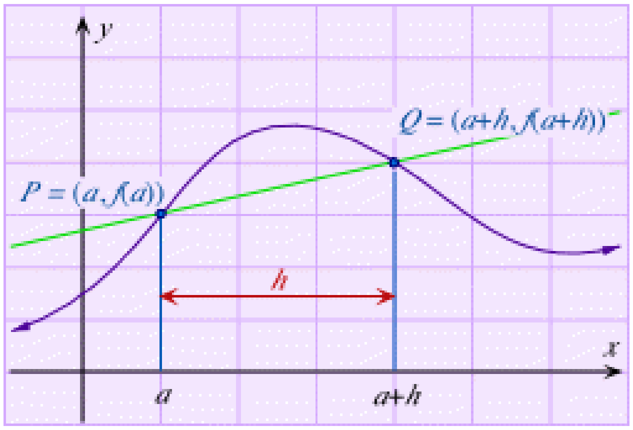
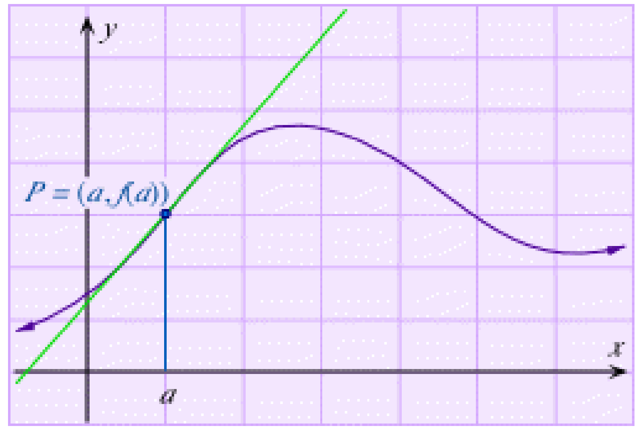
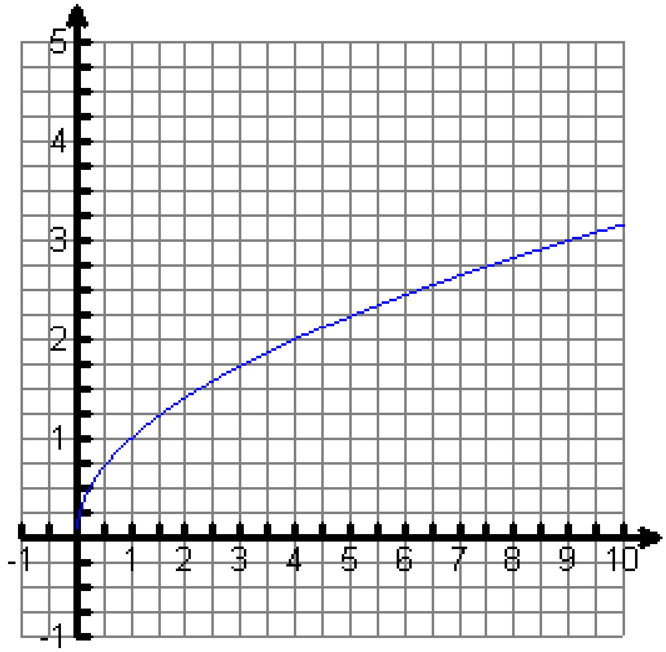
MA 181 Calculus I (Scott)
Derivatives & Rates of Change (2.6)

http://www.zweigmedia.com/RealWorld/tutorials/frames2_3.html





Slope of Secant Line (Average Rate of Change)	**Slope of tangent line (Instantaneous Rate of Change)**
$m_{PQ} =$ slope of secant line PQ	$\lim_{Q\to P} m_{PQ} = m =$ slope of the tangent line at P
Definition 1	
Equation 2	



Example 1:

Given $f(x) = \sqrt{\boldsymbol{x}}$

a. Sketch the tangent line at $a = 4$, and estimate the slope of this tangent line.
$m \approx$ ______

b. Set up and simplify the difference quotient for this function.

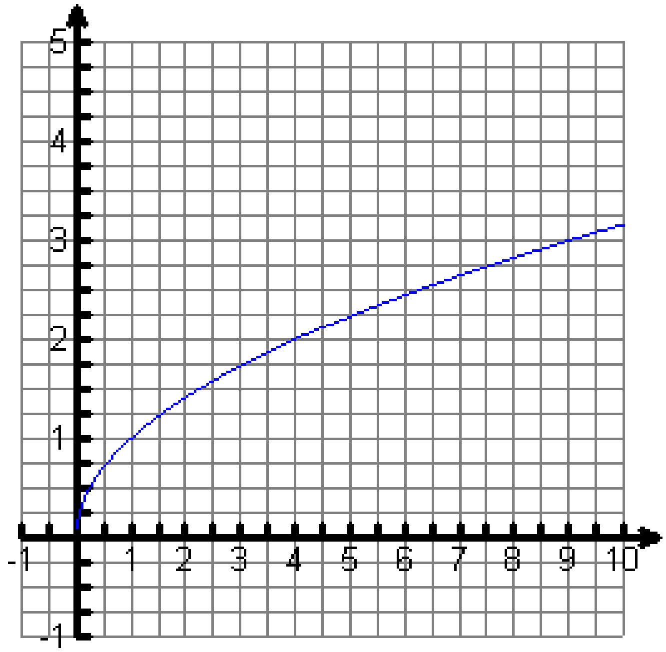
$$m_{sec} = \frac{f(x)-f(a)}{x-a} = \frac{f(x)-f(4)}{x-4}$$

c. Evaluate the limit of the difference quotient, $m_{tan} = \lim_{x \to 4} \frac{f(x)-f(4)}{x-4}$, using the limit laws. Compare the result to the estimate from part a above.

d. $f'(4) =$

Example 2:

Given $f(x) = \sqrt{\boldsymbol{x}}$



a. Sketch the tangent line at $a = 4$, and estimate the slope of this tangent line.
$m \approx$ ______

b. Set up and simplify the difference quotient for this function.

$$m_{sec} = \frac{f(a+h)-f(a)}{h} = \frac{f(4+h)-f(4)}{h}$$

c. Evaluate the limit of the difference quotient, $m_{tan} = \lim_{h\to 0} \frac{f(4+h)-f(4)}{h}$, using the limit laws. Compare the result to the estimate from part a above.

d. $f'(4) =$

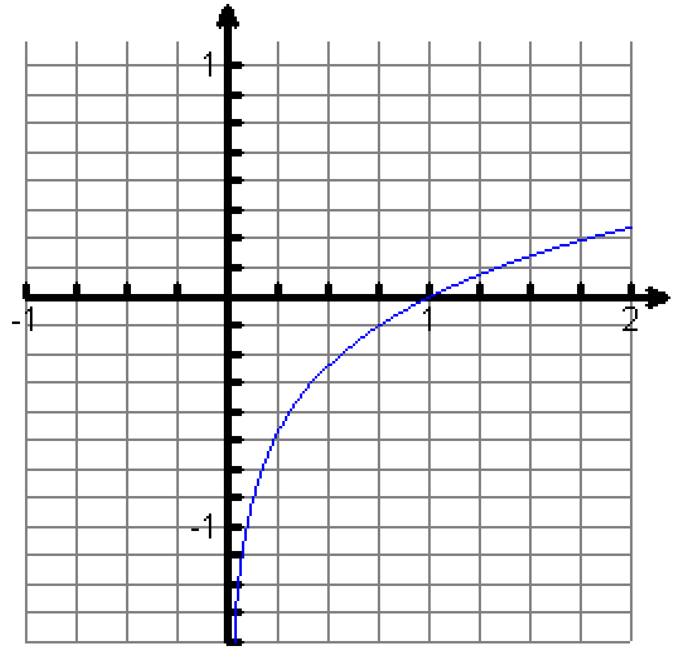
Example 3:

Given $f(x) = \log x$

Sketch the tangent line at $a = 1$, and estimate

a. the slope of this tangent line.

$m \approx$ ______



b. Set up and simplify the difference quotient for this function.

$$m_{sec} = \frac{f(a+h)-f(a)}{h} = \frac{f(1+h)-f(1)}{h}$$

c. Evaluate the limit of the difference quotient, $m_{tan} = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$, numerically. Compare the result to the estimate from part a above.

h	$\frac{f(1+h)-f(1)}{h}$	h	$\frac{f(1+h)-f(1)}{h}$
.1		-.1	
.01		-.01	
.001		-.001	
.0001		-.0001	

$$m_{tan} = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} \approx$$

d. $f'(1) \approx$

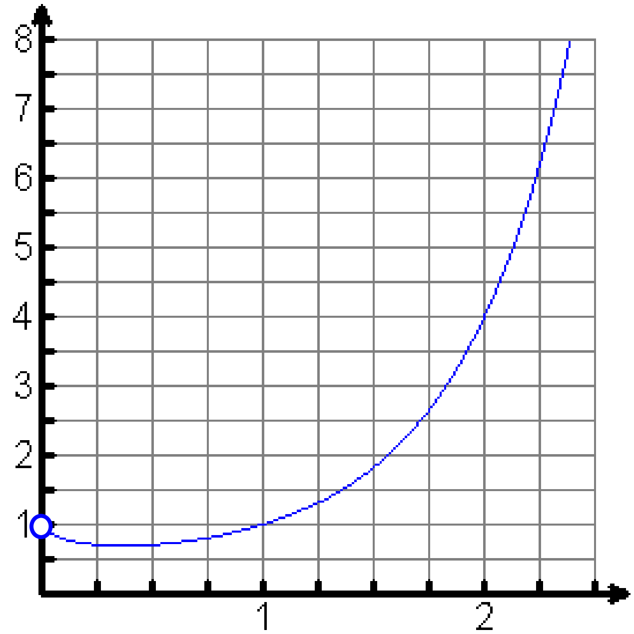
Example 4:

Given $f(x) = x^x$

Sketch the tangent line at $a = 2$, and estimate
a.

the slope of this tangent line.

$m \approx$ ______



b. Set up and simplify the difference quotient for this function.

$$m_{sec} = \frac{f(a+h)-f(a)}{h} = \frac{f(2+h)-f(2)}{h}$$

c. Evaluate the limit of the difference quotient, $m_{tan} = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$, numerically. Compare the result to the estimate from part a above.

h	$\frac{f(2+h)-f(2)}{h}$	h	$\frac{f(2+h)-f(2)}{h}$
.1		-.1	
.01		-.01	
.001		-.001	
.0001		-.0001	

$$m_{tan} = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} \approx$$

d. $f'(2) \approx$

- We sometimes refer to the slope of the tangent line to a curve at a point as the ***slope of the curve***. The idea is that if we zoom in far enough toward the point, the curve looks almost like a straight line.
  (Look here)

- The tangent line $y = f(x)$ at $(a, f(a))$ is the line throught $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.

- The *velocity* measures how quickly (rate) your position changes.

$$average\ velocity = \frac{displacement}{time} = \frac{f(a+h) - f(a)}{h}$$

$$velocity\ (or\ instantaneous\ velocity) = v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

  This means that the velocity at time $t = a$ is equal to the slope of the tangent line at the point $(a, f(a))$.

- The **derivative of a function *f* at a number *a***, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad \text{or} \qquad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

  if this limit exists.

- The tangent line $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.

1. Let $H(t)$ be the height of snow in inches as a function of time in hours after midnight. How would a weatherman describe the following statements?

(a) $H(3) = 4$

(b) $H'(t) = 2$

(c) $H'(3) = 0.5$

(d) $H'(14) < 0$

2. Let $S(t)$ be a child's distance from home as a function of time. Is $S'(t)$ positive, negative or zero if:

(a) The child is at home.

(b) The child is at school.

(c) The child is coming home.

(d) The child is going to school.

3. Let $h(t)$ be a person's height in inches at age *t* years. Write a sentence, using appropriate units, explaining the meaning of each of the following.

(a) $h(12) = 56$

(b) $h'(12) = 2.5$