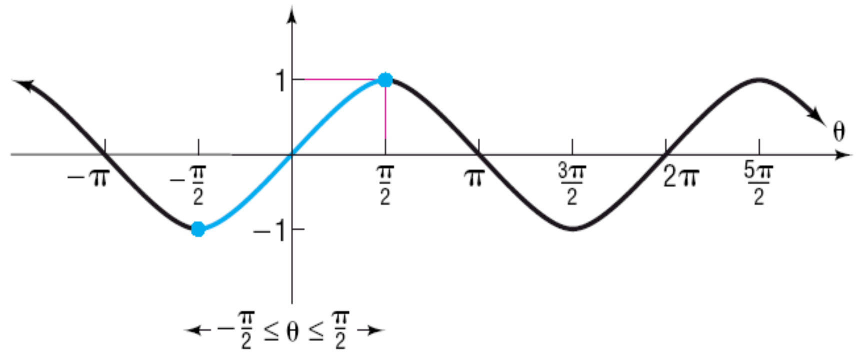
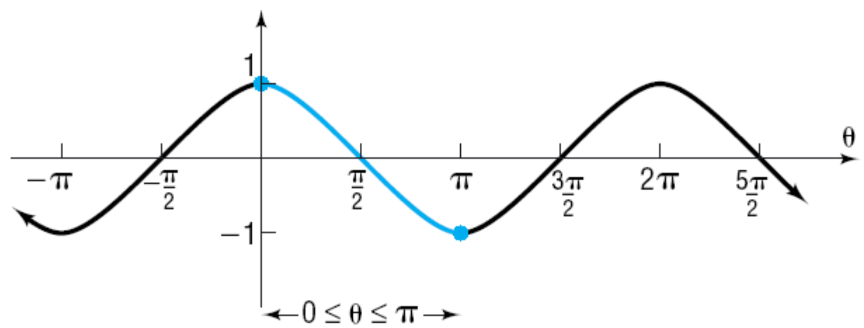
Scott
MA180/Precalculus
Chapter 6: Analytic Trigonometry

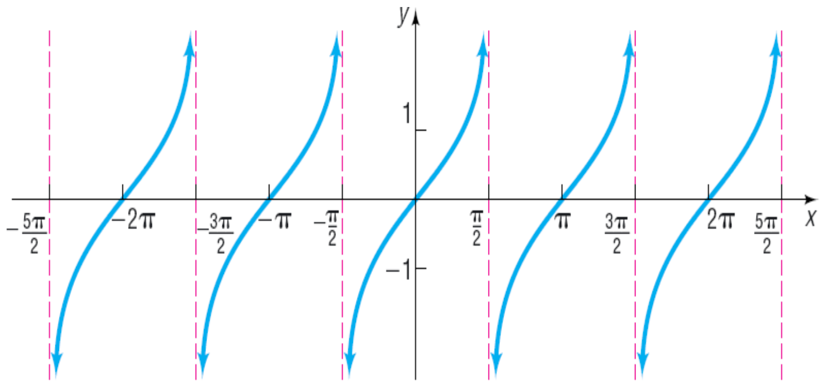
Section 7.1 and 7.2: Inverse Trigonometric Functions



$f(x) = \sin(x)$



$f(x) = \cos(x)$



$f(x) = \tan(x)$

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1. In your own words, EXPLAIN what each of the following expressions means. Evaluate each expression for $x = 0.5$. Give an exact answer if possible.
   a) $sin^{-1} x$ b) $sin\,(x^{-1})$ c) $(sin\,x)^{-1}$

2. Find the *exact* value of the following:

$\tan^{-1}\left(-\sqrt{3}\right)$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\sin^{-1}\left(-\frac{1}{2}\right)$

$\csc^{-1} 2$

Use your calculator to approximate to nearest hundredth.

$\sec^{-1} 3 =$

$\csc^{-1}(-4) =$

$\cot^{-1}(-2) =$

INVERSE TRIGONOMETRIC PROPERTIES

$\sin^{-1}(\sin u) = u,\ \text{where } -\frac{p}{2} \le u \le \frac{p}{2}$

$\cos^{-1}(\cos u) = u,\ \text{where } 0 \le u \le p$

$\tan^{-1}(\tan u) = u,\ \text{where } -\frac{p}{2} < u < \frac{p}{2}$

$\sin(\sin^{-1} v) = v,\ \text{where } -1 \le v \le 1$

$\cos(\cos^{-1} v) = v,\ \text{where } -1 \le v \le 1$

$\tan(\tan^{-1} v) = v,\ \text{where } -\infty < x < \infty$

3. Evaluate the following expressions exactly. If the expression is undefined, say so.
   a. $cos^{-1}\left(cos\left(5\pi/3\right)\right)$ b. $\cos\left(\cos^{-1}\left(1/2\right)\right)$

   c. $\sin\left(\sin^{-1} p\right)$ d. $\sin^{-1}\left(\sin p\right)$

4. Evaluate the following expressions exactly: $\tan\left(\arccos\frac{2}{3}\right)$

5. Write the following as an expression in x, $0 < x \le \frac{1}{3}$: $\sin\left(\arccos 3x\right)$

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Using Trigonometric Identities and Formulas to:

- ✓ Verify/Establish Identities (7.3 – 7.6)
- ✓ Simplify Trigonometric Expressions (7.3 – 7.6)
- ✓ Find the Exact Values of Trigonometric Expressions (7.3 – 7.6)
- ✓ Solve Trigonometric Equations (7.7 & 7.8)

Two functions f and g are said to be **identically equal** if

$$f(x) = g(x)$$

for every value of x for which both functions are defined. Such an equation is referred to as an **identity**. An equation that is not an identity is called a **conditional equation**.

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Even-Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

Guidelines for Establishing Identities

1. It is almost always preferable to start with the side containing the more complicated expression.
2. Rewrite sums or differences of quotients as a single quotient.
3. Sometimes rewriting one side in terms of sines and cosines only will help.
4. Always keep your goal in mind. As you manipulate one side of the expression, you must keep in mind the form of the expression on the other side.

Establish the identity: $\csc\theta \cdot \tan\theta = \sec\theta$

Establish the identity: $\dfrac{1+\tan\theta}{1+\cot\theta} = \tan\theta$

Establish the identity: $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\csc\theta$

Sum and Difference Formulas(7.4):

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Prove the identity: $\tan\left(\theta+\dfrac{\pi}{2}\right) = -\cot\theta$

If it is known that $\sin\alpha = \dfrac{4}{5}$, $\dfrac{\pi}{2} < \alpha < \pi$, and that $\sin\beta = -\dfrac{2}{\sqrt{5}} = -\dfrac{2\sqrt{5}}{5}$, $\pi < \beta < \dfrac{3\pi}{2}$, find the exact value of

(a) $\cos\alpha$ (b) $\cos\beta$ (c) $\cos(\alpha+\beta)$ (d) $\sin(\alpha+\beta)$

Find the exact value of the $\csc\left(\dfrac{7p}{12}\right)$

Write $\sin(\sin^{-1}u + \cos^{-1}v)$ as an algebraic expression containing u and v (that is, without any trigonometric functions).

Double-Angle & Half-Angle Formulas (7.5)

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

If $\sin\theta = \frac{3}{5}$, $\frac{\pi}{2} < \theta < \pi$, find the exact value of:

(a) $\sin(2\theta)$ (b) $\cos(2\theta)$

(c) $\tan\left(\frac{q}{2}\right)$

Product-to-Sum & Sum-to-Product Formulas (7.6)

Product-to-Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

Sum-to-Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

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Solving Trigonometric Equations (7.7 – 7.8)

Learning Objectives:

1. Solve Equations Involving a Single Trigonometric Function (p. 489)
2. Solve Trigonometric Equations Quadratic in Form (p. 495)
3. Solve Trigonometric Equations Using Identities (p. 496)
4. Solve Trigonometric Equations Linear in Sine and Cosine (p. 498)
5. Solve Trigonometric Equations Using a Graphing Utility (p. 500)

Solve each equation on the interval $0 \le q < 2p$ **.**

1. $4\sin^2 q - 3 = 0$
2. $\cot(2q) = 1$
3. $\sin\left(3q + \frac{p}{18}\right) = 1$

Solve the equation, give the general formula for all the solutions, and list six solutions.

4. $2\cos q + 1 = 0$
5. $\tan\left(\frac{q}{2}\right) = -1$

Solve each equation on the interval $0 \le q < 2p$ **.**

6. $\cos x + \cos x \tan x = 0$
7. $2\cos^2 x + 2\cos x + \sqrt{2}\cos x = -\sqrt{2}$
8. $2\cot^2 x + 5\cot x - 12 = 0$
9. $\cot^2 x - \csc x - 1 = 0$

10. Use a calculator to solve the equation on the interval $0 \le q < 2p$.

    $5\csc q = -6$