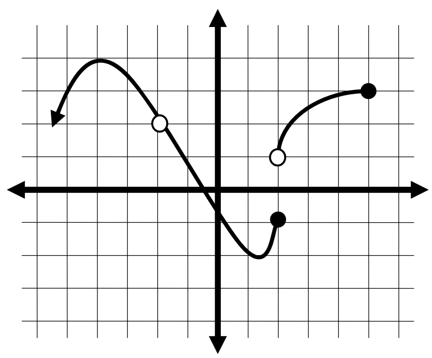
General Directions: When asked for EXACT SOLUTIONS, leave answers in fractional or radical form - not decimal form. That is, leave numbers like $2/3$, $\sqrt{3}$, π, and *e* as part of your answer.

1. State the domain of each of the following functions.

(a) $f(x)=\dfrac{\sqrt{7x+3}}{x-1}$ (b) $g(x)=\dfrac{2}{\sqrt{4-x^2}}$ (c) $h(x)=\dfrac{x^2+4x+5}{2x^2+13x-7}$

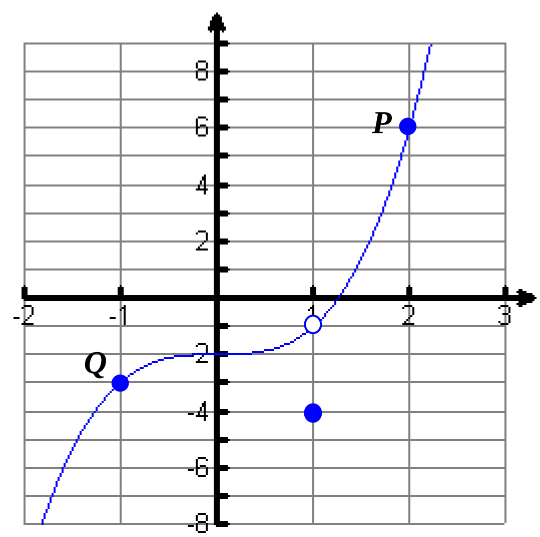
(d) $k(x)=\ln\left(\dfrac{3x-2}{x+4}\right)$ (e)



Assume x-scale and y-scale equal 1

(f) $m(x)=\dfrac{\sin x}{e^x}$

2. Use the graph of function *f* at the right to answer the following:



(a) Evaluate $f(1)$.

(b) Evaluate $f(0)$.

(c) Solve $f(x)=6$ for x

(d) Estimate the value x when $f(x) = 0$.

(e) Solve $f(x) > 0$.

(f) Find the average rate of change of f from P to Q. This represents the slope of the _____________.

(g) Find the equation of the secant line, $\overleftrightarrow{PQ}$.

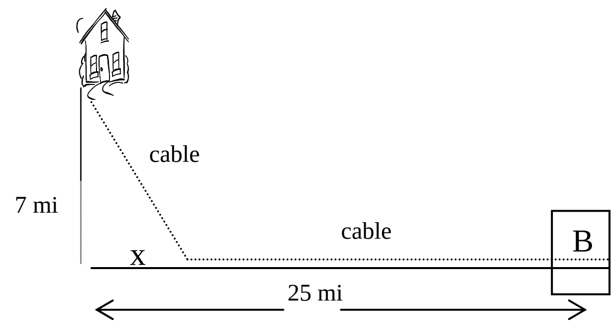
3. If $f(x)=3x^2+5x-8$, evaluate and simplify $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$.

4. Find two functions f and g such that $h(x) = f(g(x))$. *Do not use the identity function.*

(a) $h(x) = \sqrt[3]{x-5}$ (b) $h(x) = e^{5x}$ (c) $h(x) = \sin(2x)$ (d) $\ln(7x^3 - 4x)$

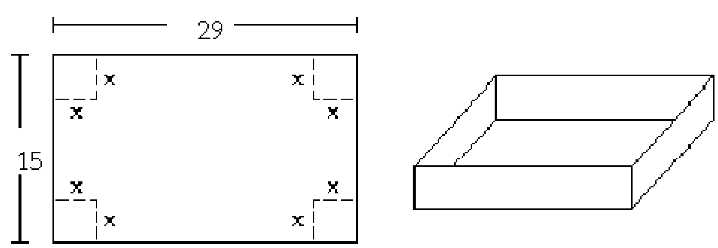
5. Given $f(x) = 35x^4 - 25x^2$, use your graphing calculator to approximate the following. Round your answers to three decimal places.
   (a) Find any local minima/maxima.
   (b) Find intervals where *f* is increasing and/or decreasing. Use interval notation.

6. A media company is going to install cable from a house to their connection box B. The house is located at one end of a driveway 7 miles back from a road (see diagram). The connection box is 25 miles down the road. The cost of installing the cable is $656 per mile off the road and $375 per mile along the road.
   Let x be the distance from where the driveway meets the road to where the cable comes to the road.



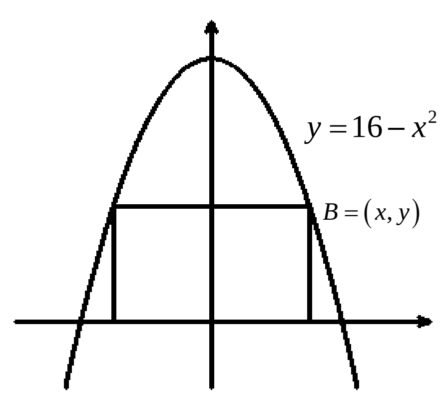
   a) Develop a function $C(x)$ that expresses the total installation cost as a function of x.
   b) Now use your calculator to graph C.
   c) Use the graph to determine the value of x that will produce the minimum cost. Round to the nearest three decimal places.
   (Tip: Use the window [0, 20, 1] x [12,000 , 20,000 , 1000].)
   d) State the minimum cost for that installation, rounded to the nearest cent.

7. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 15 cm by 29 cm by cutting out equal squares of side x at each corner and then folding up the sides as in the figure.



   a. Express the volume V of the box as a function of x.
   b. Find the maximum volume **and** the dimensions of the box of maximum volume. *Use appropriate units.*
   (Tip: Use the window: [0, 10] for x and [0, 700] for y.

8. The inscribed rectangle in the sketch always has its base on the x-axis and its upper corners on the parabola $y = 16 - x^2$.



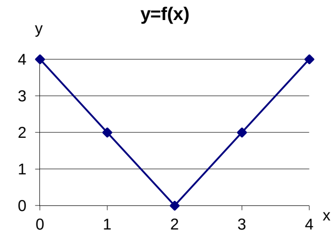
   a) Express the area A of the rectangle as a function of x.
   b) Determine the value of x for which the area A is largest.
   c) Express the distance *d* from the point B to the origin as a function of x. For what value of x is d the smallest?

9. The surface area of a balloon is given by $S(r) = 4\pi r^2$, where r is the radius of the balloon. If the radius is increasing with time t, as the balloon is being blown up, according to the formula $r(t) = \frac{2}{3}t^3, t \geq 0$ find the surface area S as a function of the time t.

10. Use the functions f and g to evaluate the following.

(a) $(f + g)(3)$ (b) $(f - g)(1)$ (c) $\left(\frac{f}{g}\right)(2)$

(d) $(f\,g)(4)$ (e) $(f \circ g)(2)$ (f) $(g \circ f)(2)$



x	$y = g(x)$
0	1
1	-2
2	0
3	-4
4	-3

11. If $f(x) = x^2 + 1$, and $g(x) = \sqrt{x-5}$, find and simplify

(a) $f(g(f(3)))$ (b) $f(g(x))$ and state the domain (c) $(g \circ f)(x)$ and state the domain

(d) $g^{-1}(x)$ and state its domain (e) $(f \circ g^{-1})(2)$

12. For each of these functions: I. $f(x) = 5x^3 - 17x^2 + 4x + 14$

II. $g(x) = 3x^4 - 20x^3 + 51x^2 - 86x + 40$.

(a) Find the zeros and identify them as rational, irrational, or non-real complex.

(b) Factor the polynomial completely over the complex numbers.

13. Given $f(-x) = -f(x)$ and $g(x) = g(-x)$, determine if the following functions are even, odd, or neither.

(a) $g(x-4)$ (b) $-f(x)+3$ (c) $g(x)-8$ (d) $-2f(x)$

____________ ____________ ____________ ____________

(e) Which of the above functions in (a) – (d) is symmetric across the origin?

(f) Which of the above functions in (a) – (d) is symmetric across the y-axis?

14. The data below represents the number of students enrolled in grades 9 through 12 in private institutions.

Year	Enrollment in 1000's
1975	1300
1980	1339
1981	1400
1982	1400
1983	1400
1984	1400
1985	1362
1986	1336
1987	1247
1988	1206
1989	1193
1990	1137
1991	1125
1992	1163
1993	1191
1994	1236
1995	1260
1996	1297

(a) Using a graphing utility, draw a scatter diagram of the data. Determine if the data appears to be linear, quadratic, cubic, exponential, logarithmic or sinusoidal. (Let 1975 correspond to $t = 0$)

(b) Using a graphing utility, find the cubic function of best fit and graph.

(c) Use the function found in part (b) to predict the number of students enrolled in private institutions for grades 9 to 12 in the year 2000. Round your answer to the nearest thousand students.

(d) Use the function found in part (b) to predict the year in which enrollment in private institutions for grades 9 through 12 will reach 2 ½ million students.

(e) The function found in part (b) does approximate the data quite nicely for the years that we have information. Explain why, for this situation, a cubic function might not be a good model over the long term. Include the phrase "end behavior" in your discussion.

15. True/False. Use the given facts about the polynomial function *f* to determine if the following statements are true or false. Circle *True* only if the statement must always be true.

Facts: $f(x) = 5x^3 + ax^2 + bx + 6$, *a* and *b* are real numbers

$f(0) = 6$ and $f(1) = -12$

(a) True False $f(x) \to \infty$ as $x \to -\infty$

(b) True False $\frac{5}{6}$ is a possible rational zero.

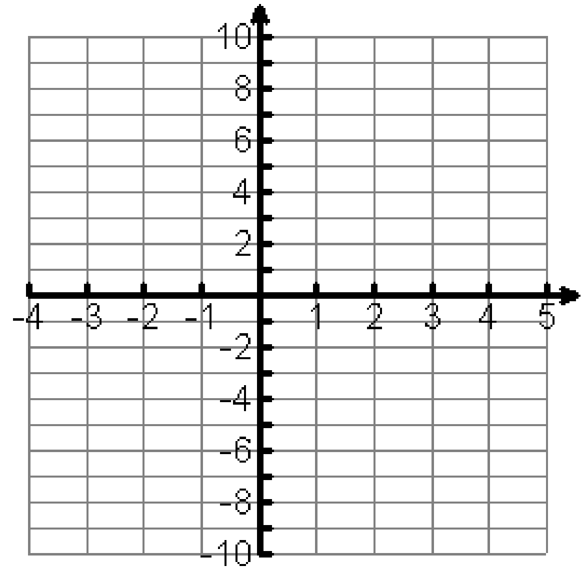
(c) True False *f* has at most three local extrema [turning points].

(d) True False *f* has three real zeros.

(e) True False *f* has at least one zero on the interval [-1, 2]

(f) True False *f* is an odd function.

16. Given $g(x)=\begin{cases} -x^2+1, & for \quad x\le 0 \\ -\frac{1}{2}x+4, & for \quad x>0 \end{cases}$



17. Consider the rational function $h(x)=\dfrac{(3x+2)(x-2)}{x(x-2)}$:

(a) State the domain of the function.

(b) Complete this statement: As $x\to-\infty$, $h(x)\to$ _______ .

(c) Write the equation of any vertical asymptotes.

(d) Write the equation of any horizontal asymptotes.

(e) Write the coordinates of any holes.

(f) List any *x*-intercepts.

(g) Graph and label any asymptotes, intercepts, and holes.

(h) State the range of this function.

18. The Parks and Wildlife commission introduces 80,000 fish into a large man-made lake. The population of the fish, in thousands, is given by $N(t)=\dfrac{20(4+3t)}{1+0.05t}$, $0\le t$, where *t* is the time in years.

(a) Approximate the population when *t* is 5, 10, and 25.

(b) What is the limiting number of fish in the lake as time increases?

19. Find the exact solution of $f(x)=\dfrac{x(3x-5)}{(7x+3)(x+4)}\ge 0$.Give the exact values for answer(s) using interval notation.

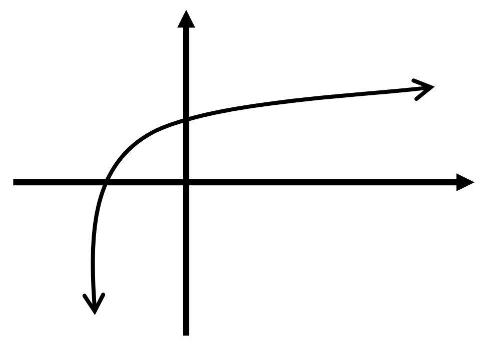
20. Solve: $x^3-5x^2+6x<0$. Write your answer using interval notation.

21. The value of an automobile depreciates. It is originally worth $40,000, but then it loses one-tenth of its value every year.
    (a) How much is it worth at the end of the first year?
    (b) How much is it worth after 3 years?
    (c) How much is it worth after *t* years?
    (d) How many years would pass before it was worth only half of its original value? *(This would be its "half-life".)* Round this value to the nearest tenth of a year.

22. Graph $g(x) = 1 - 5e^{x-3}$. (a) Find all intercepts and asymptotes. Round to two decimal places.
    (b) State the domain and range of $g(x)$.
    (c) Find $g^{-1}(x)$.
    (d) State the domain and range of $g^{-1}(x)$.

23. The unlabeled sketch at right shows the graph of $y = \log_3(x+3)$. The sketch correctly shows the shape of the graph, but it is not to scale.

    (a) Find the equation of any asymptotes and sketch.
    (b) Find the *x*-intercept and label.
    (c) Find the *y*-intercept and label.



24. Solve for *x*: $\log_5(8-18x) - 2\log_5 x = 1$

25. Approximate the value of y to three decimal places: y = $\log_5 40$

26. Exponential Growth Applications:
    Assume the function $P(t) = Ce^{kt}$ describes the population P of a certain country where the time is measured in *t* years.

    (a) What is the growth rate constant (*k*) if the population has tripled in 23 years? Approximate *k* to a tenth of a percent.
    (b) Use the *exact* value of *k* to find the population in 50 years if the initial population was 10,000.

27. Given $\sin\theta = 4/7$ and $\pi/2 < \theta < \pi$, find the EXACT values of the following:

    (a) $\tan\theta$ (b) $\sec\theta$ (c) $\cos\left(\theta + \frac{\pi}{2}\right)$ (d) $\csc(2\theta)$

28. Graph two full periods of each of the following functions.

$F(x) = -3\cos(4x) + 5$ $\qquad$ $G(t) = \frac{1}{3}\sin(2\pi t - \pi)$

For each function determine the following:

(a) domain (b) range (c) period (d) amplitude (e) phase shift

29. Establish the identity by verifying that $\frac{\csc\theta}{1+\csc\theta} - \frac{\csc\theta}{1-\csc\theta} = 2\sec^2\theta$

30. Use trigonometric identities to find the exact value of $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$, given:

$\tan\alpha = \frac{5}{12},\ \pi < \alpha < \frac{3\pi}{2};\sin\beta = -\frac{1}{2}, \pi < \beta < \frac{3\pi}{2}$

31. Use trigonometric identities to find the exact value of tan(2θ), where (–3, 4) is a point on the terminal side of θ.

32. Given $\sin(\theta) = -0.81$, find $\theta, \quad 0 \le \theta \le 2\pi$

33. Given $\cos 2x = -\sin x$:

(a) Solve the equation algebraically for the exact value of the solution(s) on the interval [0, 2π).

(b) Verify the answer(s) in part (a) using the ZERO or INTERSECT features of your graphing calculator.

Round answers to the nearest tenth of a radian.

34. Write TRUE or FALSE in the blank before each statement. [*Recall that if a statement in mathematics is not always true, then it is considered false*.]
If it is false, write a *corrected* true statement in the blank. ***t***
If it is already true, write "Identity" in the blank.

(a)____ sin (A + B) = sin(A) + sin (B) ____________________________________

(b)____ $\sin^2(A) = (\sin A)^2$ ____________________________________

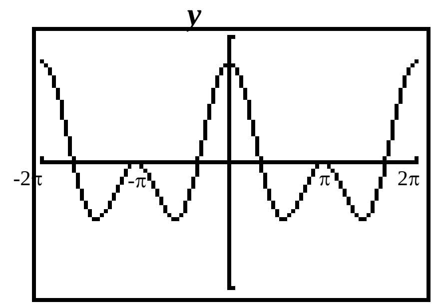
(c)____ $(\sin A + \cos A)^2 = \sin^2 A + \cos^2 A$ ____________________________________

(d)____ cos (A – B) = cos A – cos B ____________________________________

(e)____ sin (2A) = 2 sin A ____________________________________

(f)____ $\tan^{-1}(A) = \frac{1}{\tan(A)}$ ____________________________________

35. The graph at right shows two cycles of the graph of
$y = 2\cos^2 t + \cos t - 1$.
Algebraically find all zeros to the equation over the domain of all reals. Express your answer in terms of π.



36. In traveling across flat land, you notice a mountain directly in front of you. The angle of elevation [to the peak] is 3.5°. After you drive 13 miles closer to the mountain, the angle of elevation is 9°. Approximate the height of the mountain to the nearest foot. (5280 feet = 1 mile)

37. Two points A and B are on opposite sides of a building. A surveyor selects a third point C to place a transit. Point C is 53 feet from point A and 70 feet from point B. The angle ACB is 51°. How far apart are points A and B?

38. Sketch the curve described by the parametric equations by hand, showing its orientation. Then verify using a graphing calculator:
$x = 3\sin(t),\ y = -5\cos(t);\ \ 0 \le t \le 2\pi$

MA 180 Course Review   ANSWER KEY

1. a) $\{x \mid x \geq -\frac{3}{7} \text{ and } x \neq 1\}$
   b) $\{x \mid -2 < x < 2\}$
   c) $\left\{x \mid x \neq -7 \text{ or } x \neq \dfrac{1}{2}\right\}$
   d) $(-\infty, -4) \cup (\frac{2}{3}, \infty)$
   e) $\{x \mid x < -2 \text{ or } -2 < x \leq 5\}$
   f) $(-\infty, \infty)$

2. a) -4 b) -2 c) 2 d) $x \approx 1.25$
   e) $(1.25, \infty)$ f) 3; slope of secant line $\overleftrightarrow{PQ}$ g) $y = 3x$

3. $6x + 3h + 5$

4. a) $f(x) = \sqrt[3]{x}$ and $g(x) = x - 5$ b) $f(x) = e^x$ and $g(x) = 5x$
   c) $f(x) = \sin x$ and $g(x) = 2x$ d) $f(x) = \ln(x)$ and $g(x) = 7x^3 - 4x$

5. a) local minima: $f(-0.598) = -4.464$ and $f(0.598) = -4.464$
   local maximum: $f(0) = 0$

   b) Decreasing: $(-\infty, -0.598) \cup (0, 0.598)$
   Increasing: $(-0.598, 0) \cup (0.598, \infty)$

6. $C(x) = 656\sqrt{x^2 + 49} + 375(25 - x)$
   $13,142.74

7. a) $V(x) = x(15 - 2x)(29 - 2x)$
   b) The maximum volume is approximately 622.09 cm^3. The dimensions of the box of maximum volume are approximately 3.15 cm $\times$ 8.7 cm $\times$ 22.7 cm

8. a) $A(x) = 2x(16 - x^2) = 32x - 2x^3$
   b) $x \approx 2.31$
   c) $d(x) = \sqrt{x^2 + (16 - x^2)^2} = \sqrt{x^4 - 31x^2 + 256}$ d is smallest when $x \approx \pm 3.94$

9. $S(r(t)) = \frac{16}{9}\pi t^6$

10. a) -2 b) 4 c) undefined d) -12 e) 4 f) 1

11. a) 6
    b) $x - 4, \ x \geq 5$
    c) $\sqrt{x^2 - 4}, \ x \leq -2 \text{ or } x \geq 2$
    d) $g^{-1}(x) = x^2 + 5$ domain: $\{x \mid x \geq 0\}$
    e) 82

12. I. a) rational: $x=\frac{7}{5}$; irrational: $x=1\pm\sqrt{3}$

b) $f(x)=5\left(x-\frac{7}{5}\right)\left(x-1+\sqrt{3}\right)\left(x-1-\sqrt{3}\right)$

II. a) rational: $x=\frac{2}{3},4$; nonreal complex: $x=1\pm 2i$

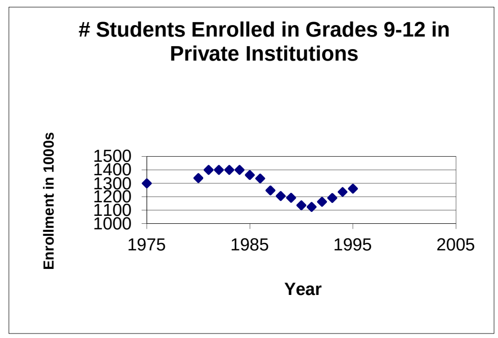
b) $g(x) = 3\left(x-\frac{2}{3}\right)(x-4)(x-1-2i)(x-1+2i)$

13. (a) neither (b) neither (c) even (d) odd (e) d (f) c

14. a) The data appears to fit a cubic or a sinusoidal model.

b) If you elect to use year 1975 as $x=0$, and rounding coefficients to 6 decimal places, the regression function is: $f(x)=0.307723x^3-9.790235x^2+71.198866x+1277.177287$.

c) 1,746,000 students
d) The closest year is 2004.
e) Because the end behavior of this function is increasing as x→∞, the enrollment at these institutions would increase without bounds as time goes by. Realistically, this is unlikely to happen.



15. (a) False; f has the same end behavior as $y=5x^3$, therefore $f(x)\to\infty$ as $x\to\infty$ and $f(x)\to-\infty$ as $x\to-\infty$.

(b) False; $\frac{6}{5}$ is a possible rational zero.

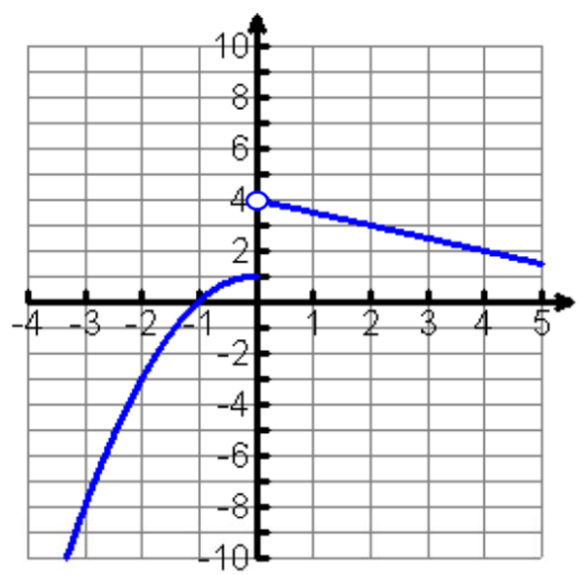
(c) False; f has at most two local extrema because it is a third degree polynomial.

(d) False; f has ***at most*** three real zeros. Two zeros could be nonreal.

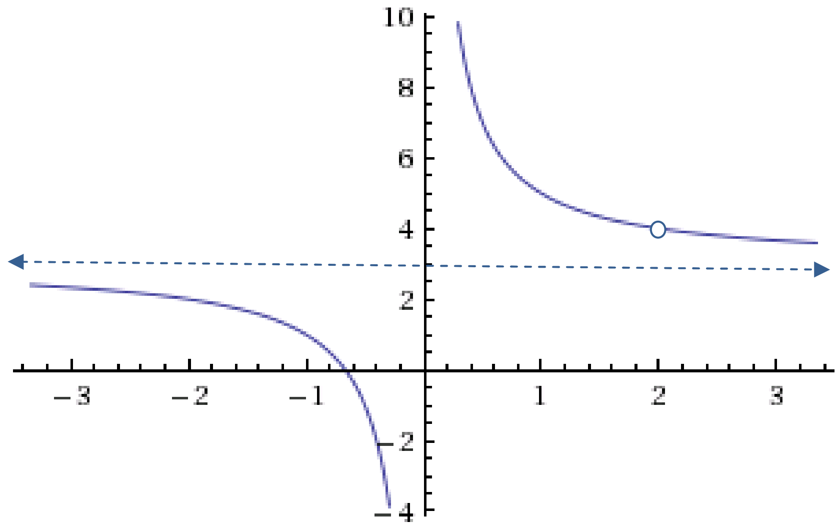
(e) True; Intermediate Value Theorem

(f) False; f is an odd degree function which does NOT necessarily mean f is an odd function.

16. see graph ➔



17. a) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$
    b) as $x \to -\infty$, $h(x) \to 3$
    c) $x = 0$
    d) $y = 3$
    e) $(2,4)$
    f) $\left(-\frac{2}{3}, 0\right)$
    g) See graph at right
    h) $\{y \mid y \neq 3 \text{ and } y \neq 4\}$



18. a) $t = 5$ 304,000 fish
    $t = 10$ 453,333 fish
    $t = 25$ 702,222 fish
    b) 1,200,000 fish

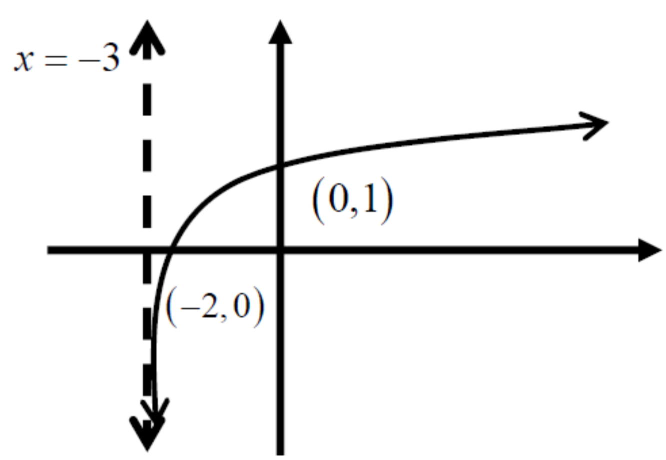
19. $(-\infty, -4) \cup \left(\frac{-3}{7}, 0\right] \cup \left[\frac{5}{3}, \infty\right)$

20. $(-\infty, 0) \cup (2, 3)$

21. a) \$36,000
    b) \$29,160
    c) $40,000(0.9)^t$ dollars
    d) 6.6 years

22. a) (1.39,0), (0,0.75) $y = 1$
    b) domain of g: all reals range of g: $\{y \mid y < 1\}$
    c) $g^{-1}(x) = \ln\left(\frac{1-x}{5}\right) + 3$
    d) domain of g^{-1}: $\{x \mid x < 1\}$ range of g^{-1}: all reals

23. a) No horizontal asymptote
    vertical asymptote at $x = -3$
    b) *x*-intercept at –2
    c) *y*-intercept at 1



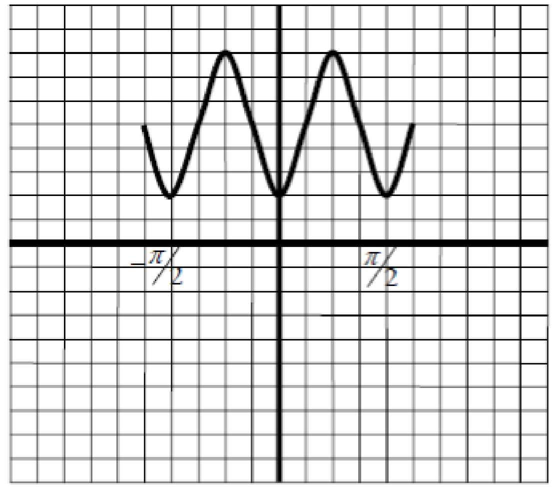
24. $x = 0.4 = \frac{2}{5}$

25. 2.292

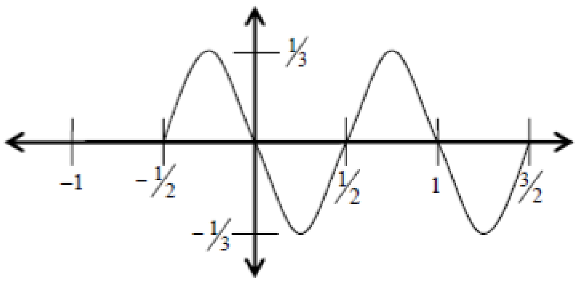
26. a) 4.8% b) 108,948

27. a) $\frac{-4}{\sqrt{33}}$ b) $\frac{-7}{\sqrt{33}}$ c) $-\frac{4}{7}$ d) $-\frac{49}{8\sqrt{33}} = -\frac{49\sqrt{33}}{264}$

28. F: (a) all reals (b) $[2,8]$ (c) $\frac{\pi}{2}$ (d) 3
(e) none



G: (a) all reals (b) $\left[-\frac{1}{3},\frac{1}{3}\right]$ (c) 1 (d) $\frac{1}{3}$
(e) $\frac{1}{2}$



29. solutions vary

30. a) $\dfrac{5\sqrt{3}+12}{26}$, $\dfrac{12\sqrt{3}-5}{26}$

31. $\dfrac{24}{7}$

32. $\theta \approx 4.09$ or $\theta \approx 5.34$

33. a) $\dfrac{\pi}{2}$, $\dfrac{7\pi}{6}$, $\dfrac{11\pi}{6}$ b) 1.6, 3.7, 5.8

34. a) False $\sin(A+B)=\sin A\cos B+\cos A\sin B$
b) True
c) False $(\sin A+\cos A)^2=\sin^2 A+2\sin A\cos A+\cos^2 A=1+2\sin A\cos A$
d) False $\cos(A-B)=\cos A\cos B+\sin A\sin B$
e) False $\sin(2A)=2\sin A\cos A$
f) False $\dfrac{1}{\tan A}=\cot A$

35. $\dfrac{\pi}{3}+2k\pi,\ \dfrac{5\pi}{3}+2k\pi,\ \pi+2k\pi$

36. 6839 feet

37. 55.1 ft

38. *y* is <u>not</u> a function of *x*