Whenever the function $y = f(x)$ has a specific interpretation in one of the sciences, its derivative will have a specific interpretation as a rate of change. The units for $\frac{dy}{dx}$ are the units for y divided by the units for x.

Example ①: **Physics - Analyzing the motion of a particle**.

A particle moves according to a law of motion $s = f(t)$, $t \geq 0$.

(a) Find the average velocity on the interval $[t_1, t_2]$.

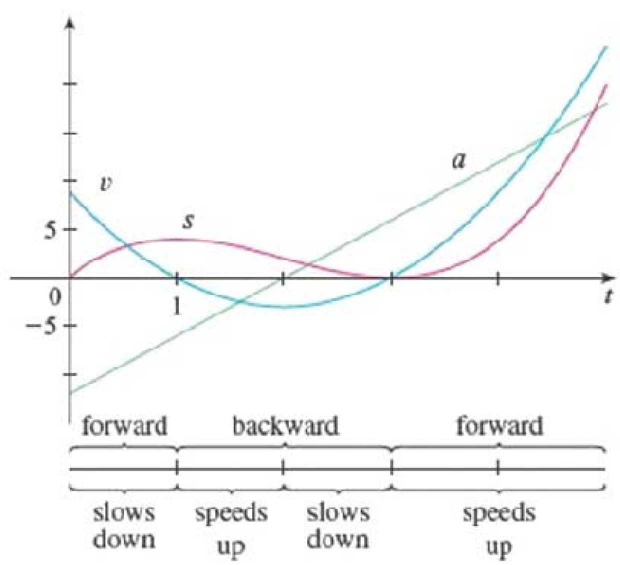
(b) Find the [instantaneous] velocity.

(c) When is the particle at rest?

(d) When is the particle moving in a positive direction? Negative direction?

(e) When is the particle speeding up? When is it slowing down?

A particle speeds up when the velocity is positive and increasing (v and a are both positive) and also when the velocity is negative and decreasing (v and a are both negative). In other words, the particle speeds up when the velocity and acceleration have the same sign. (The particle is pushed in the same direction it is moving.) The particle slows down when the velocity and acceleration have opposite signs.



A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where t is measured in seconds and s in feet.

(a) Find the average velocity on the interval $[2, 3.2]$.

(b) Find the [instantaneous] velocity at time t.

(c) What is the velocity after 3 s?

(d) When is the particle at rest?

(e) When is the particle moving in the positive direction?

(f) Find the total distance traveled during the first 8 s.

(g) Draw a diagram to illustrate the motion of the particle.

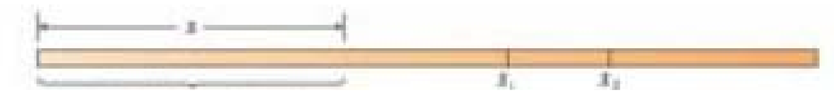
(h) Find the acceleration at time t and after 3 s.

(i) Graph the position, velocity, and acceleration functions for $s(t)$.

(j) When is the particle speeding up? When is it slowing down?

Example ② Linear Density

The **linear density** of a rod is given by taking the mass divided by the length if the mass of the rod is uniform throughout [homogeneous]. Suppose, however, that the rod's mass varies [it is NOT homogeneous]. Let $m = f(x)$ be the mass of the rod from one end to the point x. Then the linear density at x is the derivative $\rho = f'(x)$.



Complete: Suppose $f(x) = \sqrt{x}$, where x is measured in meters and m in kilograms. Find the average density of the rod from $x = 1$ to $x = 1.2$ meters, and then find the linear density at $x = 1$ meters.

Example ③: A current exists whenever electric charges move. The current is the rate at which charge flows through a surface. If $Q(t)$ is the quantity of electric charge that has passed through a point a wire at time t, then the current I is given by the derivative $I = \frac{dQ}{dt}$. It is measured in units of charge per unit time [often coulombs per second, called amperes]. $1\,A = 1\frac{C}{s}$

The quantity of charge in coulombs (C) that has passed through a point in a wire up to time (measured in seconds) is given by $Q(t) = t^3 - 2t^2 + 6t + 2$. Find the current when (a) $t = 0.5\,s$ and (b) $t = 1s$

Applications to Chemistry:

Example ④: If the temperature of a substance is kept constant, then the volume V depends on the pressure P. One quantity of interest in thermodynamics is the isothermal compressibility $\beta = -\frac{1}{V}\frac{dV}{dP}$, which measures how fast, per unit volume, the volume of a substance decreases as the pressure increases at constant temperature.

Complete: The volume in cubic meters of air at 27℃ is related to the pressure in kilopascals by the equation $V = \frac{5.3}{P}$. Find the compressibility of air when the pressure is 50 kPa.

Applications to Biology:

Example ⑤: Let $n = f(t)$ be the number of individuals in an animal or plant population at time t. The instantaneous growth rate is the derivative $\frac{dn}{dt} = f'(t)$.

Suppose a population of bacteria triples every hour and starts with 400 bacteria. Find an expression for the number of bacteria after hours and use it to estimate the rate of growth of the bacteria population after 2.5 hours.

Example ⑥: Suppose we model the flow of blood through a blood vessel by treating the blood vessel as a cylindrical tube with radius R and length l. We can find the velocity of the blood flow at a distance r from the center of the blood vessel by the law of laminar flow $v = \frac{P}{\eta l}(R^2 - r^2)$, where η is the viscosity of the blood and P is the pressure difference between the ends of the tube. In this case, the velocity gradient is the derivative $\frac{dv}{dr}$.

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Application to Physics:

Example ⑦: Newton's Law of Gravitation says that the magnitude F of the force [in Newtons] exerted by a body of mass m on a body of mass M is $F = \frac{GmM}{r^2}$ where G is the gravitational constant and r is the distance between bodies.

a. Find $\frac{df}{dr}$ and explain its meaning. What does the minus sign indicate?

b. Suppose it is known that Earth attracts an object with a force that decreases at the rate of $2\ \mathrm{N/km}$ when $r = 20{,}000\ \mathrm{km}$. How fast does this force change when $r = 10{,}000\ \mathrm{km}$.