10.3 - Arclength and Curvature

Arclength of Parameterized functions from Chapter 6

Arclength of a space curve defined by $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

Example 1

Find the length of the circular helix $\vec{r}(t) = \cos t\, \vec{\imath} + \sin t\, \vec{\jmath} + t\, \vec{k}$ from the point (1,0,0) to the point $(1,0,2\pi)$.

Independence of Parameterization

Arc Length Function

Parameterize with respect to Arc Length

Example 2

Parameterize the circular helix $\vec{r}(t) = \cos t\,\vec{\imath} + \sin t\,\vec{\jmath} + t\,\vec{k}$ with respect to arc length measured from the point $(1,0,0)$ in the direction of increasing t.

Curvature

Example 3

Show that the curvature of a circle of radius a is $1/a$

Theorem

The curvature of the curve given by the vector funtion $\vec{r}(t)$ is:

$$\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}$$

Example 4

Find the curvature of the twisted cubic $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ at a general point and at $(0,0,0)$.

Curvature of plane curves

Example 5

Find the curvature of the parabola $y = x^2$.

The Normal and Binormal Vectors

Example 6

Find the unit normal and binormal vectors for the circular helix $\vec{r}(t) = \cos t\,\vec{\imath} + \sin t\,\vec{\jmath} + t\,\vec{k}$.

Normal Plane and Osculating Plane

Example 7

Find the equations of the normal and osculating planes of the helix at the point $\left(0,1,\frac{\pi}{2}\right)$.

Example 8

Find and graph the osculating circle of the parabola $y = x^2$ at the origin.

HW 10.3 # 1, 3, 7, 9, 11, 15, 17, 21, 25, 29