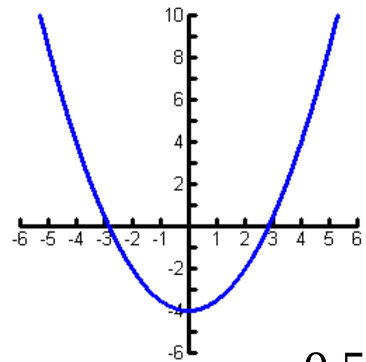
Properties of Functions (2.3)

1. Determine Even and Odd Functions from a Graph (p. 79)
2. Identify Even and Odd Functions from the Equation (p. 80)
3. Use a Graph to Determine Where a Function Is Increasing, Decreasing or Constant (p. 81)
4. Use a Graph to Locate Local Maxima and Local Minima (p. 82)
5. Use a Graphing Utility to Approximate Local Maxima and Local Minima and to Determine Where a Function Is Increasing or Decreasing (p. 83)
6. Find the Average Rate of Change of a Function (p. 84)

A function f is **even** if, for every number x in its domain, the number $-x$ is also in the domain and

$$f(-x) = f(x)$$

For an **even** function, for every point (x, y) on the graph, the point ($-x$, y) is also on the graph.



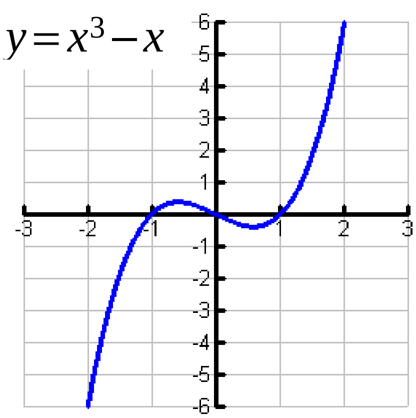
$y=0.5x^2-4$

A function f is **odd** if, for every number x in its domain, the number $-x$ is also in the domain and

$$f(-x) = -f(x)$$

So for an **odd** function, for every point (x, y) on the graph, the point ($-x$, $-y$) is also on the graph.

$y=x^3-x$



Theorem

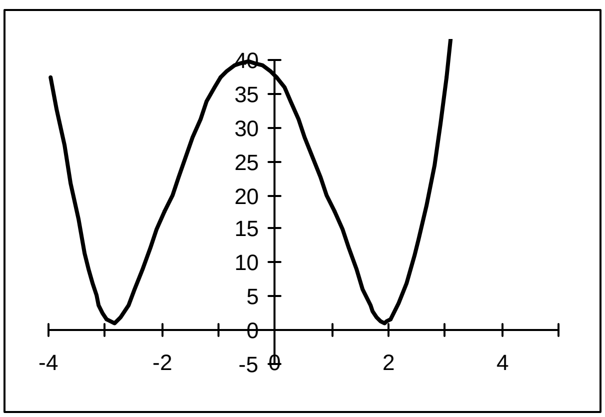
A function is even if and only if its graph is symmetric with respect to the y-axis.

A function is odd if and only if its graph is symmetric with respect to the origin.

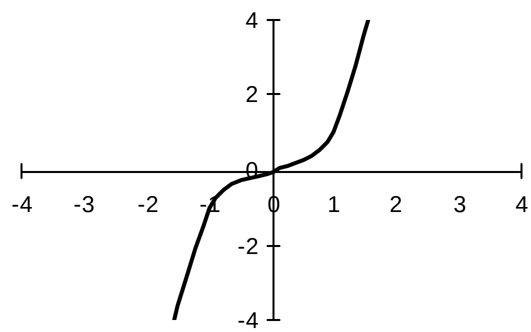
Part 2: Identify the following functions as even, odd, or neither.

1. $f(x) = 2x^3 + 3x^2$
2. $f(x) = 4x^3 + 3x$
3. $f(x) = 3x^2 - 6$

4.



5.

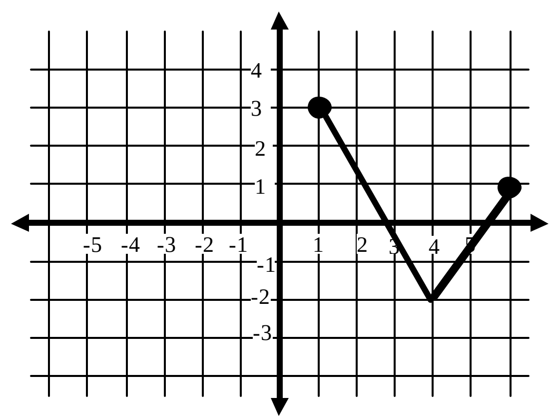


Part 2: Complete the graph.

6. A partial graph of $f(x)$ is shown .

   Complete the graph if $f(-x) = -f(x)$.

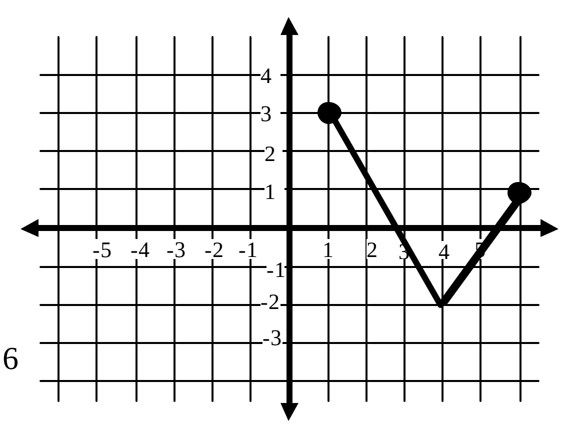
   Is $f(x)$ even, odd, or neither?



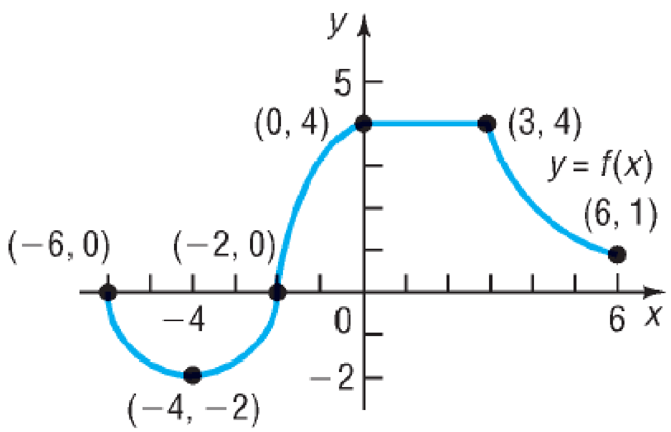
7. A partial graph of $g(x)$ is shown.

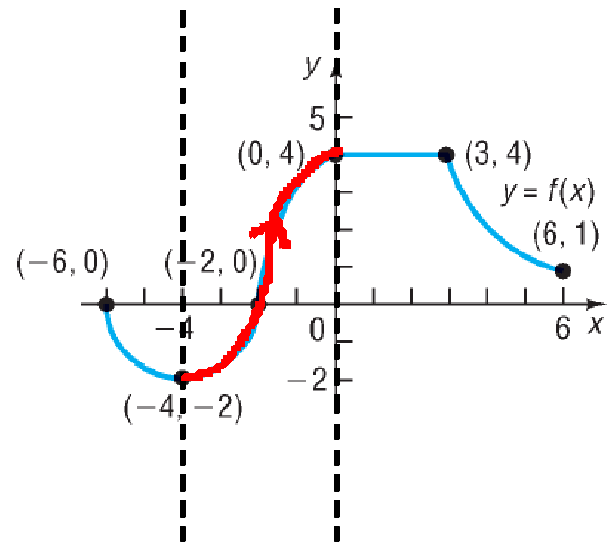
   Complete the graph if $g(-x) = g(x)$.

   Is $g(x)$ even, odd, or neither?

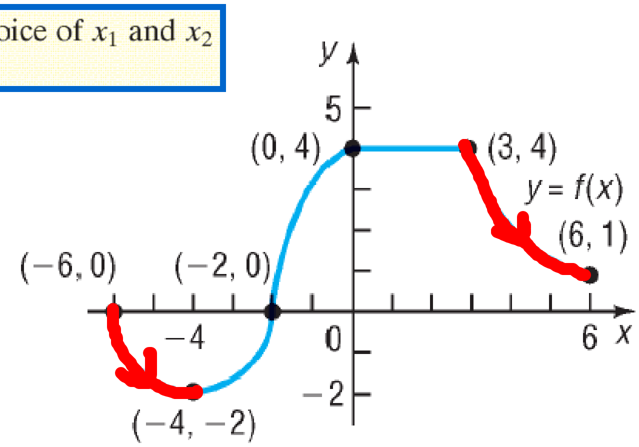


A function f is **increasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

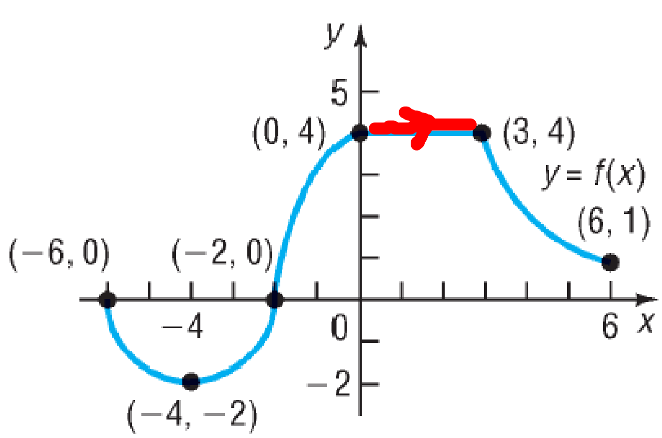




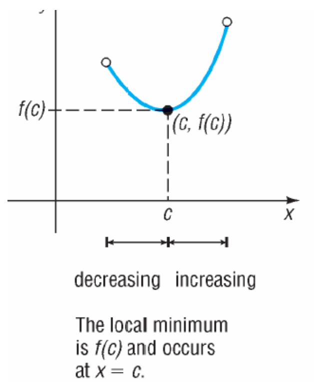
A function f is **decreasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.



A function f is **constant** on an interval I if, for all choices of x in I, the values $f(x)$ are equal.



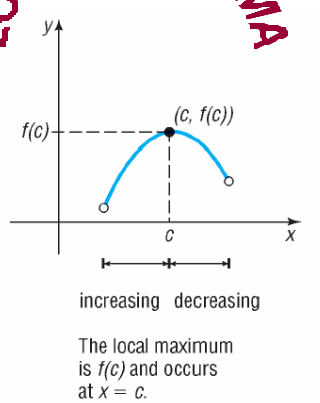
A function f has a **local minimum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) \geq f(c)$. We call $f(c)$ a **local minimum of f**.



LOCAL MINIMA

A function f has a **local maximum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) \leq f(c)$. We call $f(c)$ a **local maximum of f**.

LOCAL MAXIMA

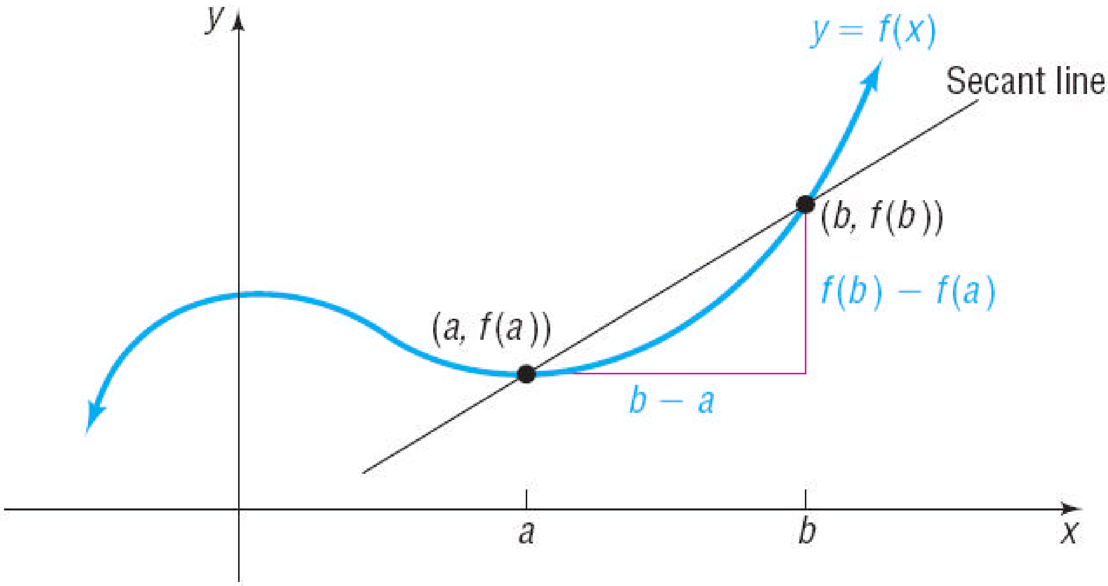


EXAMPLE

Using a Graphing Utility to Approximate Local Maxima and Minima and to Determine Where a Function Is Increasing or Decreasing

Use a graphing utility to graph $f(x) = 2x^3 - 3x + 1$ for $-2 < x < 2$.

Determine where f is increasing and where it is decreasing.



If a and b, $a \neq b$, are in the domain of a function $y = f(x)$, the **average rate of change of f** from a to b is defined as

Slope of the Secant Line

The average rate of change of a function from a to b equals the slope of the

EXAMPLE

Finding the Equation of a Secant Line

Suppose that $g(x) = -2x^2 + 4x - 3$.

(a) Find the average rate of change of g from -2 to 1.

(b) Find an equation of the secant line containing $(-2, g(-2))$ and $(1, g(1))$.

(c) Using a graphing utility, draw the graph of g and the secant line obtained in part (b) on the same screen.