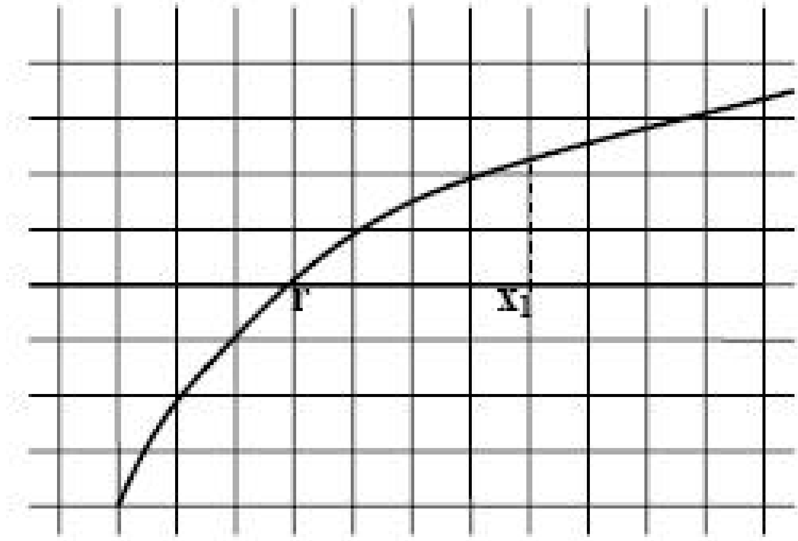
MA 181 Calculus I
Newton's Method (4.8)



1. Draw a line tangent to the curve at $x = x_1$.
2. Extend the tangent line so that it intersects the x-axis. Call the x-intercept [of the tangent line) x_2 .
3. Now draw a new line tangent to $y = f(x)$ at $x = x_2$
4. Again, extend this new tangent line to the x-axis and call its x-intercept x_3 .
5. Describe the numbers $x_{1,}$ x_2 and x_3 and their relationship to *r*, whic*r* is the x-intercept of $y = f(x)$. What is happening to the values of $x_{1,}$ x_2 and x_3 in terms of their relationship to *r*, the x-intercept of *f(x)*?
6. Write a formula for x_2 in terms of x_1 using the fact that the slope of L_1 [the tangent line to the curve $y = f(x)$ at $x = x_1$] is $f'(x_1)$.
7. Write a formula for x_3 in terms of x_2 using the fact that the slope of L_2 [the tangent line to the curve $y = f(x)$ at $x = x_2$] is $f'(x_2)$.
8. Guess the formula for x_{n+1}, the x-intercept of the $(n+1)$ tangent line, as a function of x_n , the x-intercept of n^{th} tangent line.

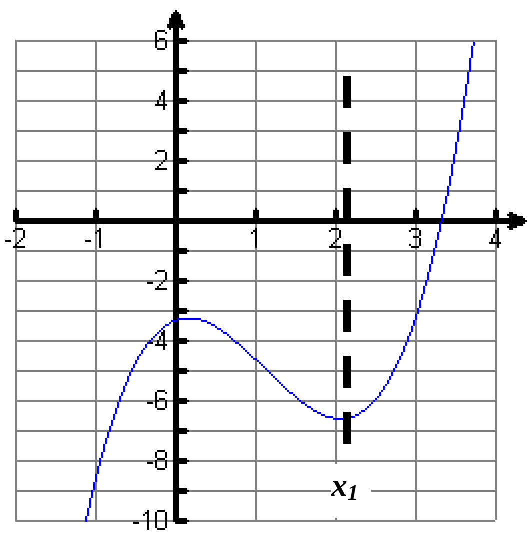
http://archives.math.utk.edu/visual.calculus/3/newton.5/

9. Let **f** be a differentiable function. Choose a point $\mathbf{x_1}$ near a root of **f**. Define recursively

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

If the point $\mathbf{x_1}$ is chosen sufficiently close to the root then the $\mathbf{x_n}$'s are successively better approximations of the root.

When does Newton's Method fail?



10. Use Newton's method to approximate the indicated root of the equation correct to four decimal places.

a. $e^{2x} + 5x = 0$, $x_1 = 0$

b. $\cos(x) - x^2 = 0$, $x_1 = 1$

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The easiest way to use Newton's Method on a TI calculator is as follows:

(1) Store $f(x)$ as Y1 and $f'(x)$ as Y2.

(2) Enter $x_1 \to x$ (using the STO key).

(3) Enter $x - \frac{y_1}{y_2} \to x$ repeatedly. Stop when the values shown on the calculator stop changing.