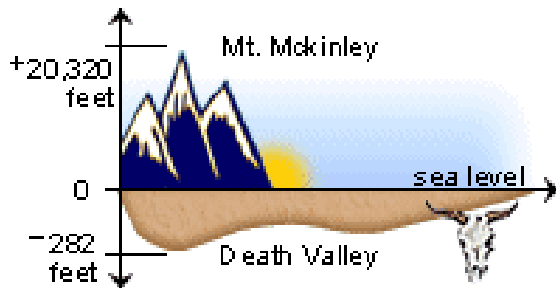


Introduction to Integers (2.1)

Integers are the set of whole numbers and their opposites.

$$\{ \dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots \}$$

Example: The highest elevation in North America is Mt. McKinley, which is 20,320 feet above sea level. The lowest elevation is Death Valley, which is 282 feet below sea level.



We can represent each elevation as an integer:

Elevation	Integer
20,320 feet above sea level	20,320
sea level	0
282 feet below sea level	-282

Write an integer for each of the following:

7° F below zero _____

Owing \$75 _____

150 meters above sea level _____

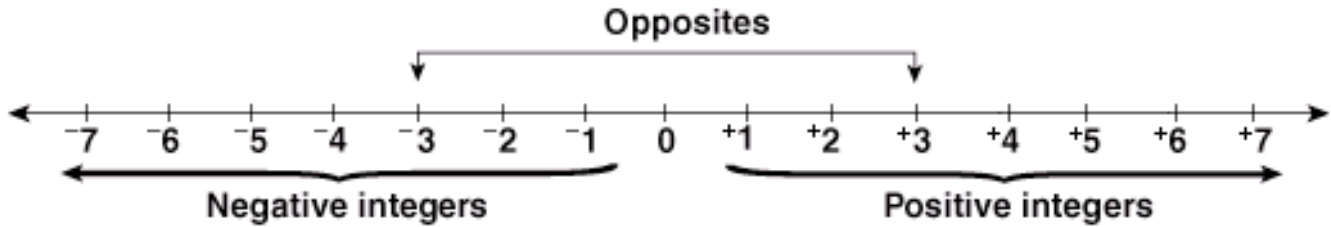
5 shots under par _____

sea level _____

loss of 10 pounds _____

net loss of \$1.45 thousand _____

The US national debt is approximately \$10.6 trillion _____



- The number line goes on forever in both directions. This is indicated by the arrows.
- Whole numbers greater than zero are called **positive integers**. These numbers are to the right of zero on the number line.
- Whole numbers less than zero are called **negative integers**. These numbers are to the left of zero on the number line.
- The integer zero is neutral. It is neither positive nor negative.
- The sign of an integer is either positive (+) or negative (-), except zero, which has no sign.
- Two integers are opposites if they are each the same distance away from zero, but on opposite sides of the number line. One will have a positive sign, the other a negative sign. In the number line above, +3 and -3 are labeled as opposites.
- We compare integers just as we compare whole numbers. For any two numbers graphed on a number line, the number to the right is the greater number and the number to the left is the smaller number.



The **absolute value** of a number is the number's *distance* from 0 on the number line. The symbol for absolute value is _____. Since a distance is always zero or positive, the absolute value of a number is always _____.

① **Simplify**

- a. $|-8|$ b. (-24) c. $-|-5|$ d. $-(-3)$

② **Insert $<$, $>$, or $=$ between each pair of numbers to make a true statement.**

- a. $|-4|$ 4
b. $|-20|$ $-(-20)$
c. $|-4|$ $|-7|$
d. -4 -7
e. $|-8|$ $-|8|$

③ **Write in order from least to greatest.**

- a. 0, 4, -5, -25, -1 b. 0, -4, $|-5|$, $-|25|$, -1

_____, _____, _____, _____, _____

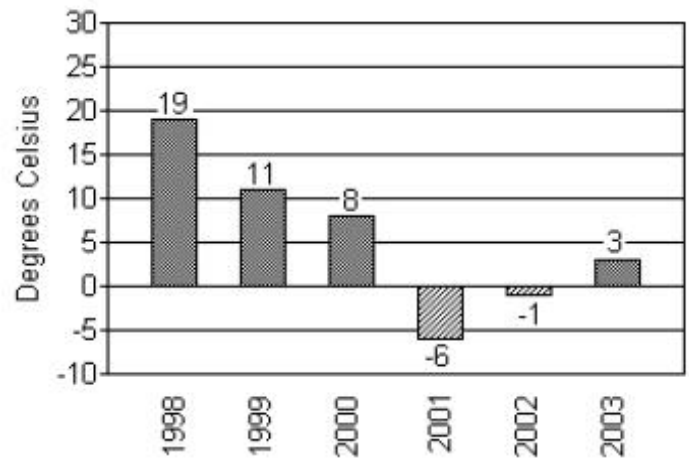
_____, _____, _____, _____, _____

④ **Which is warmer: -3° F or 0° F?**

⑤ **Which is a greater elevation -20 feet or -100 feet?**

⑥ The bar graph below shows the temperatures recorded as the high temperature in Little City on Brianna's birthday for the indicated years.

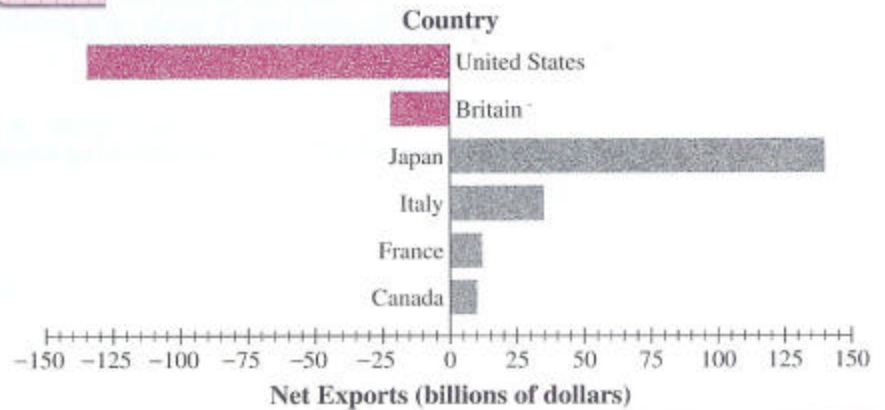
- In which year was the temperature closest to 0°C ?
- In which year was the recorded temperature the highest?
- In which year was the temperature closest to 5°C ?



⑦ The number line in the following graphic represents net exports in billions of dollars for six countries. Net exports are obtained by subtracting total imports from total exports; a negative export means the country imported more goods than it exported.



The World of Exports



- Estimate the net amount of exports for Japan. **Explain** what this number tells you about the imports and exports of Japan.
- Estimate the net amount of exports for the United States. **Explain** what this number tells you about the imports and exports of US.

Adding Integers (2.2)

Adding Integers with the Same Sign

Problem: Jamal owes his friend Katrina \$3. If he borrows another \$6, how much will he owe her altogether?



Solution: This problem is quite simple: just add \$3 and \$6 and the result is \$9.

The problem above can be solved using addition of integers. Owing \$3 can be represented by -3 and owing \$6 can be represented by -6 . The problem becomes:

$$-3 + -6 = -9$$

Look at the number line below. If we start at 0, and move 3 to the left, we land on -3 . If we then move another 6 to the left, we end up at -9 .



Rule: The sum of two negative integers is a negative integer.

(We already know the sum of two positive integers is a positive integer)

To Add Two Numbers With the Same Sign.....

- **Step 1.** Find the sum of their absolute values.
- **Step 2.** Use their common sign as the sign of the sum.

Add

① $-2 + (-3) =$

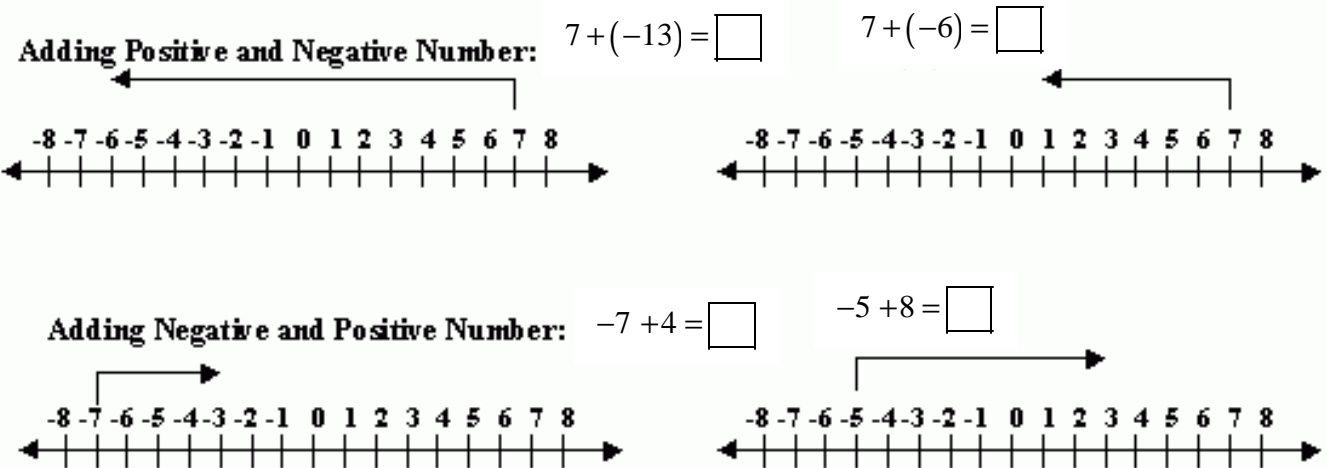
② $2 + 3 =$

③ $-20 + (-60) + (-5) =$

④ Evaluate $a + b$ for $a = -6$ and $b = -12$

- ⑤ A deep-sea diver dives from the surface to 128 feet below the surface. She then dives down 20 more feet. Find the diver's depth.

Adding Integers with Different Signs



To Add Two Numbers with Different Signs

1. Find the [absolute value](#) of each integer.
2. Subtract the smaller number from the larger number you get in Step 1.
3. The result from Step 2 takes the sign of the integer with the greater absolute value.

Add

① $-8 + 6 =$

② $8 + (-6) =$

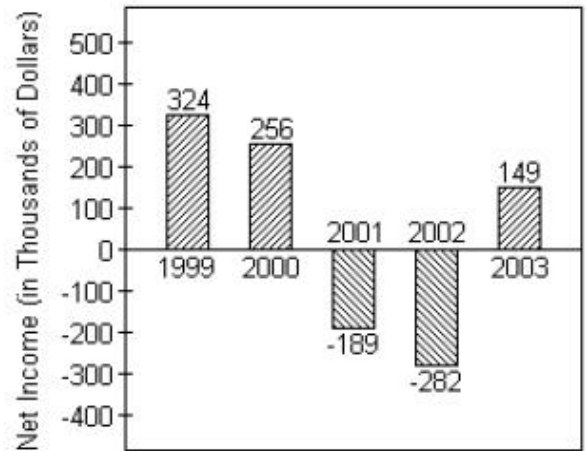
③ $20 + (-50) =$

④ $-100 + 30 + (-20) + 10$

Remember we can add in any order we wish

- ⑤ In four rounds of a card game, you get scores of 1, -5, 10, and -4. What is your final score?

- ① The bar graph shows the yearly net income for Widgets, LTD.
- What was the net income (in dollars) for Widgets, LTD in 1999?
 - Find the total net income for years 2002 and 2003.
 - Find the total net income for all years shown.



- ② On part of a scenic tour of underground caves, Dave and Neil started at an elevation of -53 feet. They then rose 15 feet. What was their elevation at this point?

Subtracting Integers (2.3)

- ① a) $3 - 2 =$ b) $3 + (-2) =$ c) $(-2) + 3 =$
② a) $6 - 4 =$ b) $6 + (-4) =$ c) $(-4) + 6 =$
③ a) $10 - 6 =$ b) $10 + (-6) =$ c) $(-6) + 10 =$

Addition is
commutative
 $a+b = b+a$

Let a and b be any real number. Then $a - b$ can be rewritten as _____

$$a - b = \underline{\hspace{2cm}}$$

To subtract integers, we rewrite the subtraction problem as an _____ problem.

Subtraction can always
 be rewritten as addition

Remember..... $a - b = a + (-b)$

LCC (Leave-Change-Change)

- ① Suppose you use your checking account to manage all your expenses. You keep very accurate records. There are times when you must write a check for an amount that is greater than your balance. When you record this check, your new balance is represented by a negative number.

CHECK NUMBER	DATE	DESCRIPTION	SUBTRACTIONS (-)		ADDITIONS (+)		BALANCE	
					120		64	
381	3/2/03	Daycheck						
	3/4/03	Car repairs	167					
382	3/8/03	Phone	25					
383	3/15/03	Clothes	22					

What is the current balance?

② $-3 - 6 = \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$

③ $4 - 8 = \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$

④ $7 - 5 = \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$

⑤ $20 - 35 = \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$

⑥ $40 - (-5) = \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$

⑦ $33 - (-7) = \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$

⑧ $-15 - (-20) = \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$

⑨ $-10 - (-5) = \underline{\hspace{2cm}} = \underline{\hspace{1cm}}$

⑩

The temperature on a December morning is -5°F at 5 a.m. If the temperature drops 3° by 6 a.m., rises 7° by 7 a.m., and then drops 2° by 8 a.m., find the temperature by 8 a.m.

Multiplying and Dividing Integers (2.4)

Multiplying Integers:

Case 1: The product of two integers with opposite signs.

Complete the following patterns of products.

What happens to the first factor each time?

$$5 \cdot 2 = \underline{\quad}$$

$$4 \cdot 2 = \underline{\quad}$$

$$3 \cdot 2 = \underline{\quad}$$

$$2 \cdot 2 = \underline{\quad}$$

$$1 \cdot 2 = \underline{\quad}$$

$$0 \cdot 2 = \underline{\quad}$$

$$-1 \cdot 2 = \underline{\quad}$$

$$-2 \cdot 2 = \underline{\quad}$$

$$-3 \cdot 2 = \underline{\quad}$$

$$\underline{\quad} \cdot 2 = \underline{\quad}$$

$$\underline{\quad} \cdot 2 = \underline{\quad}$$

$$\underline{\quad} \cdot 2 = \underline{\quad}$$

$$\underline{\quad} \cdot 2 = \underline{\quad}$$

What happens to the product each time?

The product of two numbers having different (opposite) signs is a _____ number.

Case 2: The product of two integers with the same signs.

What happens to the first factor each time?

$$5 \cdot (-3) = \underline{\quad}$$

$$4 \cdot (-3) = \underline{\quad}$$

$$3 \cdot (-3) = \underline{\quad}$$

$$2 \cdot (-3) = \underline{\quad}$$

$$1 \cdot (-3) = \underline{\quad}$$

$$0 \cdot 2 = \underline{\quad}$$

$$-1 \cdot (-3) = \underline{\quad}$$

$$-2 \cdot (-3) = \underline{\quad}$$

$$-3 \cdot (-3) = \underline{\quad}$$

$$\underline{\quad} \cdot (-3) = \underline{\quad}$$

$$\underline{\quad} \cdot (\quad) = \underline{\quad}$$

$$\underline{\quad} \cdot (\quad) = \underline{\quad}$$

$$\underline{\quad} \cdot (\quad) = \underline{\quad}$$

What happens to the product each time?

The product of two numbers having the same sign is a _____ number.

Dividing integers is related to multiplication of integers. The sign rules for division can be discovered by writing a related multiplication statement.

$$\frac{6}{2} = 3 \quad \text{because } 3 \cdot 2 = 6$$

$$\frac{-6}{2} = -3 \quad \text{because } -3 \cdot 2 = -6$$

$$\frac{8}{-2} = \quad \text{because } \quad \cdot -2 = 8$$

$$\frac{-12}{\quad} = -4 \quad \text{because } \quad \cdot (-4) = -12$$

Dividing Integers:

The quotient of two integers having the same sign is _____.

The quotient of two integers having opposite (different) signs is _____.

① Evaluate the following:

$$x^2 \text{ for } x=3 \text{ and } x=-3$$

$$-x^2 \text{ for } x=3 \text{ and } x=-3$$

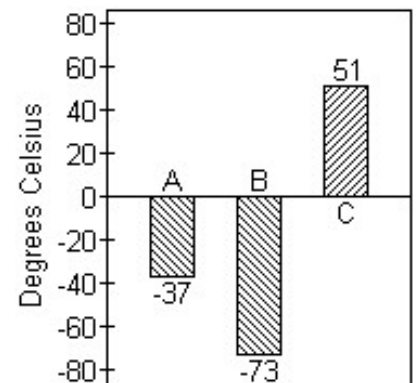
② Multiply:

$$-4(-2)(6)$$

$$-2(-3)(-7)$$

③ The graph shows the melting points in degrees Celsius of three compounds: Compound A, Compound B and Compound C.

The melting point of Compound D is -3 times the melting point of Compound A. Find the melting point of Compound D.



Order of Operations (2.5)

1. Do all operations within grouping symbols such as parentheses or brackets.
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Simplify

$$24 \div (-6) \cdot 2 - 2^3 + (-12)$$

HELPFUL HINT

When simplifying expressions with exponents, parentheses make an important difference.

$(-5)^2$ and -5^2 do not mean the same thing.

$(-5)^2$ means $(-5)(-5) = 25$.

-5^2 means the opposite of $5 \cdot 5$, or -25 .

Only with parentheses is the -5 squared.