

Simplifying Algebraic Expressions (3.1)

- 1) In algebra, a variable is an unknown quantity that changes [or varies] in value from one situation to another. A single letter usually represents the variable
- 2) The addends of an algebraic expression are called the terms.
- 3) A term that is only a number is called a constant term, or simply a constant. A term that contains a variable is called a variable term.
- 4) The number factor of a variable term is called the numerical coefficient. A numerical coefficient of 1 is usually not written.

What are the numerical coefficients of the terms below?

$5x$

x

$-7y$

$-x$

$3y^2$

- 5) Terms that are exactly the same, except that they may have different numerical coefficients are called like terms.

Like Terms {

Unlike Terms {

- 6) A sum or difference of like terms can be simplified using the Distributive Property.

Distributive Property

If **a**, **b**, and **c** are numbers, then $ac + bc = (a + b)c = c(a + b)$

Also, $ac - bc = (a - b)c = c(a - b)$

- The Distributive Property guarantees that no matter what number x is,

$$7x + 2x = (7 + 2)x = 9x \quad \text{[for example]}$$

This is an example of **combining like terms**.

- An algebraic expression is simplified when all like terms have been combined.
- We can use the distributive property to combine like terms or to multiply expressions.
- This property says that multiplication distributes over addition and subtraction.

Simplify the expressions

① $2b - 3b - b$

② $2(x + 12)$

③ $2(x + 12) - 5x$

④ $3(a + 5) + 4(a - 6)$

⑤ $3x - 2y - 4x + 5y$

⑥ $3 + 5(d - 4) - 6$

Solving Equations (Section 2.6)

1) An **equation** is mathematical statement that says that two expressions are equal.

$$\textit{expression} = \textit{expression}$$

2) The **solution** to an equation is a number that when substituted for the unknown quantity [variable] in the equation will result in a true statement when all the arithmetic has been performed.

3) An **equation** is **solved** when the unknown quantity [variable] is determined.

4) To solve an equation, we will use properties of equality to write simpler equations, all equivalent to the original equation, until the final equation has the form

$$x = \text{number} \quad \text{or} \quad \text{number} = x$$

Examples:

① Is -6 a solution of $-3k = 12 - k$?

② Is 1 a solution of $2(b - 3) = 10$?

Properties of equality for solving equations state that in performing the same operation on both sides of the equal sign in a true equation, the resulting equation is still true.

Addition Property of Equality (2.6)

Let a , b , and c represent numbers.

If $a = b$, then $a + c = b + c$

In other words, the same number may be added to both sides of an equation without changing the solution of the equation.

Solve for x :

$$x - 4 = 3$$

To solve the equation for x , we need to rewrite the equation in the form $x = \text{number}$.

To do so, we add 4 to both sides of the equation.

$$x - 4 = 3$$

$$x - 4 + 4 = 3 + 4 \quad \text{Add 4 to both sides.}$$

$$x = 7 \quad \text{Simplify.}$$

Check:

$$x - 4 = 3 \quad \text{original equation}$$

$$7 - 4 = 3 \quad \text{Replace } x \text{ with } 7.$$

$$3 = 3 \quad \text{Simplify. True.}$$

Since $3 = 3$ is a true statement, 7 is the solution of the equation.

✧ Remember that we can get the variable alone on either side of the equation. $x = 3$ or $3 = x$

Solve. Check each solution.

Write an equation that can be solved using the addition property of equality.

Write an equation that can be solved using the multiplication property of equality.

Multiplication Property of Equality (2.6)

Let a , b , and c represent numbers and let $c \neq 0$.

If $a = b$, then

$$a \cdot c = b \cdot c \quad \text{and} \quad \frac{a}{c} = \frac{b}{c}, c \neq 0$$

In other words, both sides of an equation may be multiplied or divided by the same nonzero number without changing the solution of the equation.

Solve for x :

$$-4x = 8$$

To solve the equation for x , we need to rewrite the equation in the form $x = \text{number}$. To do so, we divide both sides of the equation by -4 and then simplify.

To check, replace the x in the original equation with the solution.

$$4x = 8 \quad \text{original equation}$$

$$4 \cdot \square = 8 \quad \text{Replace } x \text{ with } 2.$$

$$8 = 8 \quad \text{Simplify. True statement.}$$

Since $8 = 8$ is a true statement, 2 is the solution of the equation.

SIMPLIFY vs. SOLVE

- We simplify expressions and solve equations.
- To **simplify** an **expression** we write it in a less complicated form.
- To **solve** an **equation** means to find the number(s) that make the equation true when substituted for its variable.
- Quite often we must simplify expressions on the left- or right-hand sides of an equation when solving it.

Steps for Solving Linear Equations in One

- Step 1. If parentheses are present, use the distributive property to remove parentheses.
- Step 2. Combine any like terms on each side of the equation.
- Step 3. Use the addition property to rewrite the equation so that the variable terms are on one side of the equation and constant terms are on the other side.
- Step 4. Use the multiplication property of equality to divide both sides by the numerical coefficient of x to solve.
- Step 5. Check the solution in the original equation.

Translating Word Phrases into Expressions

the product of 5 and a number: $5x$

twice a number: $2x$

a number decreased by 3: $x - 3$

a number increased by 2: $x + 2$

four times a number: $4x$

the sum of a number and 7: $x + 7$

three times the sum of a number and 7: $3(x + 7)$

three times a number plus seven:

the quotient of 5 and a number:

four times the difference of a number and 5:

the difference of four times a number and 5:

the quotient of negative two and 4 times a number:

Linear Equations in One Variable and Problem Solving (3.4)

Keywords and phrases suggesting addition, subtraction, multiplication, division or equals.

Addition	Subtraction	Multiplication	Division	Equal Sign
sum	difference	product	quotient	equals
plus	minus	times	into	gives
added to	less than	of	per	is / was
more than	less	twice	divide	yields
total	decreased by	multiply	divided by	amounts to
increased by	subtracted from	double		is equal to

Problem-Solving Steps

1. Understand the problem. During this step, become comfortable with the problem. Some ways of doing this are:
 - Read and reread the problem.
 - Choose a variable to represent the unknown.
 - Construct a drawing.
 - Propose a solution and check it. Pay careful attention to how you check your proposed solution. This will help when writing an equation to model the problem.
2. Translate the problem into an equation.
3. Solve the equation.
4. Interpret the results. *Check* the proposed solution in the stated problem and *state* your conclusion.

1. Four times the difference of 12 and a number equals 72. Find the number.
2. The difference of a number and 9 is 37 less the number. Find the number.
3. Four times the sum of a number and 2 is 24 less than the number times 8. Find the number.
4. The product of 10 and a number is the same as 36 less twice that same number. Find the number.
5. In a recent year, the two top-selling PC games were *World of Warcraft* and *The Sims 2: University Expansion Pack*. The average price of *World of Warcraft* is \$14 more than the average price of *Sims*. If the total of these two prices is \$80, find the price of each game.
6. An Xbox 360 game system and several games are sold for \$560. The cost of the Xbox 360 is 3 times as much as the cost of the games. Find the cost of the Xbox 360 and the cost of the games.