MA 103 Exponential and Logarithmic Models
Section 9.7

All of these problems should be solved algebraically, using logarithms when appropriate. You should check your answers whenever possible, and state all answers using appropriate units.

1. The population of Mexico is increasing exponentially according to the model $f(t) = 100(1.018)^t$, where t represents years since 2001 and $f(t)$ is measured in millions.

(a) What was the population of Mexico in 2001?

(b) Use the model to predict the population of Mexico in 2010.

(c) In what year will the population be 140 million?

(d) How long will it take the population to double? This is called the **doubling time**.

\

2. The number of children N enrolled in preschool, in millions, t years after 1970 can be approximated by $N(t) = 4(1.025)^t$.

(a) How many children were enrolled in preschool in 1970?

(b) How many children were enrolled in preschool in 2000?

(c) In what year were 7.5 million children enrolled in preschool?

(d) What is the doubling time for the number of children enrolled in preschool?

In many situations, base e is used for exponential growth and the **exponential growth model** is a function of the form $P(t) = P_0 e^{kt}$, $k > 0$, where $P(t)$ is the amount at time t, P_0 is the amount at time t = 0, and k is the **exponential growth rate** for the situation. Use this function in #3 and #4 below.

3. The value of a painting is increasing with an exponential growth rate of 6.2 % per year. The painting was worth $50 million in 1980.

(a) Let t = 0 correspond to 1980 and write the exponential growth function for the value of the painting.

(b) Using this model, what was the value of the painting in 2008?

(c) In what year would the value of this painting reach $600 million?

(d) What is the doubling time for the value of the painting?

4. The population of Florida is increasing with an exponential growth rate of 2.1% per year. The population was 12.9 million in 1990.

(a) Let t = 0 correspond to 1990 and write the exponential growth function for the population of Florida.

(b) What was the population of Florida in 2005?

(c) When will the population be 20 million?

(d) What is the doubling time?

5. When an amount of money P_0 is invested at interest rate k **compounded continuously**, interest is calculated as though it were computed every "instant" and is added to the original amount. Thus, the larger the balance, the faster interest is earned and the faster the balance grows. The balance $P(t)$ after t years is given by the exponential growth model $P(t) = P_0 e^{kt}$, where k is the continuous compound interest rate.

Suppose that $10,000 is invested at 3.8% interest compounded continuously.

(a) Write the exponential growth function.

(b) What is the balance after 3 years?

(c) After how many years will the investment double?

6. Blue fin tuna are large fish that are used for sushi and are worth a great deal of money. As a result, the number of western Atlantic blue fin tuna has declined dramatically. The number of blue fin tuna in thousands between 1974 and 1991 can be modeled by the function $f(x) = 230(0.1)^{0.055t}$ where t is the number of years since 1974.

(a) Evaluate $f(0)$ and write a sentence explaining what this means.

(b) Determine the year in which Atlantic blue fin tuna numbered 140 thousand.

OVER →

7. The number of people living in industrialized urban regions throughout the world has not grown exponentially. Instead, it has grown logarithmically and is modeled by

$$f(x) = 0.36 + 0.15 \ln x$$

In this formula, $f(x)$ is in billions of people and x is in years since 1949, where the domain of the function is $1 \le x \le 81$.

(a) What was the population in industrialized urban regions in 2000?

(b) In what year will the population in industrialized urban regions reach 1 billion?

8. Students were tested at the end of a course with a final exam. They were then tested again at monthly intervals later. The average score was determined to be

$$S(t) = 78 - 15 \log(t+1)$$

where $S(t)$ is the average score t months after taking the final exam.

(a) What was the average score when the students first took the exam (at t = 0)?

(b) What was the average score after 6 months?

(c) After how many months would the function predict that the average score would be 60?

Answers

1(a) 100 million	(b) 117.4 million	(c) in 2020	(d) 38.5 years
2(a) 4 million	(b) 8.4 million	(c) in 1995	(d) 28.1 years
3(a) $P(t) = 50e^{0.062t}$	(b) \$283.7 million	(c) in 2020	(d) 11.2 years
4(a) $P(t) = 12.9e^{0.021t}$	(b) 17.7 million	(c) 2011	(d) 33.0 years
5(a) $P(t) = 10,000e^{0.038t}$	(b) \$11207.52	(c) in 18.2 years	

6(a) f(0) = 230 In 1974, there were 230,000 blue fin tuna. (b) in 1978

7(a) .95 billion	(b) in 2020	
8(a) 78	(b) 65	(c) 15 months