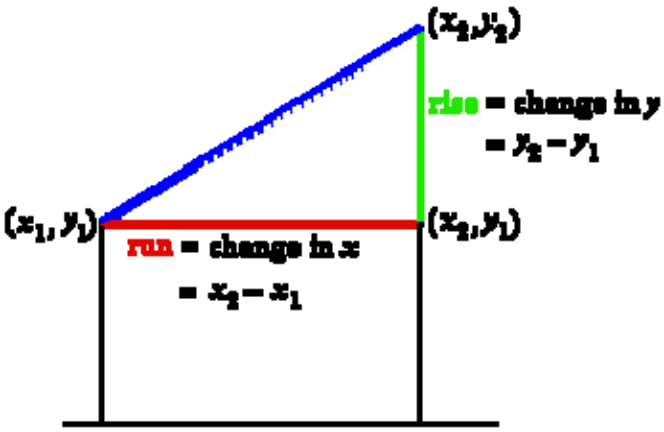
MA 181 (Calculus I)

Rate of Change

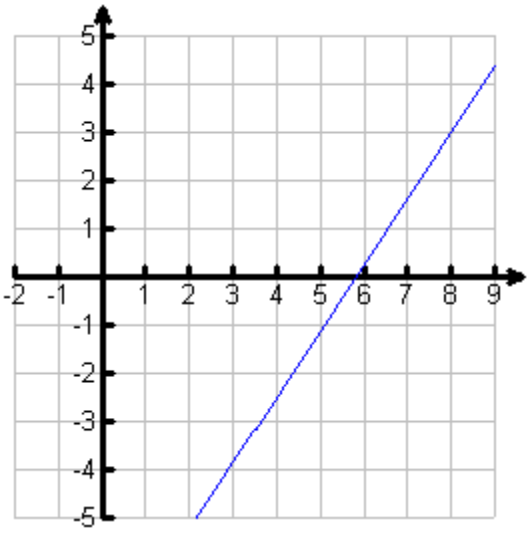


The slope of the line connecting two points (x_1, y_1) and (x_2, y_2) is the ratio described by each of the following expressions — all of which say the same thing:

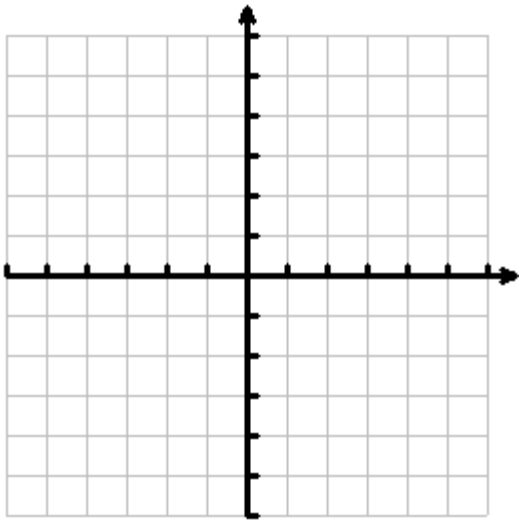
- slope
- rise over run
- change in y over change in x
- rate of change
- $\frac{y_2 - y_1}{x_2 - x_1}$

Lines are very special curves. The essence of linearity is that the slope of a line does not depend on which two points are selected for the computation. The ratio of rise to run is the same no matter where you start and no matter how long the run is or in which direction.

1. Estimate the slope of the line and find the equation of the line.



2. Sketch a line with a slope of $-\frac{3}{4}$.



Adapted from *Calculus Modeling & Application, 2nd Ed.*
http://www.math.duke.edu/education/calculustext/Chapter2/Contents2.html

Rate of Change

In calculus we study rates of change in general, not just for linear functions. Our first step is to rewrite the equation $slope = \frac{rise}{run}$ in the equivalent form

$$average\ rate\ of\ change = \frac{change\ in\ dependent\ variable}{change\ in\ independent\ variable}$$

A linear function has only one rate of change: the slope of its graph. In this case, we may talk about the rate of change. However, the average rate of change of a more general function varies from one interval to another.

3. An object is dropped from a height. The table records at each second the distance it has fallen.

Falling Body Data	
Time (seconds)	**Distance (meters)**
1.0	5.0
2.0	19.4
3.0	44.1
4.0	78.0
5.0	122.8
6.0	175.8
7.0	240.0
8.0	312.8
9.0	396.1
10.0	489.0

   a. What is the average speed of the object for the first five seconds? For the first three seconds? For the first second?

   b. Estimate the average speed for the first 4.5 seconds. (Answers may vary)

   c. Suppose the height from which the object is dropped is 489 meters; what is the average speed for the last two seconds of its fall?

   d. Use your graphing calculator to graph the data in Table 2.1. Change your window to plot just the portion of the graph that that represents the last two seconds. How does your answer to (c) relate to what you see in the graph?

   e. Does it make sense to talk about a single rate of change for this function? Why or why not?

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