

Algebraic Atrocities

Many of the mistakes students make (and therefore many of the points lost on quizzes and tests) are a result of algebraic errors. The purpose of this problem set is to remind you of certain algebraic properties so that you will use these properties correctly throughout the semester.

Part I. Decide whether each of the following is true or false. In either case, justify your answer with an explanation or a numerical example. If a statement is false, what would the correct statement be?

1. $(x + y)^2 = x^2 + y^2$

2. $\sqrt{x^2 + 4} = x + 2$

3. $(\ln x)^3 = 3 \ln x$

4. $\frac{3}{x+2} - \frac{x+1}{x+2} = \frac{4-x}{x+2}$

5. $\frac{3(x-4)+x}{(x-4)^2} = \frac{3+x}{x-4}$

6. $\frac{\sin(2x)}{2} = \sin x$

7. $x(x+2)^3 = (x^2 + 2x)^3$

8. $\frac{x}{\sqrt{x}} = \sqrt{x}$

9. $\sin^2 x = (\sin x)^2$

10. $\log x = \ln x$

11. If $x(x-3) = 12$, then $x = 12$ or $x - 3 = 12$

12. $\sin^{-1} x = \frac{1}{\sin x}$