MA 103 Solving Systems of Linear Equations Algebraically
Section 3.2

Systems of linear equations can be solved algebraically using either the ***Substitution Method*** or the ***Elimination Method***.

The Substitution Method
To solve a system of equations using the substitution method,

- Solve one of the equations for one of the variables. If possible, choose the variable that has a coefficient of one or negative one in one of the equations.
- Substitute this expression into the other equation.
- Solve for the remaining variable
- Substitute the value you just found into one of the equations to get the other variable
- Check the answer in both of the original equations

Example: Solve the system of equations $\begin{aligned} x - y &= 11 \\ 2x + 3y &= 12 \end{aligned}$ using the substitution method.

• Solve the first equation for one of the variables. In this case, solve for x in the first equation because it has a coefficient of one.	$x = y + 11$
• Substitute this expression into the second equation.	$2(y + 11) + 3y = 12$
• Solve for the remaining variable.	$2y + 22 + 3y = 12$ $5y + 22 = 12$ $5y = -10$ $y = -2$
• Substitute the value you just found into one of the equations to get the other variable.	$x = y + 11$ *so* $x = -2 + 11 = 9$
• Check the answer in both of the original equations.	$x - y = 11$ $9 - (-2) = 11$ $2x + 3y = 12$ $2(9) + 3(-2) = 18 - 6 = 12$

Both equations check, so the answer is (9, -2)

1. Use the substitution method to solve each of the following systems of equations. If it is not possible to find a unique solution for the system, why not?

(a) $\begin{aligned} 2y - 3x &= 13 \\ y - 4x &= 9 \end{aligned}$

(b) $\begin{aligned} x-4y&=5 \\ 3&=8y-2x \end{aligned}$

The Elimination Method

To solve a system of equations using the elimination method,

- Multiply one or both equations by constants that will create a new system in which one of the variables has coefficients that are the opposite of one another, that is, the coefficients are the same number in absolute value but have opposite signs
- Add the two new equations together. One variable should be eliminated.
- Solve for the remaining variable
- Substitute the value you just found into one of the equations to get the other variable
- Check the answer in both of the original equations

Example: Solve the system $\begin{aligned} 2x-3y&=7 \\ 3x+4y&=2 \end{aligned}$ by the method of elimination.

• Multiply one or both equations by constants that will create a new system in which one of the variables has coefficients that are the opposite of one another, that is, the coefficients are the same number in absolute value but have opposite signs	$4(2x-3y=7)4$ $3(3x+4y=2)$ $8x-12y=28$ $9x+12y=6$
• Add the two new equations together. One variable should be eliminated.	$17x=34$
• Solve for the remaining variable	$x=\frac{34}{17}=2$
• Substitute the value you just found into one of the equations to get the other variable	$2x-3y=7$ $2(2)-3y=7$ $4-3y=7 \;\; so \;\; -3y=3 \;\; and \;\; y=-1$
• Check the answer in both of the original equations	$2(2)-3(-1)=4+3=7$ $3(2)+4(-1)=6-2=2$

Both equations check, so the answer is (2, -1)

2. Use the elimination method to solve each of the following systems of equations. If it is not possible to find a unique solution for the system, why not?

(a)
$$2x+3y=8$$
$$4x+5y=12$$

(b)
$$4x-3y=1$$
$$8x-6y=2$$

(c)
$$5x-7y=-16$$
$$2x+3y=11$$