Polynomial & Rational Inequalities (4.4)

Steps for Solving Polynomial and Rational Inequalities Algebraically

STEP 1: Write the inequality so that a polynomial or rational expression f is on the left side and zero is on the right side in one of the following forms:

$$f(x) > 0 \qquad f(x) \geq 0 \qquad f(x) < 0 \qquad f(x) \leq 0$$

For rational expressions, be sure that the left side is written as a single quotient.

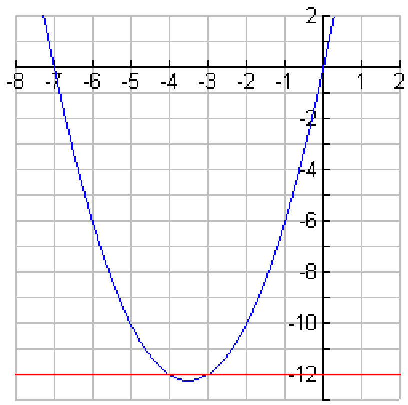
STEP 2: Determine the numbers at which the expression f on the left side equals zero and, if the expression is rational, the numbers at which the expression f on the left side is undefined.

STEP 3: Use the numbers found in Step 2 to separate the real number line into intervals.

STEP 4: Select a number in each interval and evaluate f at the number.
(a) If the value of f is positive, then $f(x) > 0$ for all numbers x in the interval.
(b) If the value of f is negative, then $f(x) < 0$ for all numbers x in the interval.
If the inequality is not strict ($\geq$ or $\leq$), include the solutions of $f(x) = 0$ in the solution set, but be careful not to include values of x where the expression is undefined.

Solve the inequalities algebraically and graphically. Use interval notation. Write the values in the solution set exactly.

① $x^2 + 7x < -12$



② $\dfrac{x(3x-5)}{(2x+3)(x+4)} \geq 0$

③ $\dfrac{x+16}{3x+2} \le 5$

④ $\dfrac{2}{x+1} \le \dfrac{3}{x-1}$

Double & Absolute Value Inequalities A.9

Solve the inequalities algebraically. Use interval notation. Write the values in the solution set exactly.

Double Inequalities:

① $0 < \frac{3x+2}{2} < 4$

② $1 \leq 1 - \frac{1}{2}x < 4$

Absolute Value Inequalities:

Theorem

If a is any positive number

$|u| < a$ is equivalent to $-a < u < a$

$|u| \leq a$ is equivalent to $-a \leq u \leq a$

In other words, $|u| < a$ is equivalent to $-a < u$ and $u < a$.

Theorem

If a is any positive number, then

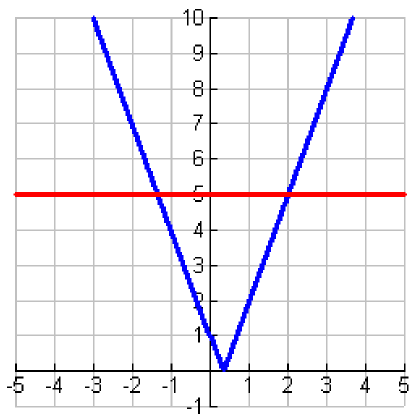
$|u| > a$ is equivalent to $u < -a$ or $u > a$

$|u| \geq a$ is equivalent to $u \leq -a$ or $u \geq a$

To solve an absolute value inequality algebraically, rewrite the inequality without the absolute value symbols using this theorem.

Solve:

③ $|3x - 1| < 5$



④ $|4x + 3| \geq 15$