

### Summary

A quadratic function is a function of the form  $f(x) = ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are real numbers and  $a \neq 0$ . Its graph is always a parabola which opens upward if  $a > 0$  and downward if  $a < 0$ .

The **vertex** or turning point of the parabola occurs when  $x = -\frac{b}{2a}$  and  $y = f(-\frac{b}{2a})$ .

The **y-intercept** occurs when  $x = 0$ ,  $y = c$ . To find it, substitute  $x = 0$  into the equation and find the corresponding value of  $y$ .

The **x-intercepts** occur when  $y = 0$ . To find the  $x$ -intercepts, substitute  $y = 0$  into the equation and find the corresponding value(s) of  $x$ . You will need to solve the equation  $ax^2 + bx + c = 0$ . You can do this either

- (a) **Algebraically** using factoring, if possible, or the quadratic formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , or
- (b) **Graphically**, finding the  $x$ -intercepts by using the CALC menu of your calculator. To do this, press 2nd TRACE on the TI-83/83+/84/84+, and then choose zero, and follow the directions on the calculator.

### Examples and Applications

1. Let  $f(x) = 3x^2 - 12x - 19$ .
  - (a) **Algebraically** determine the  $x$ -coordinate of the vertex by using the formula given above and then find the  $y$ -coordinate by substituting this  $x$ -value into the function.
  - (b) Determine the  $y$ -intercept of the function.
  - (c) Choose an appropriate window and graph the function.
  - (d) Determine the  $x$ -intercept(s) of the function correct to two decimal places.
2. Recall that if the demand function for an item is  $p(x)$ , then the **revenue  $R(x)$**  received by selling  $x$  items at a price of  $p(x)$  dollars per item is  $R(x) = (\# \text{ of items sold}) \cdot (\text{price per item}) = x p(x)$   
Suppose that the demand function for a certain item is  $p(x) = -3.1x + 524$ ,  $0 \leq x \leq 150$ .
  - (a) Write the revenue function for the item.
  - (b) **Algebraically** determine the  $x$ -coordinate of the vertex of the revenue function by using the formula given above and then find the  $y$ -coordinate by substituting this  $x$ -value into the function.

- (c) Determine the coordinates of the y-intercept of the revenue function.
- (d) Choose an appropriate window and graph the revenue function. You should see the vertex and intercepts of the function.
- (e) Suppose the cost function for the item is  $C(x) = 75x + 6200$ . Graph this function along with the revenue function. Sketch your graph in the space below.

When revenue and cost are equal, the manufacturer will not either make or lose money and is said to **break even**. Graphically, the production levels (x-values) at which this occurs are the x-coordinates of the points of intersection of the revenue and cost functions. These are called the **break-even points**. Use the intersect option on the CALC menu (2nd TRACE) to determine the break-even points for these functions. **Round each answer to the nearest whole number.**

Break-even points:  $x =$                       and  $x =$

- (f) When the revenue is greater than the cost, the company makes money and is said to have a profit. Graphically, this occurs on any x-interval for which the revenue function is **above** the cost function. Look at the graph in part (e). On what interval does this occur?

Profit interval(s):

- (g) When revenue is less than cost, the company loses money and is said to have a loss. Graphically, this occurs on any x-interval for which the revenue function is **below** the cost function. On what interval(s) does this occur?

Loss interval(s):

- 3. Another way to determine break-even points and intervals of profit and loss is to look at the graph of the profit function. Recall that profit is the difference between revenue and cost; that is **Profit = Revenue - Cost**.

- (a) Use the revenue and cost functions from problem #1 above to write the profit function for this example. Simplify the result, eliminating parentheses and combining like terms.

- (b) Algebraically determine the coordinates of the vertex of the profit function.
- (c) Determine the coordinates of the y-intercept of the profit function.
- (d) Clear the cost and revenue functions from your calculator. Then choose an appropriate window for the profit function, graph it and sketch the graph in the space below. You should see the vertex and intercepts of the function.

- (e) Recall that break-even occurs when Revenue = Cost. Since Profit = Revenue - Cost, this is equivalent to Profit = 0. Graphically, these are the x-intercepts of the profit function. To find them, use the feature on the CALC menu called zero on the TI-83/83+/84/84+. **Again, round to the nearest whole number.**

Break-even points:  $x =$                       and  $x =$

- (f) The company will make a profit when the profit function is positive; that is, above the x-axis. Use the graph to help you to write the interval on which the company makes a profit.

Profit interval(s):

- (g) The company will have a loss when the profit function is negative; that is, below the x-axis. Use the graph to help you to write the interval on which the company has a loss.

Loss interval(s):

- (h) Your answers to parts (e), (f) and (g) in this question should be the same as the answers to parts (e), (f) and (g) in question #2. Are they?

- (i) Look at the graph of the profit function again. For what production level (x-value) will a maximum profit occur?

- (j) What is the maximum profit?