MA 181
Section 3.7

Logarithmic Differentiation

1. Let $f(x)=(x+1)^x$.

As of now, none of the rules we have learned can help us find $f'(x)$.

(a) Why doesn't the Power Rule apply to this function?

(b) Why doesn't the formula $\frac{d(a^x)}{dx}=a^x\ln a$ apply to this function?

(c) Start with the equation $y=(x+1)^x$. Take the natural logarithm of each side to get $\ln y=\ln(x+1)^x$. Now use a property of logarithms to turn the right side into a product. Next differentiate each side of this equation. You will have to use implicit differentiation on the left side and the product rule on the right side.

(d) Now solve for dy/dx (or y', depending on which notation you have used).

(e) Next replace any y terms by $(x+1)^x$, since this is what y was equal to. You have now found $f'(x)$ by a process called *logarithmic differentiation.*

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2. Try following the procedure outlined above to find the derivative of $g(x) = x^{\sin x}$.

3. You can also use logarithmic differentiation to find the derivative of an expression involving a messy product and/or quotient.

For example, if $y = \frac{(x^2+3)^3(4x-3)}{\sqrt{x^4+6}}$, take the natural logarithm of each side to get

$$\ln y = \ln \frac{(x^2+3)^3(4x-3)}{\sqrt{x^4+6}}.$$

Next, use the properties of logarithms to rewrite the right side as a combination of sums and differences. Be sure also to use the rule of logarithms which enables an exponent to be written in front of the logarithm. Now differentiate each side with respect to x, solve for $\frac{dy}{dx}$, and replace y by the original expression for y to obtain an answer which is only in terms of x.

Answers:

1. $(x+1)^x \left(\frac{x}{x+1} + \ln(x+1) \right)$

2. $x^{\sin x} \left(\frac{\sin x}{x} + \cos x \ln x \right)$

3. $\frac{(x^2+3)^3(4x-3)}{\sqrt{x^4+6}} \left(\frac{6x}{x^2+3} + \frac{4}{4x-3} - \frac{2x^3}{x^4+6} \right)$