

## 9.5 Equations of Lines and Planes

### Equations of lines

#### Example 1

Find a vector equation for the line that passes through the point  $(2, -1, 5)$  parallel to the vector  $\vec{i} - 2\vec{j} + \vec{k}$ , and find two other points on the line.

### Symmetric equation form of a line

Example 2

Find parametric equations and symmetric equations of the line that passes through the points  $(-1, 5, 2)$  and  $(2, -1, 5)$ . At what point does this line intersect the  $yz$ -plane.

**General form of the line segment connecting  $r_0$  and  $r_1$ .**

Example 3

Show that the lines  $L_1$  and  $L_2$  with parametric equations

$$\begin{aligned}x &= 2 - t & y &= 3 + 2t & z &= 2t - 1 \\x &= 5 + s & y &= 3s + 1 & z &= 2s - 5\end{aligned}$$

are skew lines.

## Equation of a plane

### Example 4

Find the equation of the plane that goes through the point  $(3, -5, 2)$  with normal vector  $\vec{n} = \langle 2, 1, 3 \rangle$ . Find the intercepts and sketch the plane.

Example 5

Find the equation of the plane that passes through the points  $(0,1,1)$ ,  $(1,0,1)$ , and  $(1,1,0)$ .

Example 6

Find the angle between the planes  $x + 2y - z = 1$  and  $3x - y + z = 3$  and find the symmetric equations of the line of intersection.

Find the formula for the shortest distance  $D$  from a point  $P_1(x_1, y_2, z_1)$  to the plane  $ax + by + cz + d = 0$ .

**Example 7**

Find the distance between the parallel planes  $x + 2y - z = 5$  and  $2x + 4y - 2z = 7$ .

Example 8

In a previous example we showed that the lines  $L_1$  and  $L_2$  with parametric equations

$$x = 2 - t \quad y = 3 + 2t \quad z = 2t - 1$$

$$x = 5 + s \quad y = 3s + 1 \quad z = 2s - 5$$

are skew lines.

Find the distance between these lines.