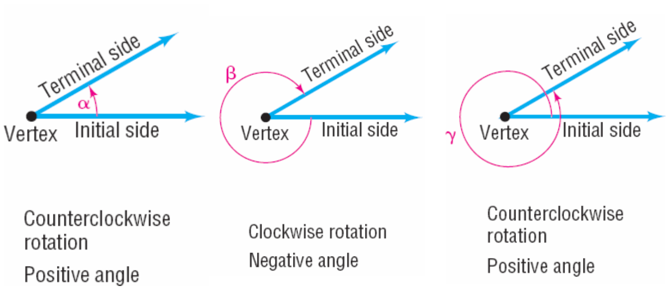
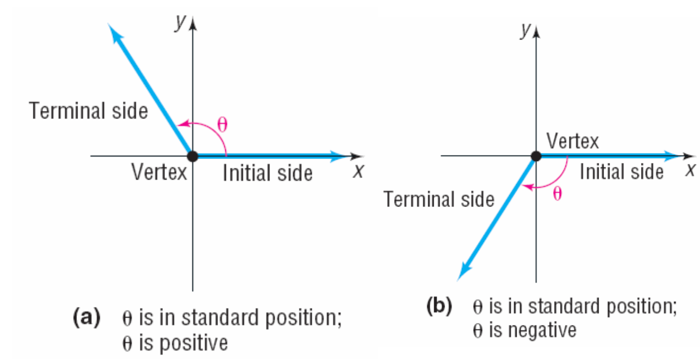
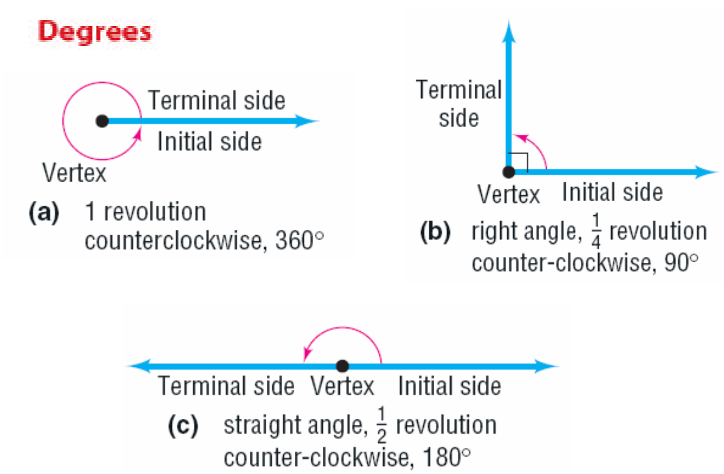
Section 6.1: Angles and Their Measure

Learning Objectives:

1. Convert between Decimals and Degrees, Minutes, Seconds Forms of Angles (p. 354)
2. Find the Arc Length of a Circle (p. 356)
3. Convert from Degrees to Radians and from Radian to Degrees (p. 356)
4. Find the Area of a Sector of a Circle (p. 359)
5. Find the Linear Speed of an Object Traveling in Circular Motion (p. 360)



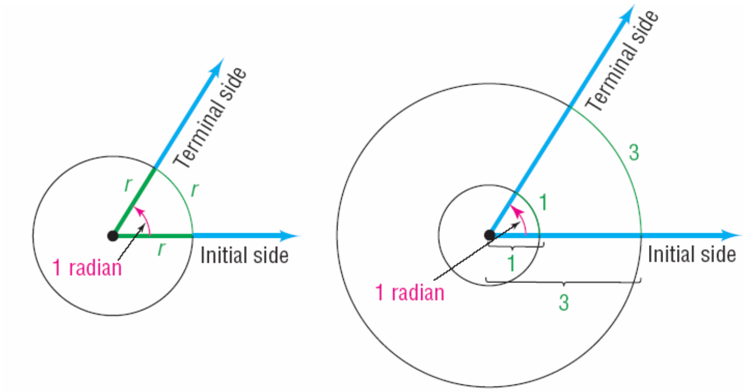




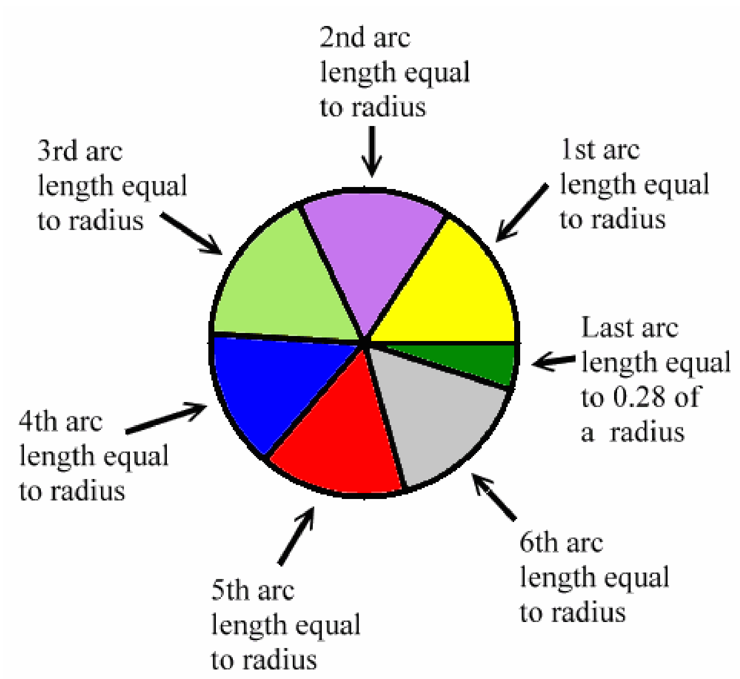
$$1 \text{ counterclockwise revolution} = 360^\circ$$
$$1^\circ = 60' \qquad 1' = 60''$$

1. Convert $87^\circ 32' 15''$ to a decimal in degrees.

Radians



http://id.mind.net/~zona/mmts/trigonometryRealms/radianDemo1/RadianDemo1.html



2. Convert $-\frac{11p}{3}$ to degree measure.

Section 6.2: Trigonometric Functions: Unit Circle Approach

Learning Objectives:

1. Find the Exact Values of the Trigonometric Functions Using a Point on the Unit Circle (p. 367)
2. Find the Exact Values of the Trigonometric Functions of Quadrantal Angles (p. 368)
3. Find the Exact Values of the Trigonometric Functions of $\frac{p}{4} = 45^\circ$ (p. 370)
4. Find the Exact Values of the Trigonometric Functions of $\frac{p}{6} = 30^\circ$ and $\frac{p}{3} = 60^\circ$ (p. 371)
5. Find the Exact values of the Trigonometric Functions for Integer Multiples of $\frac{p}{6} = 30^\circ$, $\frac{p}{4} = 45^\circ$, and $\frac{p}{3} = 60^\circ$ (p. 374)
6. Use a Calculator to Approximate the Values of the Trigonometric Functions of Acute Angles (p. 375)
7. Use a Circle of Radius *r* to Evaluate the Trigonometric Functions (p. 376)

Let t be a real number and let $P = (x, y)$ be the point on the unit circle that corresponds to t.

The **sine function** associates with t the y-coordinate of P and is denoted by

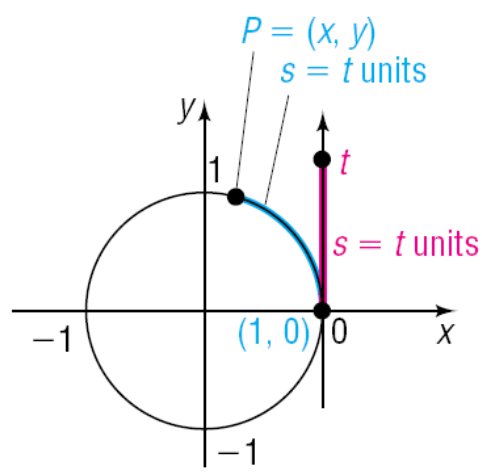
$$\sin t = y$$

The **cosine function** associates with t the x-coordinate of P and is denoted by

$$\cos t = x$$

If $x \neq 0$, the **tangent function** associates with t the ratio of the y-coordinate to the x-coordinate of P and is denoted by

$$\tan t = \frac{y}{x}$$



Let t be a real number and let $P = (x, y)$ be the point on the unit circle that corresponds to t.

If $y \neq 0$, the **cosecant function** is defined as

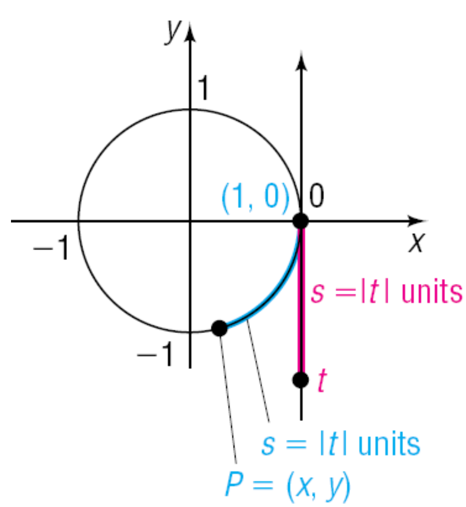
$$\csc t = \frac{1}{y}$$

If $x \neq 0$, the **secant function** is defined as

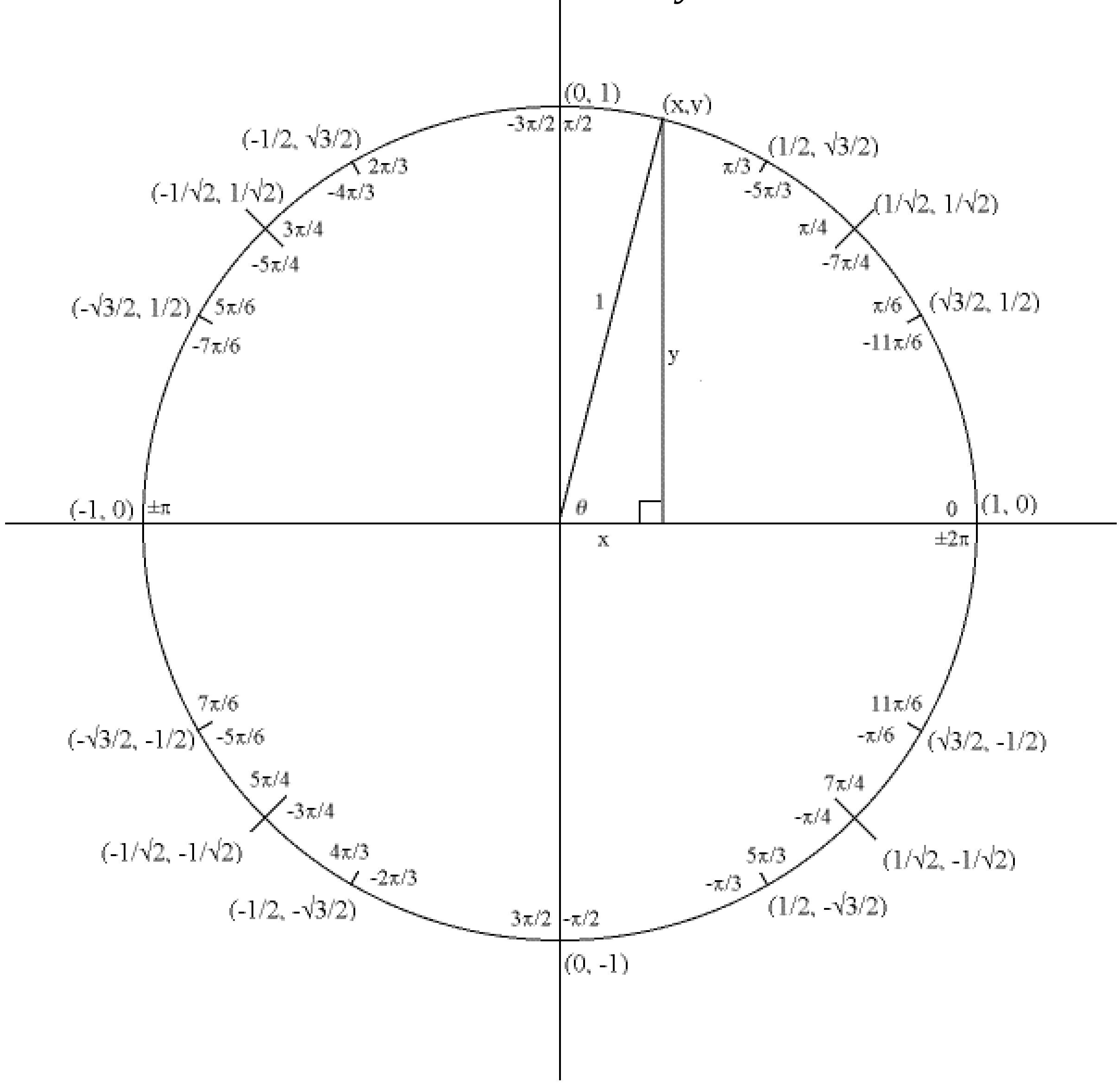
$$\sec t = \frac{1}{x}$$

If $y \neq 0$, the **cotangent function** is defined as

$$\cot t = \frac{x}{y}$$

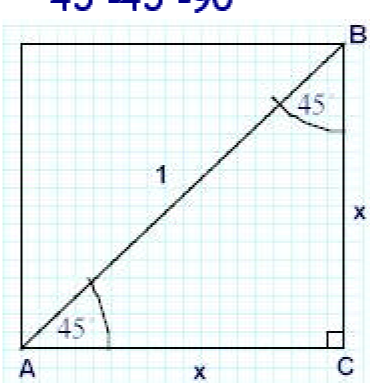


The Unit Circle: $x^2 + y^2 = 1$



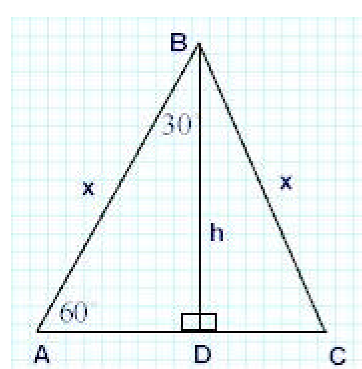
In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

45°-45°-90°



30°-60°-90°

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.



Sine Function: $f(x) = \sin x$

x	**sin x**
0	
$\frac{p}{2}$	
p	
$\frac{3p}{2}$	
$2p$	

Domain:

Range:

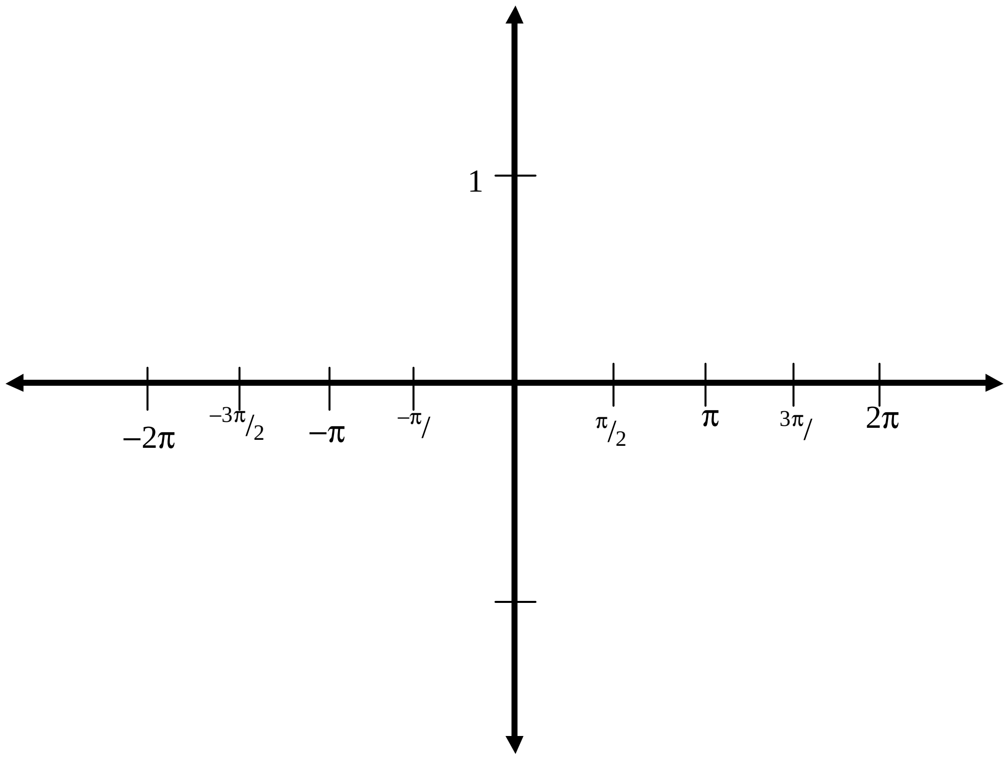
Symmetry:

Period:

x-intercepts:

y-intercept:

relative extrema



Cosine Function: $f(x) = \cos x$

x	**cos x**
0	
$\frac{p}{2}$	
p	
$\frac{3p}{2}$	
$2p$	

Domain:

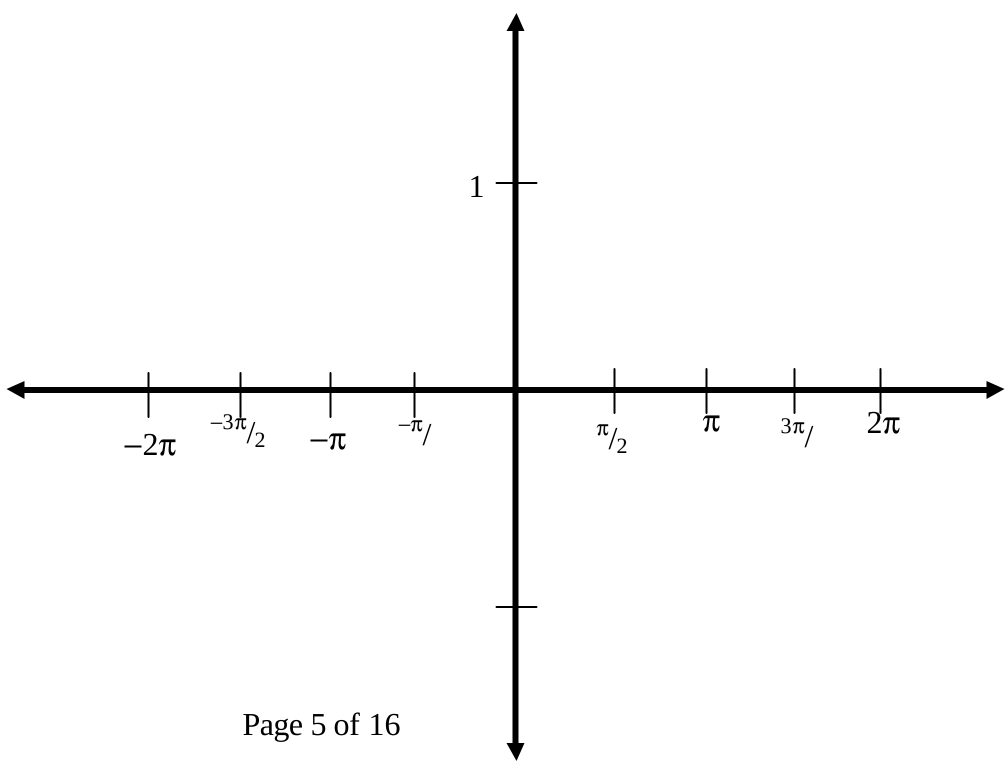
Range:

Symmetry:

Period:

x-intercepts:

y-intercept:



Fundamental Trigonometric Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

Finding the Values of the Trigonometric Functions When One Is Known

Given the value of one trigonometric function and the quadrant in which θ lies, the exact value of each of the remaining five trigonometric functions can be found in either of two ways.

Method 1 Using a Circle of Radius r

STEP 1: Draw a circle showing the location of the angle θ and the point $P = (x, y)$ that corresponds to θ. The radius of the circle is $r = \sqrt{x^2 + y^2}$.

STEP 2: Assign a value to two of the three variables x, y, r based on the value of the given trigonometric function and the location of P.

STEP 3: Use the fact that P lies on the circle $x^2 + y^2 = r^2$ to find the value of the missing variable.

STEP 4: Apply the theorem on page 382 to find the values of the remaining trigonometric functions.

Method 2 Using Identities

Use appropriately selected identities to find the value of each of the remaining trigonometric functions.

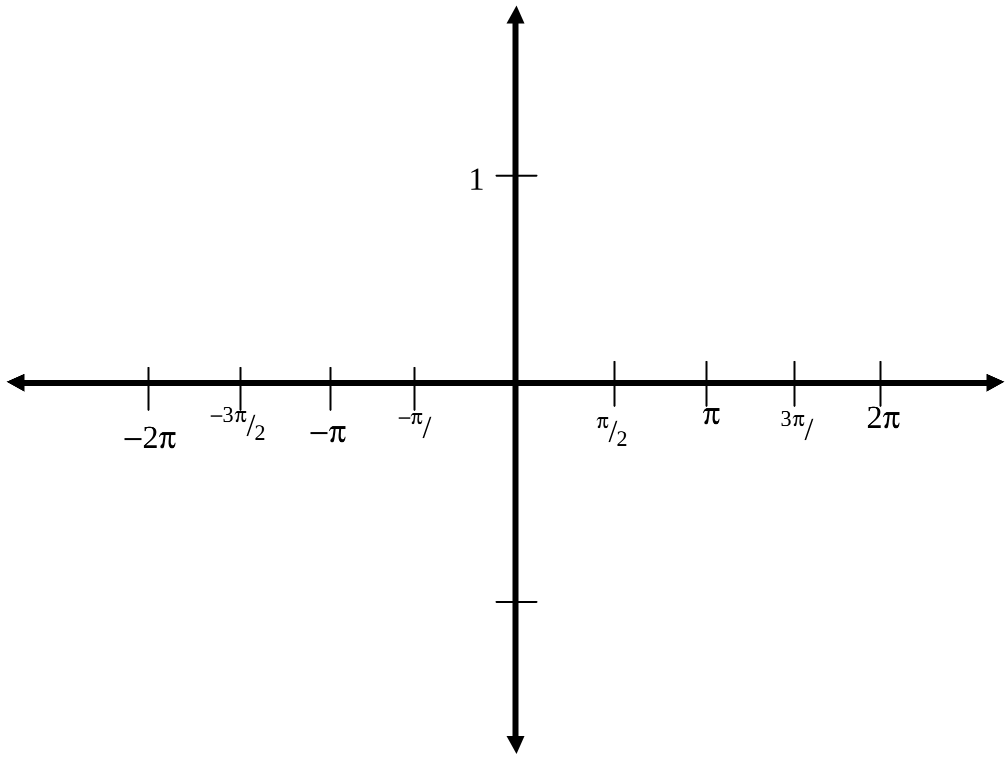
8. Find the values of the remaining trigonometric functions given: *(Use your Fundamental Trig Identities.)*

a. $\tan b = 5$ and $\cos b < 0$

b. $\cos a = \frac{1}{4}$ and $\sin a < 0$

Tangent Function: $f(x) = \tan x$

x	tan x
0	
$\frac{p}{2}$	
p	
$\frac{3p}{2}$	
$2p$	



Domain:

Range:

Vertical Asymptote(s):

Symmetry:

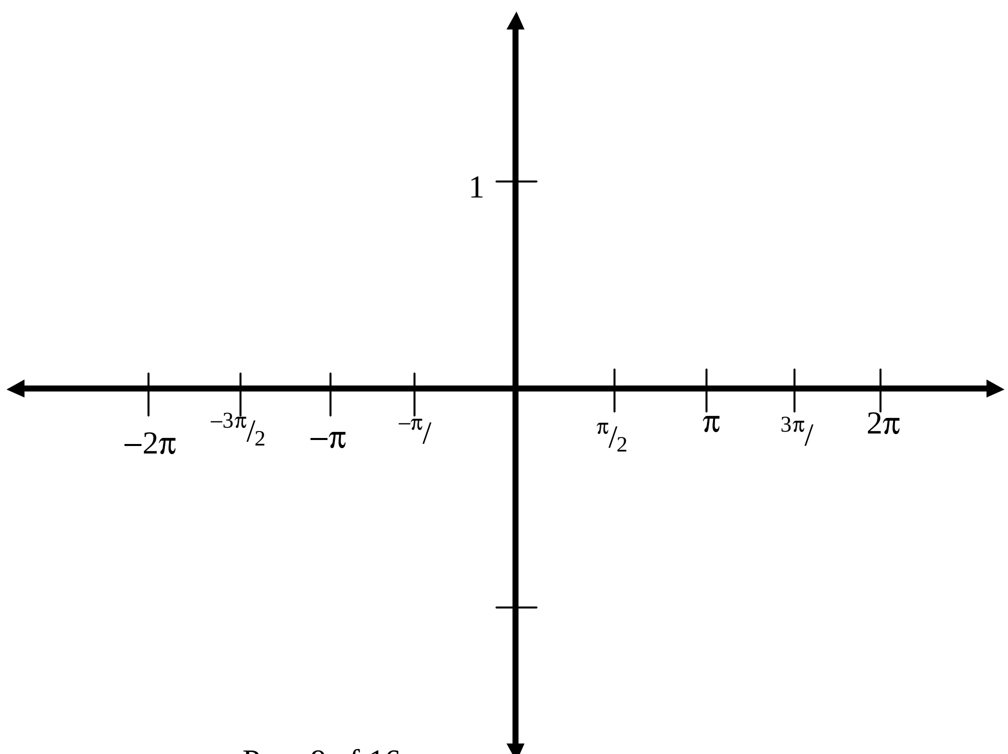
Period:

x-intercepts:

y-intercept:

Cotangent Function: $f(x) = \cot x$

x	cot x
0	
$\frac{p}{2}$	
p	
$\frac{3p}{2}$	
$2p$	



Domain:

Range:

Vertical Asymptote(s):

Symmetry:

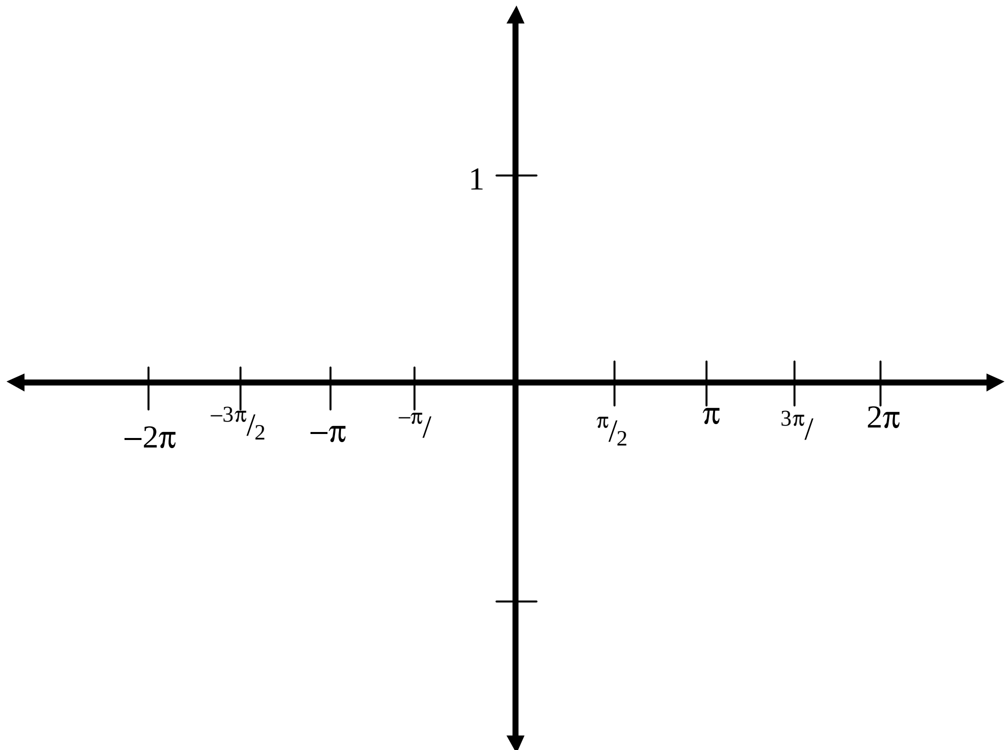
Period:

x-intercepts:

y-intercept:

Secant Function: $f(x) = \sec x$

x	cos x	sec x
0		
$\frac{p}{2}$		
p		
$\frac{3p}{2}$		
$2p$		



Domain:

Range:

Vertical Asymptote(s):

Symmetry:

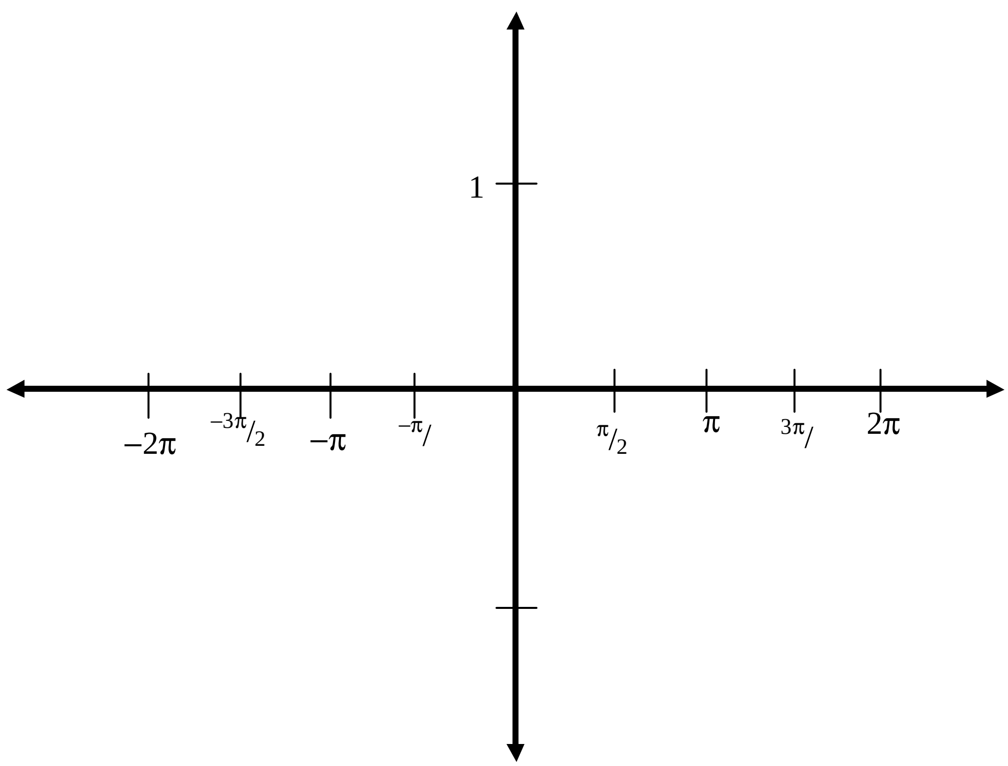
Period:

x-intercepts:

y-intercept:

Cosecant Function: $f(x) = \csc x$

x	sin x	csc x
0		
$\frac{p}{2}$		
p		
$\frac{3p}{2}$		
$2p$		



Domain:

Range:

Vertical Asymptote(s):

Symmetry:

Period:

x-intercepts:

y-intercept:

<u>Sinusoidal Graphs</u> $f(x) = A\sin(w\,x - f) + B$ **and** $f(x) = A\cos(w\,x - f) + B$

$$\sin x = \cos\left(x - \frac{p}{2}\right)$$

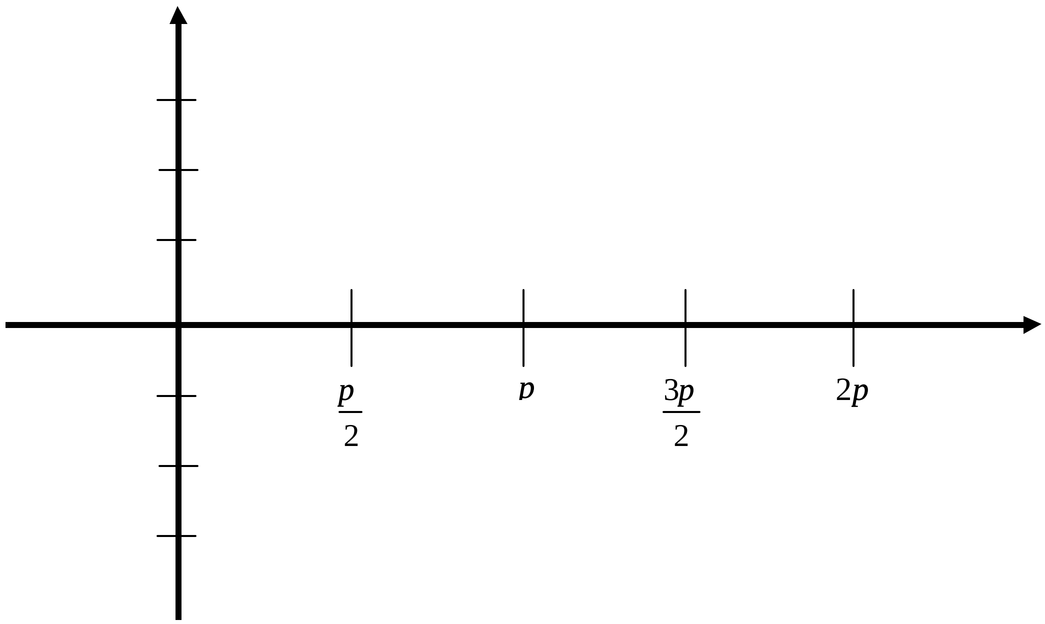
Amplitude: $f(x) = A\sin x$ and $f(x) = A\cos x$

Graph the following on the axes.

$f(x) = \sin x$

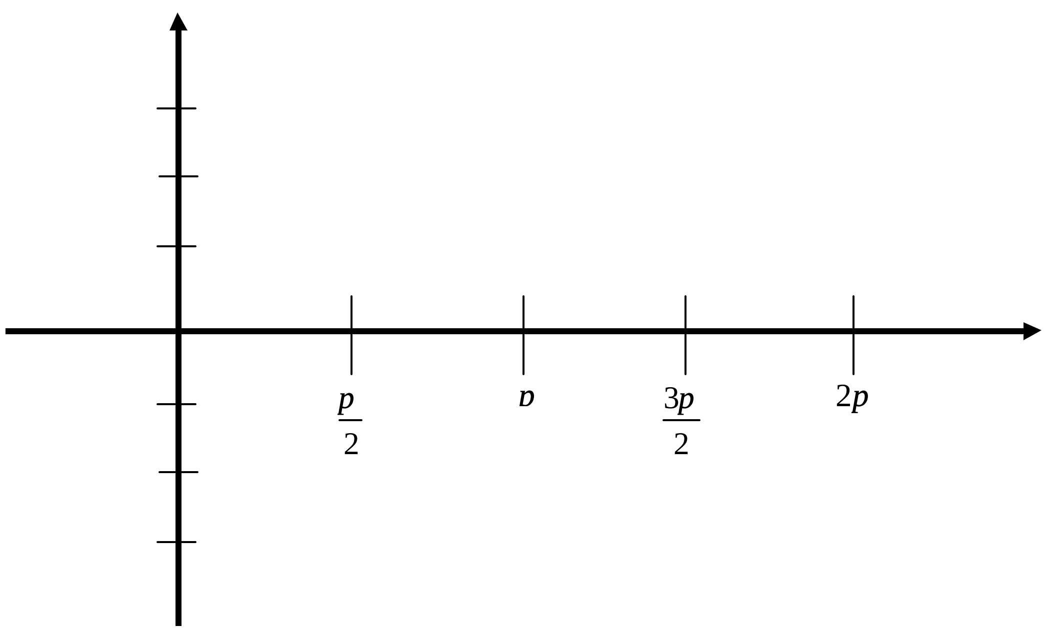
$g(x) = 2\sin x$

$h(x) = 3\sin x$

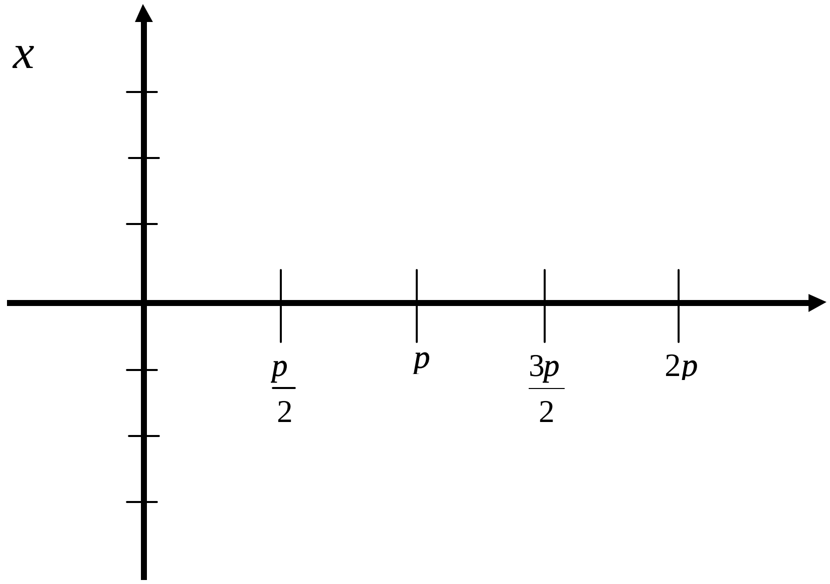


$f(x) = \cos x$

$g(x) = 3\cos x$



Period: $y = \sin w\, x$ **and** $y = \cos w\, x$

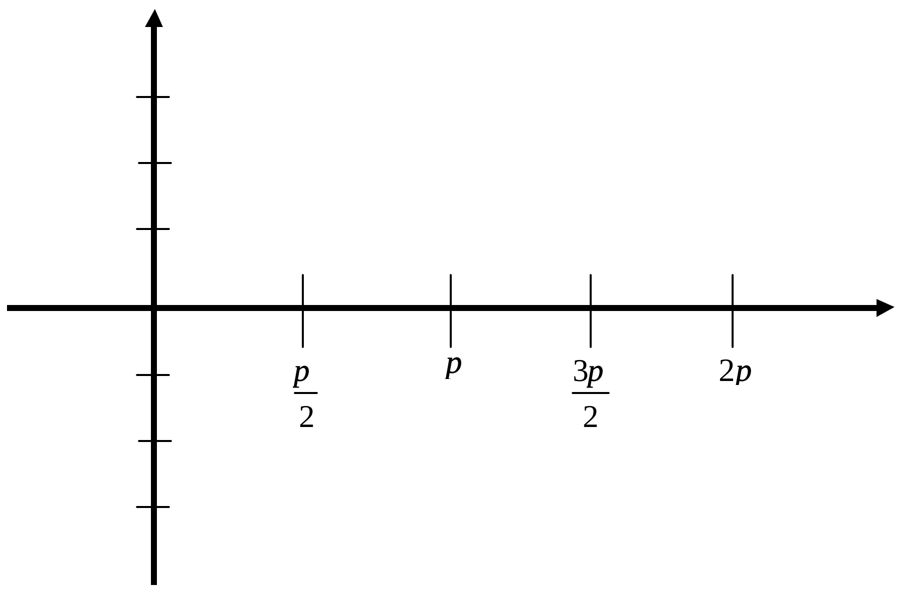


Graph the following:

$f(x) = \sin x$

$g(x) = \sin 2x$

$h(x) = \sin \dfrac{x}{2}$

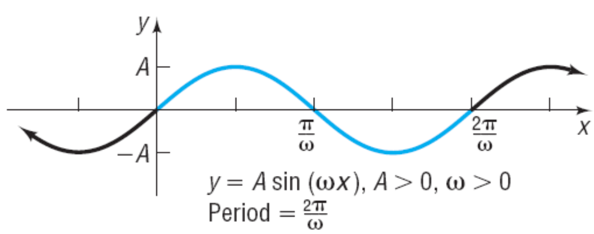


Graph the following:

$f(x) = \cos x$

$g(x) = \cos 2x$

$h(x) = \cos \dfrac{x}{2}$



Theorem

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are

Amplitude $= |A|$ Period $= T = \dfrac{2\pi}{\omega}$

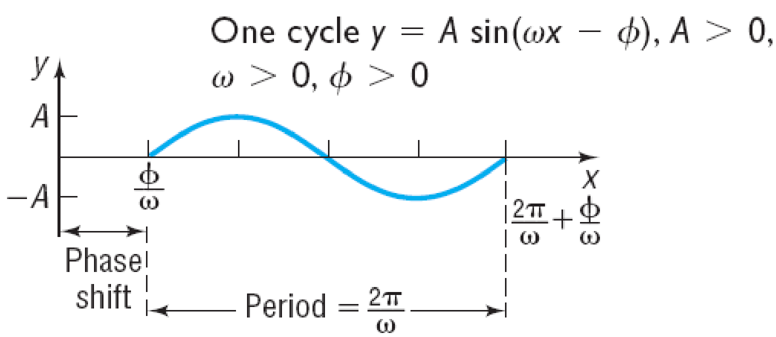
SUMMARY Steps for Graphing a Sinusoidal Function of the Form $y = A\sin(\omega x)$ or $y = A\cos(\omega x)$ Using Key Points

STEP 1: Determine the amplitude and period of the sinusoidal function.

STEP 2: Divide the interval $\left[0, \frac{2\pi}{\omega}\right]$ into four subintervals of the same length.

STEP 3: Use the endpoints of these subintervals to obtain five key points on the graph.

STEP 4: Plot the five key points with a sinusoidal graph to obtain the graph of one cycle. Extend the graph in each direction to make it complete.



For the graphs of $y = A\sin(\omega x - \phi)$ or $y = A\cos(\omega x - \phi)$, $\omega > 0$,

Amplitude $= |A|$ Period $= T = \frac{2\pi}{\omega}$ Phase shift $= \frac{\phi}{\omega}$

The phase shift is to the left if $\phi < 0$ and to the right if $\phi > 0$.

SUMMARY Steps for Graphing Sinusoidal Functions $y = A\sin(\omega x - \phi) + B$ or $y = A\cos(\omega x - \phi) + B$

STEP 1: Determine the amplitude $|A|$ and period $T = \frac{2\pi}{\omega}$.

STEP 2: Determine the starting point of one cycle of the graph, $\frac{\phi}{\omega}$. Determine the ending point of one cycle of the graph, $\frac{\phi}{\omega} + \frac{2\pi}{\omega}$. Divide the interval $\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + \frac{2\pi}{\omega}\right]$ into four subintervals, each of length $\frac{2\pi}{\omega} \div 4$.

STEP 3: Use the endpoints of the subintervals to find the five key points on the graph.

STEP 4: Plot the five key points with a sinusoidal graph to obtain one cycle of the graph. Extend the graph in each direction to make it complete.

STEP 5: If $B \neq 0$, apply a vertical shift.