3

Point charge Q at the origin and a surface that encompasses the origin.

HW # 1, 5, 7, 11, 13